

# Radio-Frequency IC Design

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## Lecture 9: Receivers & Mixers

ELEC 404

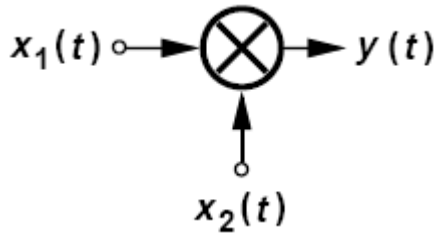
Acknowledgement: *RF Microelectronics*. B. Razavi



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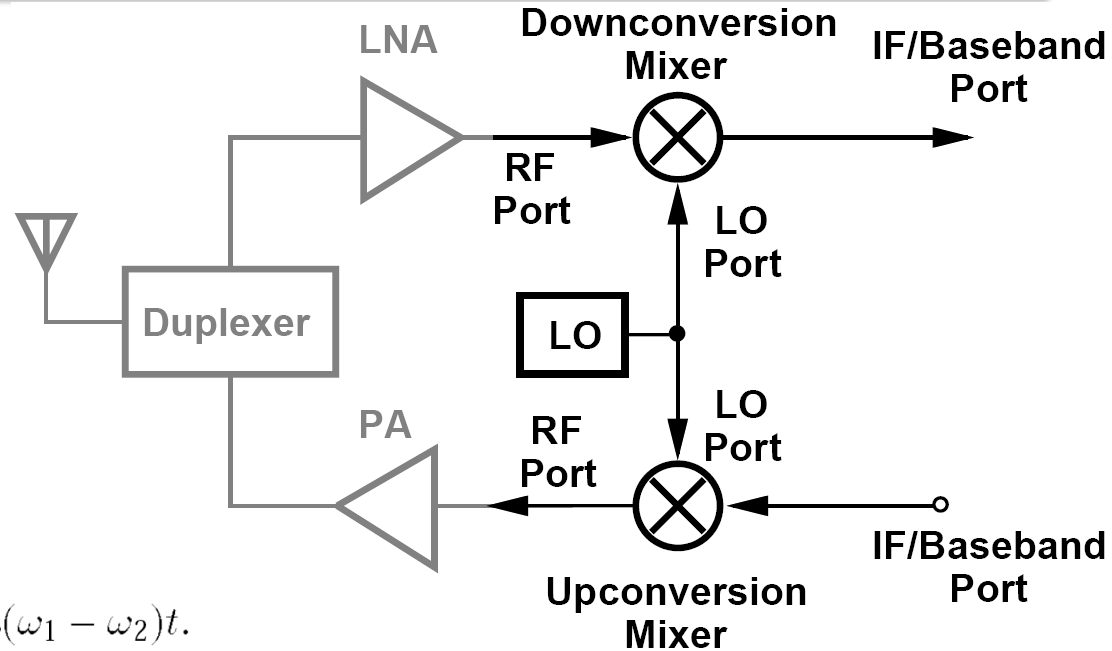
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# Mixers



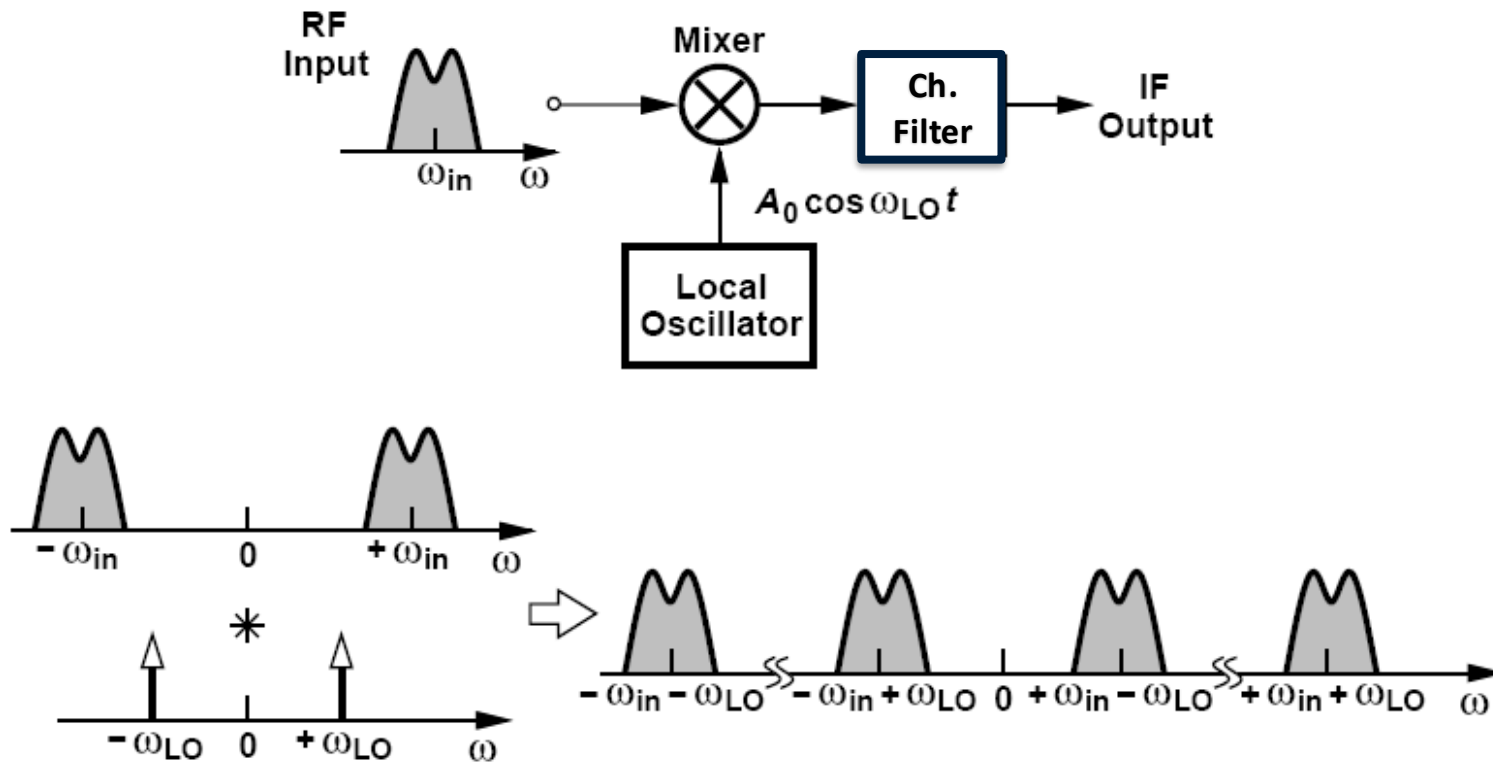
$$y(t) = k(A_1 \cos \omega_1 t)(A_2 \cos \omega_2 t)$$

$$= \frac{kA_1 A_2}{2} \cos(\omega_1 + \omega_2)t + \frac{kA_1 A_2}{2} \cos(\omega_1 - \omega_2)t.$$



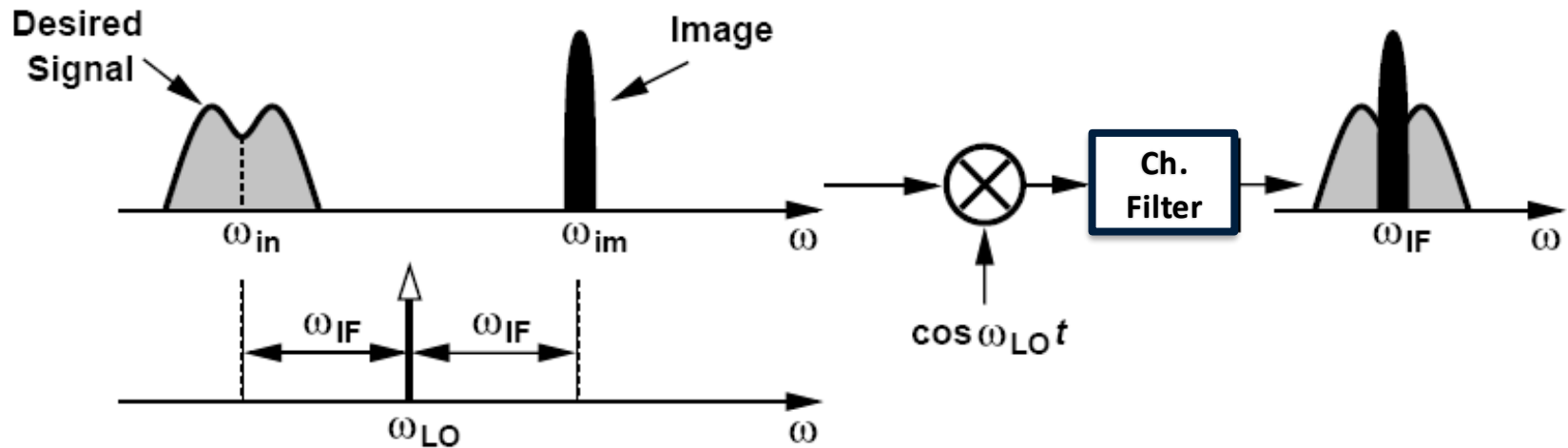
- Perform frequency translation by multiplying two waveforms (and possibly their harmonics).
- Nonlinear LO port.
- Linear RF port to minimize compression and/or intermodulation.
- Implementation – Active/passive, single-balanced/double-balanced
- Voltage Conversion Gain =  $[kA_1 A_2 / 2] / A_1 = kA_2 / 2$

# Basic Heterodyne Receivers



- Nonzero IF (Intermediate Frequency)  $\rightarrow \omega_{LO} \neq \omega_{in}$
- Due to its high noise, the downconversion mixer is preceded by an LNA

# Basic Heterodyne RX: Image Aliasing

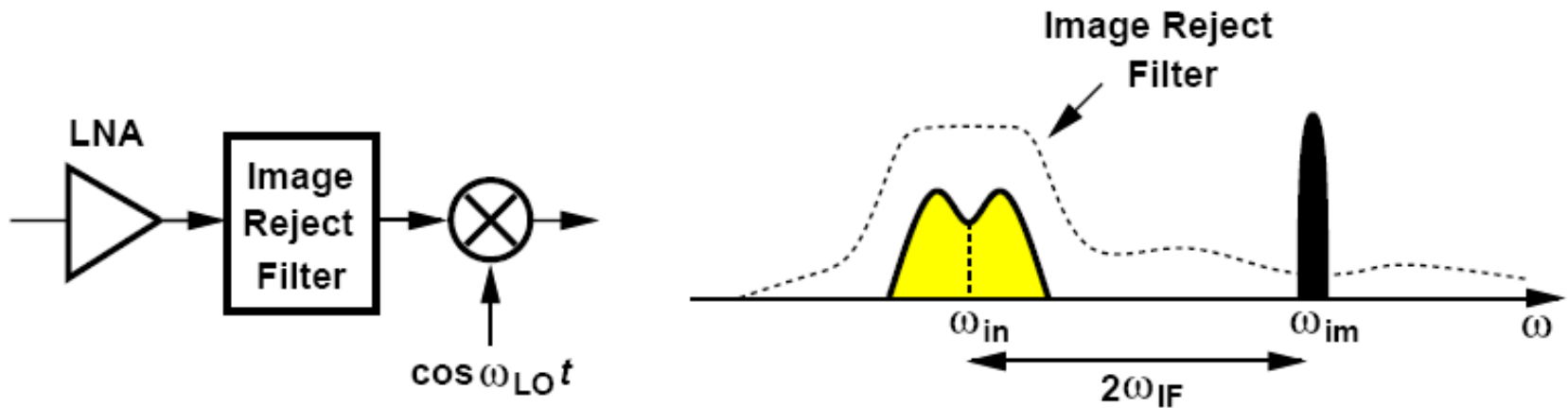


$$A \cos(\omega_{LO} - \omega_{in})t = A \cos \omega_{IF} t = A \cos(\omega_{im} - \omega_{LO})t$$

$$\omega_{im} = \omega_{in} + 2\omega_{IF} = 2\omega_{LO} - \omega_{in}$$

- Two spectra located symmetrically around  $\omega_{LO}$  downconverted to IF
- Image cannot be removed through filtering at the IF output
- Image rejection ratio:  $\text{dB}(P_{IF \text{ desired}}/P_{IF \text{ image}})$

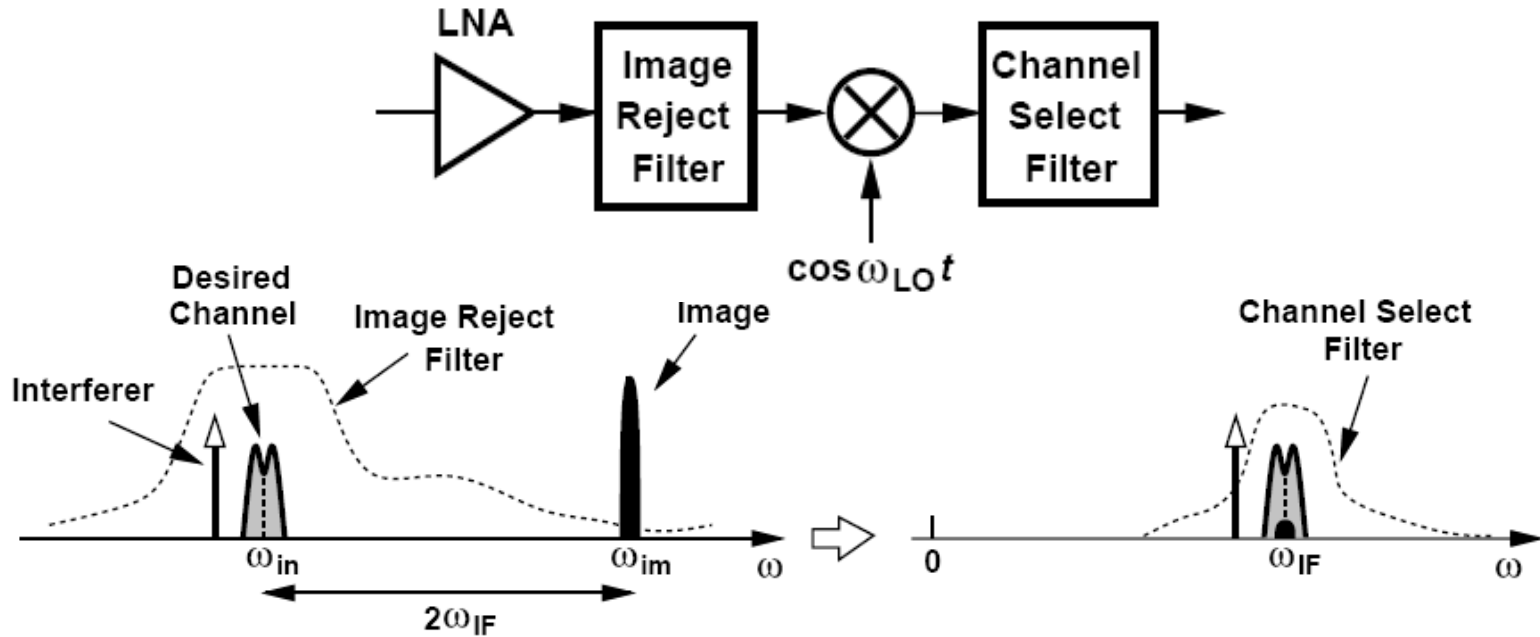
# Image Rejection



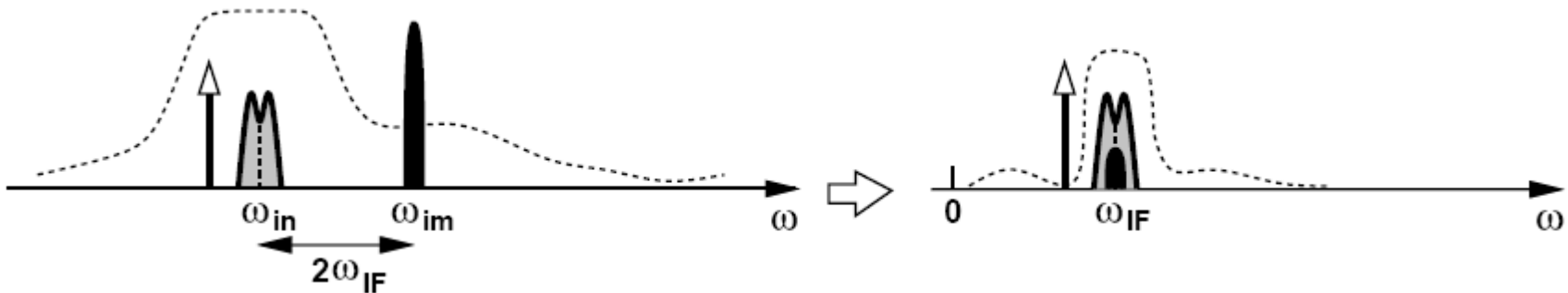
## Image-reject filter for high image rejection

- Typically between LNA & mixer
  - LNA gain lowers the filter's contribution to the RX NF
- Linearity and selectivity requirements have dictated passive, off-chip implementations

# Image Rejection vs. Channel Selection

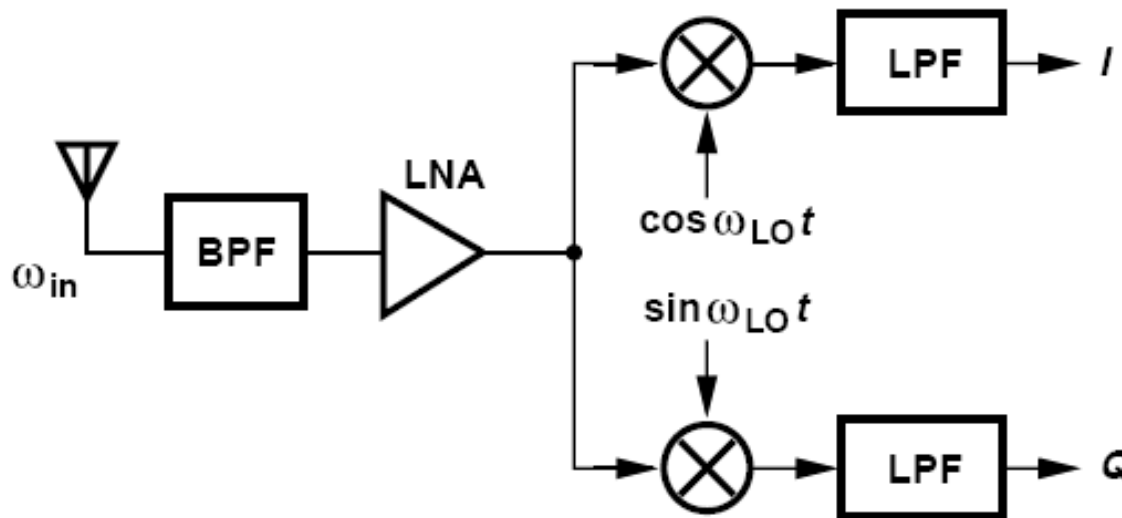


- A high IF allows substantial image-rejection but poor channel selection for given  $Q_{CSF}$ .



- A low IF helps with the channel selection and suppression of in-band interferers.

# Direct-Conversion Receivers



- Absence of an image greatly simplifies the design process
- Channel selection is performed by on-chip LPF
- Mixing spurs are considerably reduced in number
- Quadrature phases to prevent self-corruption of signal (asymmetrically-modulated) upon downconversion

# Specifications: Noise, Linearity, Gain

## LNA Gain and Mixer Noise/Linearity tradeoff

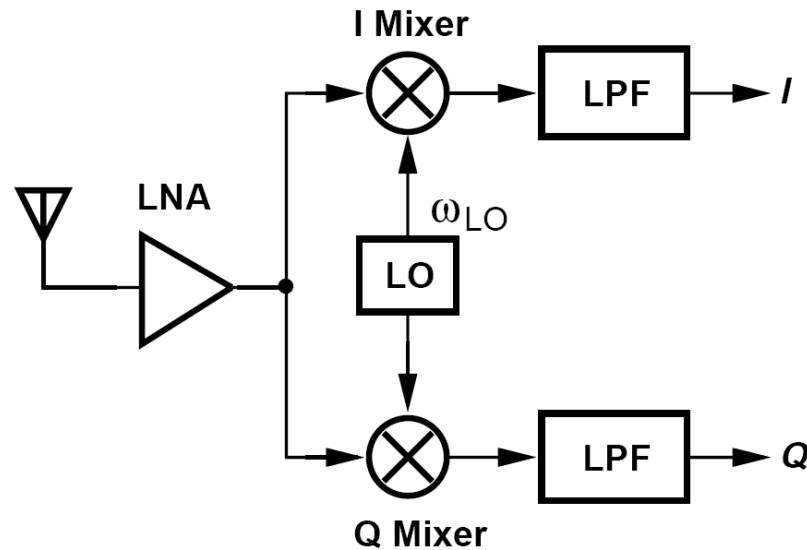
- Input noise of mixer divided by the LNA gain when referred to the RX input
- But  $IP_3$  of the mixer also scaled down by the LNA gain

$$F \approx F_{LNA} + \frac{F_{Mixer} - 1}{A_{PLNA}} + \frac{F_{substg} - 1}{A_{PLNA}A_{PMixer}}$$
$$\frac{1}{IIP3} \approx \frac{1}{IIP3_{LNA}} + \frac{G_{LNA}}{IIP3_{Mixer}} + \frac{G_{LNA}G_{Mixer}}{IIP3_{substg}}$$

## Mixer Gain and RX Noise/Linearity tradeoff

- High mixer gain important in suppressing noise from subsequent stages
- Linearity of mixer critical for RX linearity
- Both high gain and high linearity difficult to attain in Mixer
  - ~ <10dB gain while retaining linearity at low Vdd

# NF of DCR

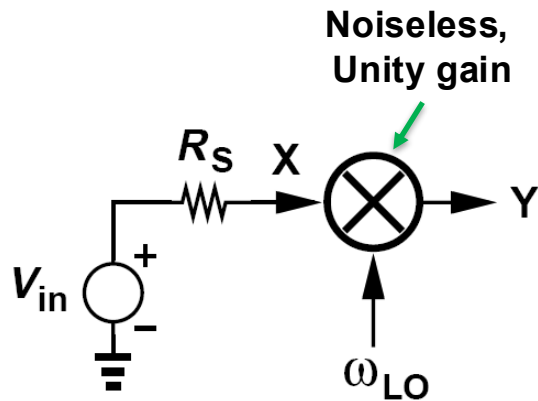


$$F = \frac{\text{SNR}_{in}}{\text{SNR}_I} = \frac{\text{SNR}_{in}}{\text{SNR}_Q}$$

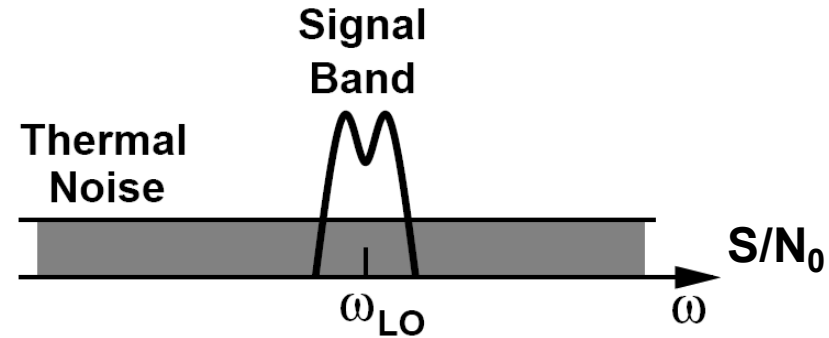
- SNR in the final combined output would serve as a more accurate measure of the noise performance, but it depends on the modulation scheme.
- Double Sideband (DSB) or Single Sideband (SSB)?

# Mixer Noise Figures: DSB NF

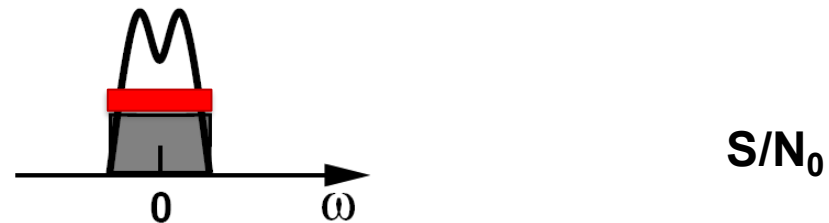
## Mixer in a DCR implementation



Spectrum at X



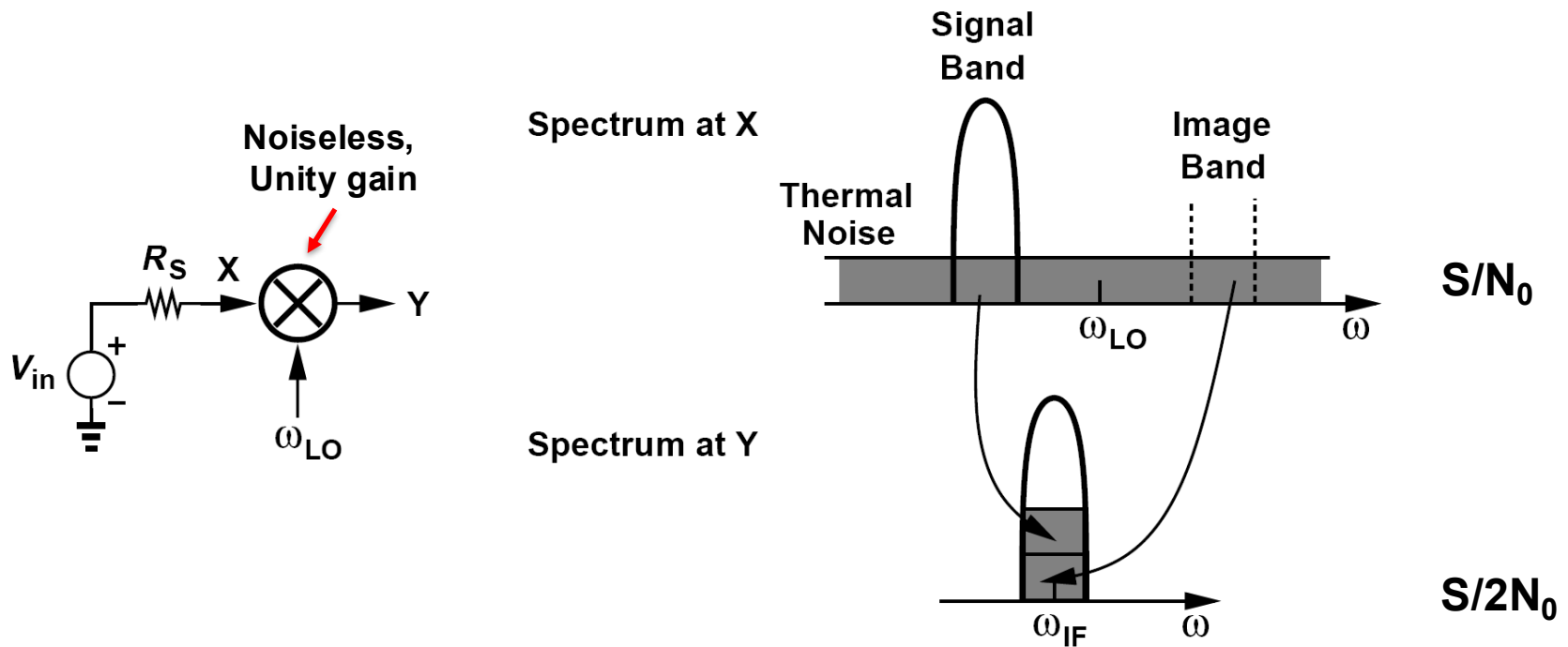
Spectrum at Y



- Signal & noise in the signal band translated to the BB  $\rightarrow SNR_{op} = SNR_{ip}$
- NF of the noiseless mixer = 0dB
- For a **noisy** mixer, NF calculated this way gives the DSB NF = DSB-NF<sub>mixer</sub>

# Mixer Noise Figures: SSB NF

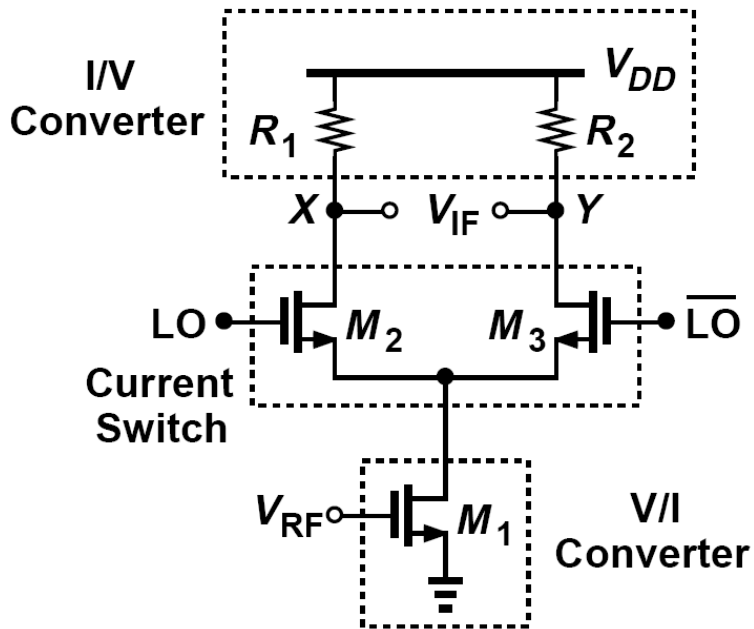
Same mixer in Heterodyne implementation, exhibiting a flat frequency response @ RF Port from the image band to the signal band.



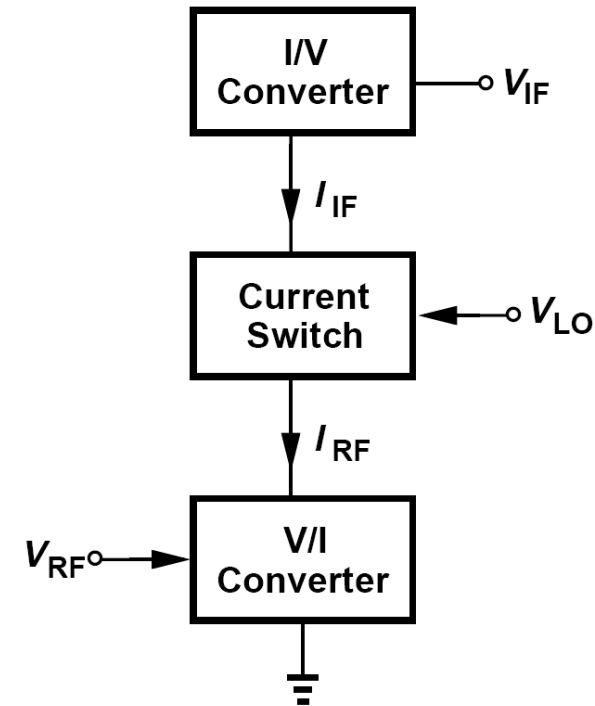
- $SNR_{op} = SNR_{ip}/2$
- NF of a noiseless mixer is 3dB!
- For a noisy mixer, NF calculated this way gives the “single-sideband” (SSB) NF
- $SSB-NF_{mixer} = 3dB + DSB-NF_{mixer}$

# Active Downconversion Mixers

- Conversion and gain in one stage.
  - Convert RF voltage  $\rightarrow$  RF current,
  - “commutate” (steer) the RF current by the LO, and
  - convert IF current  $\rightarrow$  IF voltage.



- $M_2$  and  $M_3$  “switching pair”
- Switching pair needs *large* LO swings for complete steering from one side to another, but not too large (or rail-to-rail).

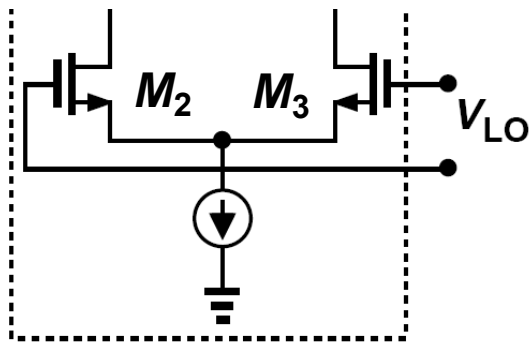


# Single Balanced Mixer – I

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$$i_{out, diff} = \text{sgn}(\cos(\omega_{LO}t)) [I_{DC} + I_{RF} \cos(\omega_{RF}t)]$$

$$\text{sgn}(\cos(\omega_{LO}t)) = \frac{4}{\pi} \left( \cos(\omega_{LO}t) + \frac{1}{3} \cos(3\omega_{LO}t) + \frac{1}{5} \cos(5\omega_{LO}t) + \dots \right)$$



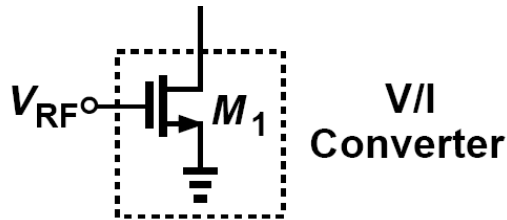
# Single Balanced Mixer – II

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$$i_{out, diff} = \frac{4}{\pi} \left( \cos(\omega_{LO}t) + \frac{1}{3} \cos(3\omega_{LO}t) + \dots \right) I_{DC} +$$
$$\frac{4}{\pi} \left( \cos(\omega_{LO}t) \cos(\omega_{RF}t) + \frac{1}{3} \cos(3\omega_{LO}t) \cos(\omega_{RF}t) + \dots \right) I_{RF}$$

- **Poor LO-IF isolation**
- **Conversion Gain (Transconductance):**

# RF transconductors – CS



$$I_D = \frac{\mu C O X \frac{W}{L}}{2} \frac{(V_{GS} - V_{TH})^2}{1 + \left( \frac{\mu_0}{2v_{sat}L} + \theta \right) (V_{GS} - V_{TH})}$$

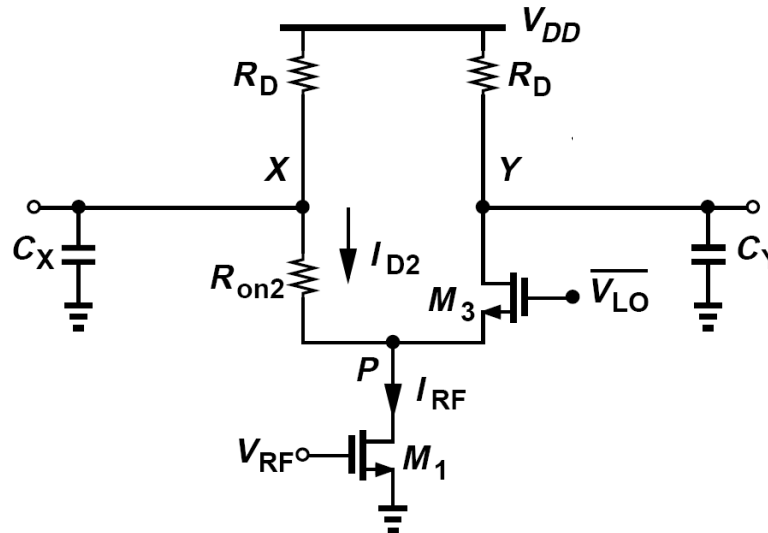
$$A_{IIP3} = \sqrt{\frac{\frac{8}{3} (V_{GS0} - V_{TH})}{\frac{\mu_0}{2v_{sat}L} + \theta} - 4(V_{GS0} - V_{TH})^2}$$

# Linearity

- The input transistor imposes a direct trade-off:

$$IIP3 \propto V_{GS} - V_{TH} \qquad G_{c, max} \propto \frac{1}{V_{GS} - V_{TH}} \qquad \overline{V_{n,in}^2} \propto \frac{1}{g_m} \propto \frac{V_{GS} - V_{TH}}{I_D}$$

- Linearity also degrades if the switching transistors enter the triode region → LO swings cannot be arbitrarily large.
  - If  $M_2$  and  $M_3$  operate in saturation,  $I_{RF}$  splits in the ratio of  $g_{m2}/g_{m3}$ , and independent of  $V_X$  and  $V_Y$
  - If  $M_2$  is in triode while  $M_3$  is still ON and in saturation,  $I_{D2}$  is a function of  $V_X$ , leading to signal-dependent current division between  $M_2$  and  $M_3$ .



# Equilibrium Overdrive of a Diffamp

Overdrive volt. seen by  $M_2$  and  $M_3$  when both of them carry a current of  $I_1/2$

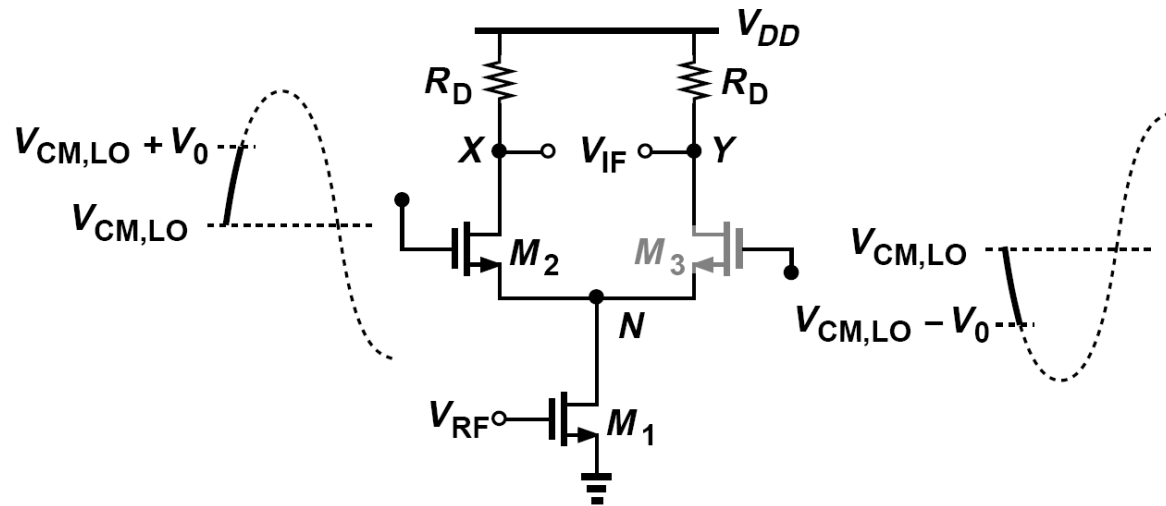
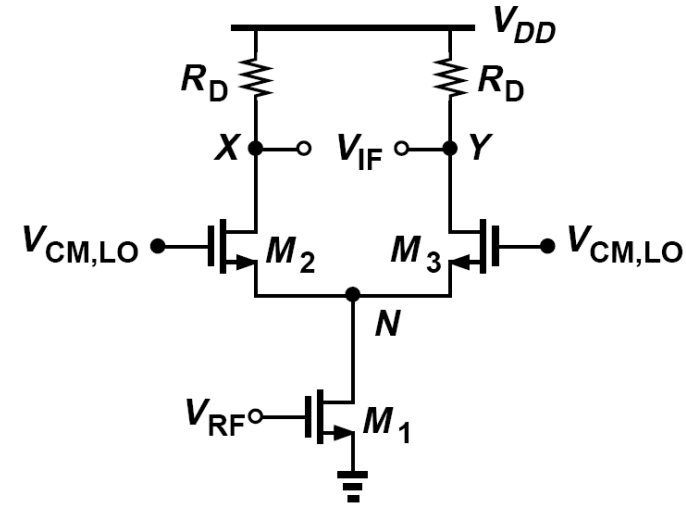
- For square law devices,

$$\frac{I_1}{2} = \frac{\beta_2 (V_{GS2} - V_{TH2})_{eq}^2}{2}$$

$$(V_{GS2} - V_{TH2})_{eq} = \sqrt{\frac{I_1}{\beta}}$$

- The diffamp steers most of its tail current if

$$V_0 = \frac{\sqrt{2} (V_{GS2} - V_{TH2})_{eq}}{2}$$



# Conversion Gain vs. Volt. Headroom

$$A_v = \frac{2}{\pi} g_{m1} R_D$$

- $G_{m1}$  is limited by current budget and IP3

$$g_{m1} = \frac{2I_{D1}}{V_{GS} - V_{TH}}$$

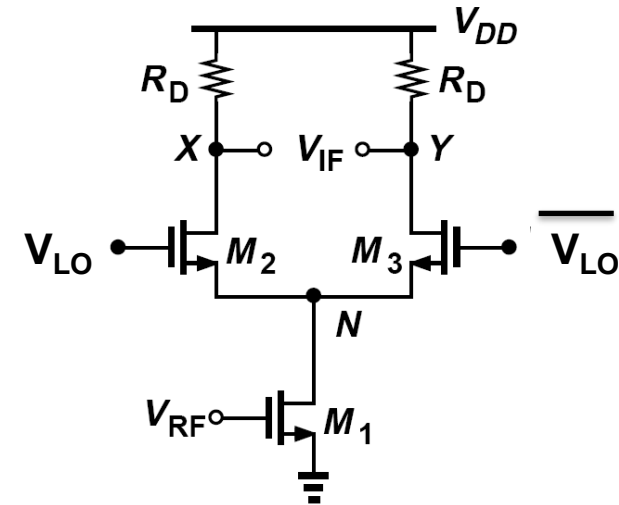
- $R_D$  is limited by max allowable DC voltage across it.

$$R_{Dmax} = \frac{V_{Rmax}}{I_{D1}/2}$$

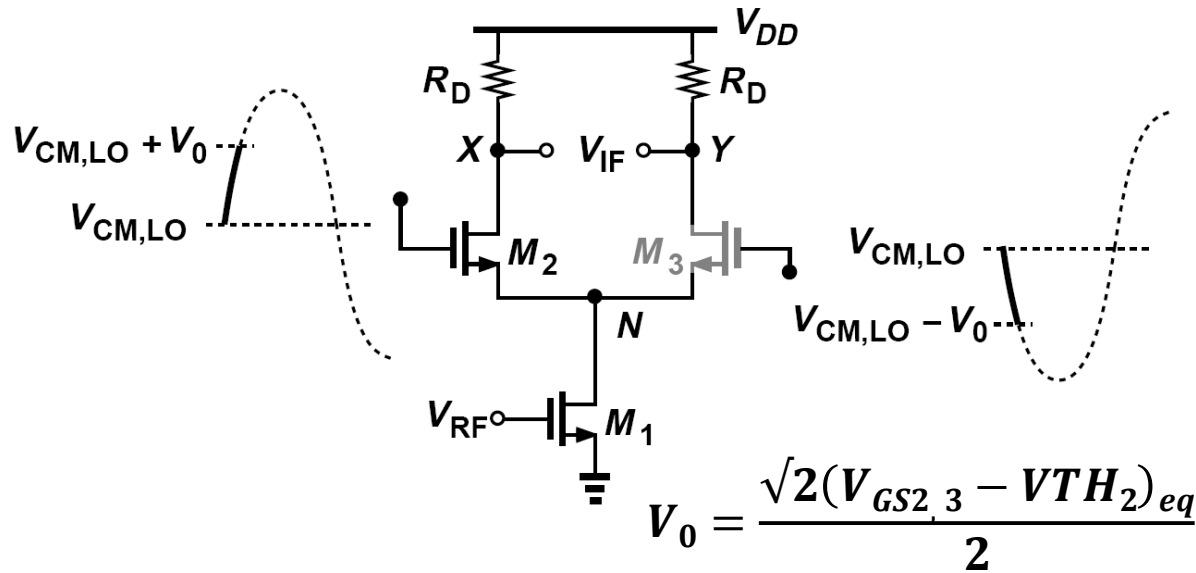
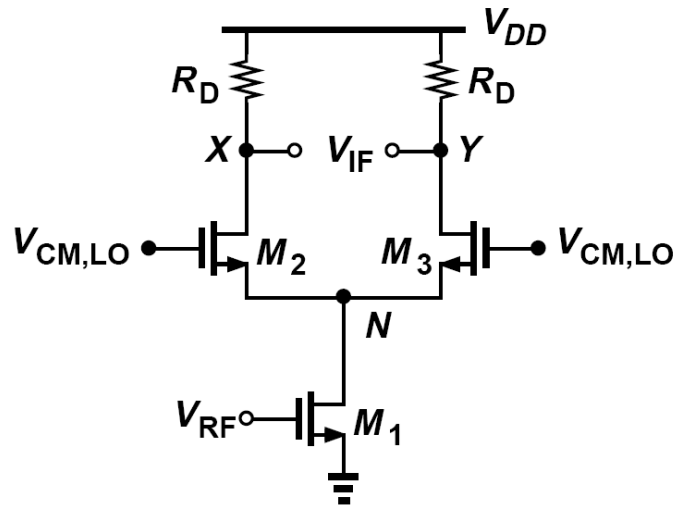
$$V_{Rmax} = V_{DD} - \left[ V_{GS1} - V_{TH1} + \left( 1 + \frac{1}{\sqrt{2}} \right) (V_{GS2} - V_{TH2})_{eq} \right]$$

$$\begin{aligned} A_{V,max} &= \frac{2}{\pi} g_{m1} R_{D,max} \\ &= \frac{8}{\pi} \frac{V_{R,max}}{V_{GS1} - V_{TH1}} \end{aligned}$$

- Low  $V_{DD}$  severely limits the conversion gain of active mixers.



# $R_{D,max} [\rightarrow V_{X,min} \text{ or } V_{Y,min}]$



If  $M_1$  is at the edge of saturation, then  $V_N \geq V_{GS1} - V_{TH1}$

$$V_{CM,LO} - V_{GS2,3} \geq V_{GS1} - V_{TH1}$$

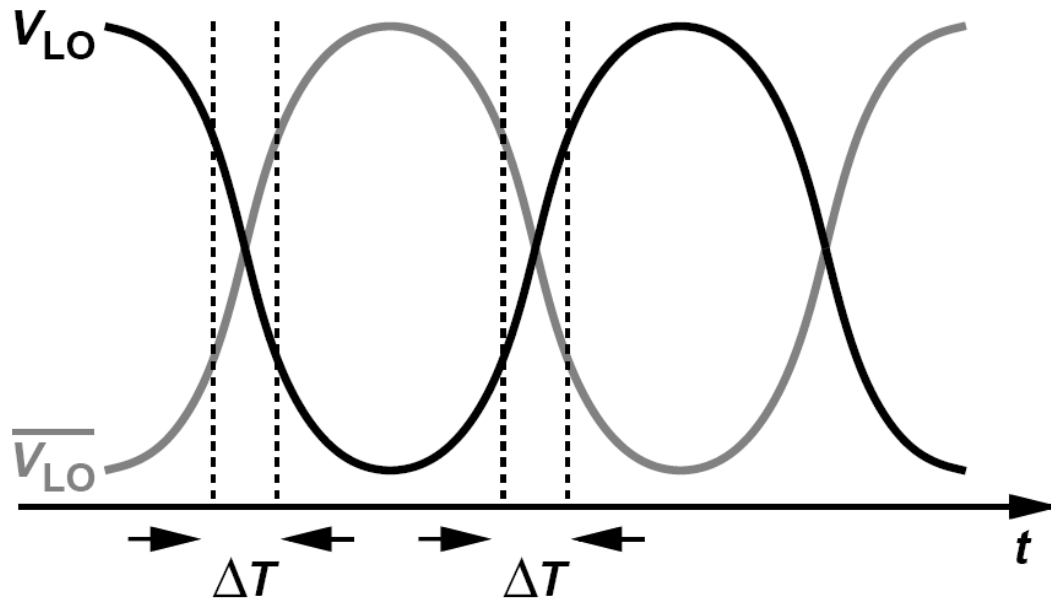
For  $M_3$  to turn off and  $M_2$  to remain in saturation up to this point,  $V_{D2} \geq V_{G2} - V_{TH2}$

$$\begin{aligned} V_{X,min} &= V_{CM,LO} + \frac{\sqrt{2}}{2}(V_{GS2,3} - V_{TH2}) - V_{TH2} \\ &= V_{GS1} - V_{TH1} + \left(1 + \frac{\sqrt{2}}{2}\right)(V_{GS2,3} - V_{TH2}) \end{aligned}$$

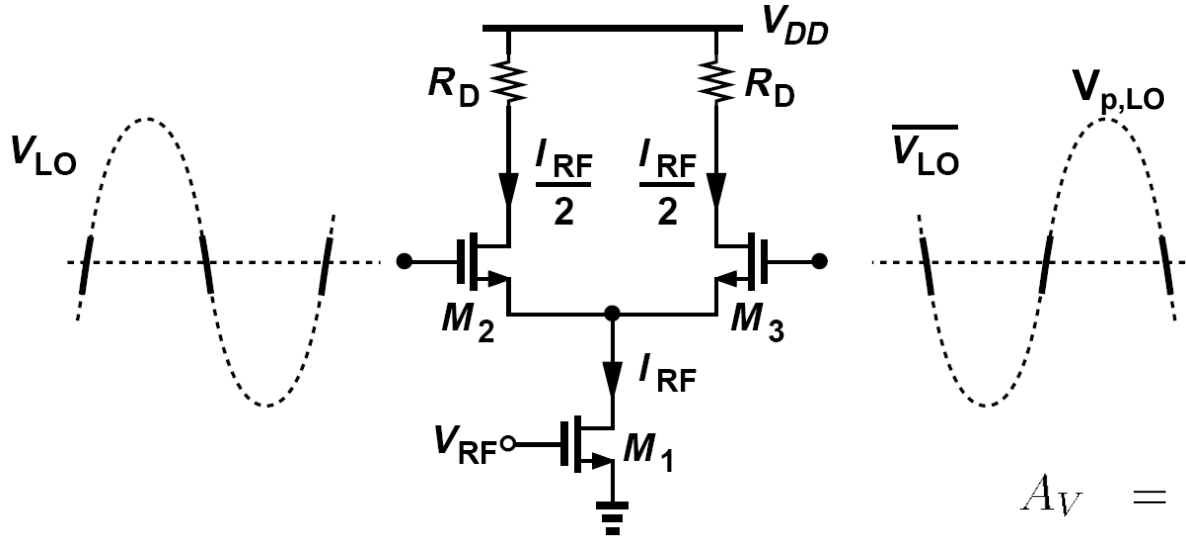
$$V_{R,max} = V_{DD} - V_{X,min}$$

# Ideal LO Waveform

- Ideal LO  $\rightarrow$  Square wave to ensure abrupt switching & max. conversion gain
- @ high RF, LO  $\rightarrow$  inevitably sinusoids
- Downconversion of interferers located at the LO harmonics is a serious issue in broadband RX.



# Gain with gradual LO transitions



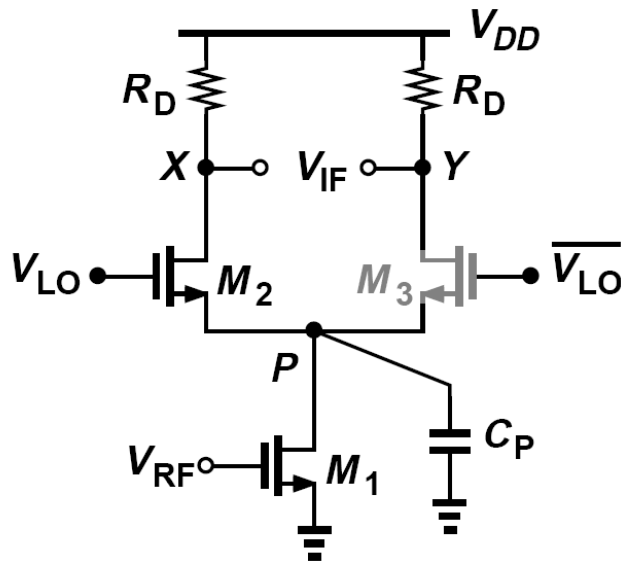
$$V_0 = \frac{\sqrt{2}(V_{GS2,3} - V_{TH2})}{2}$$

$$A_V = \frac{2}{\pi} g_{m1} R_D \left( 1 - \frac{2\Delta T}{T_{LO}} \right)$$

$$= \frac{2}{\pi} g_{m1} R_D \left[ 1 - \frac{(V_{GS} - V_{TH})_{eq}}{5\pi V_{p,LO}} \right]$$

- When  $M_2$  and  $M_3$  are near equilibrium, the RF current produced by  $M_1$  is split approximately equally between them, thus appearing as a CM current and yielding little conversion gain for that period of time ( $\Delta T$ ).
- Lower LO swing  $\rightarrow$  higher  $\Delta T$  and lower gain

# Gain with Parasitic Cap



With abrupt LO edges,  $M_2$  is on and  $M_3$  is off  $\rightarrow$

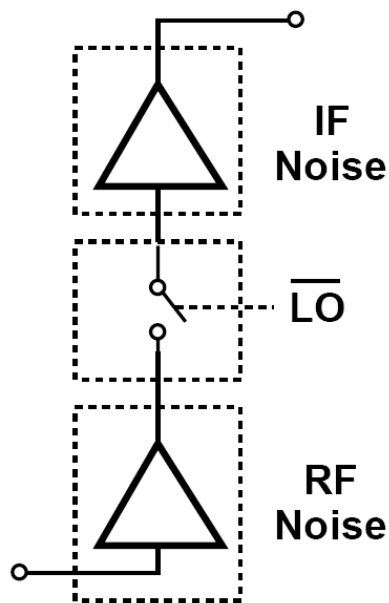
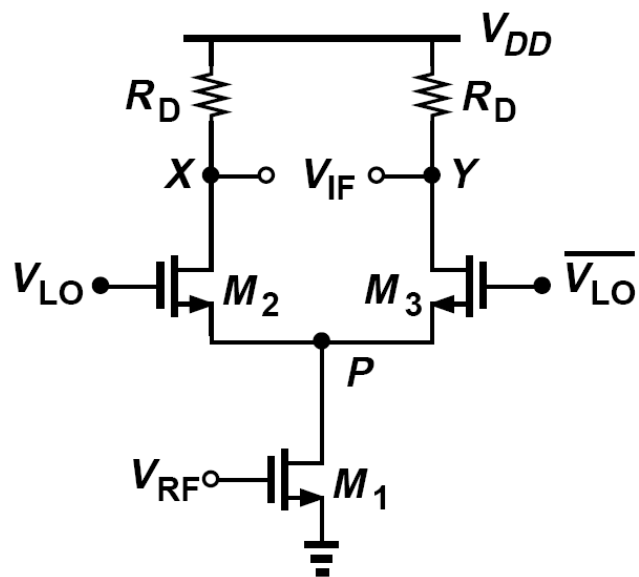
$$C_P = C_{DB1} + C_{GS2} + C_{GS3} + C_{SB2} + C_{SB3}.$$

- The RF current produced by  $M_1$  is split between  $C_P$  and  $1/g_{m2}$ .

$$A_{V,max} = \frac{2}{\pi} g_{m1} R_D \left[ 1 - \frac{(V_{GS} - V_{TH})_{eq}}{5\pi V_{P,LO}} \right] \frac{g_{m2}}{\sqrt{C_P^2 \omega^2 + g_{m2}^2}}.$$

# Noise in Active Mixers: Analysis

- The noise components of interest lie in the RF range before downconversion and in the IF range after downconversion.



Thermal	Flicker (active load)
Thermal	Flicker
Thermal	

- Total noise power @ IF o/p  
 $= \sum \text{Power}[\text{magnitude of each noise source} \times \text{its } \underline{\text{conversion gain}} \text{ to the IF o/p}]$
- Total noise power @ RF i/p  
 $= \text{Total noise power @ IF o/p} \div [\text{overall mixer } \underline{\text{conversion gain}}]^2$

# Flicker Noise Referred to I/P

$$V_{n, out}(f) = \left( \frac{I_{SS}R_D}{\pi V_{p, LO}} + 2fLOCP \right) V_{n2}(f)$$

Considering flicker noise of  $M_3$  also, ignoring  $C_p$  and dividing by conversion gain,

$$\begin{aligned} V_{n, in}(f) &= \frac{\sqrt{2}I_{SS}}{2g_{m1}V_{p, LO}} V_{n2}(f) \\ &= \frac{\sqrt{2}(V_{GS} - V_{TH})_1}{4V_{p, LO}} V_{n2}(f) \end{aligned}$$

- $V_{n2}(f)$  is typically very large because  $M_2$  and  $M_3$  are relatively small
- For active load, possibly much higher flicker noise



# Mixer Design – II

Transistor simulation curves for overdrive and  $I_D \rightarrow W_1 = 15\mu\text{m}$ ,  $W_{2,3} = 20\mu\text{m}$ ,  $g_{m1} = 12.75\text{mS}$ .

$C_1 = C_2 = 2\text{pF}$  to suppress LO component at o/p (prevent compression at the o/p).

$$\begin{aligned}A_v &= \frac{2}{\pi} g_{m1} R_D \\ &= 4.1 \quad (= 12.3 \text{ dB}).\end{aligned}$$

$$\begin{aligned}\overline{V_{n,in}^2} &= \pi^2 kT \left( \frac{\gamma}{g_{m1}} + \frac{2}{g_{m1}^2 R_D} \right) \\ &= 4.21 \times 10^{-18} \text{ V}^2/\text{Hz},\end{aligned}$$

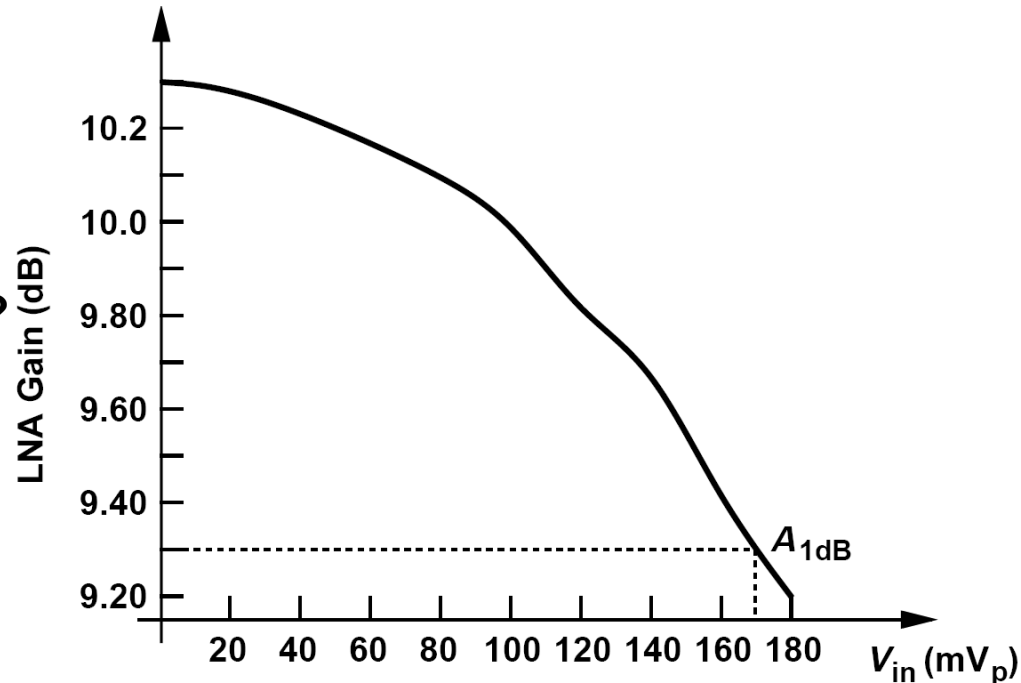
Single-sideband NF with respect to  $R_S = 50\Omega$ :

$$\begin{aligned}\text{NF}_{SSB} &= 1 + \frac{\overline{V_{n,in}^2}}{4kT R_S} \\ &= 6.1 \quad (= 7.84 \text{ dB})\end{aligned}$$

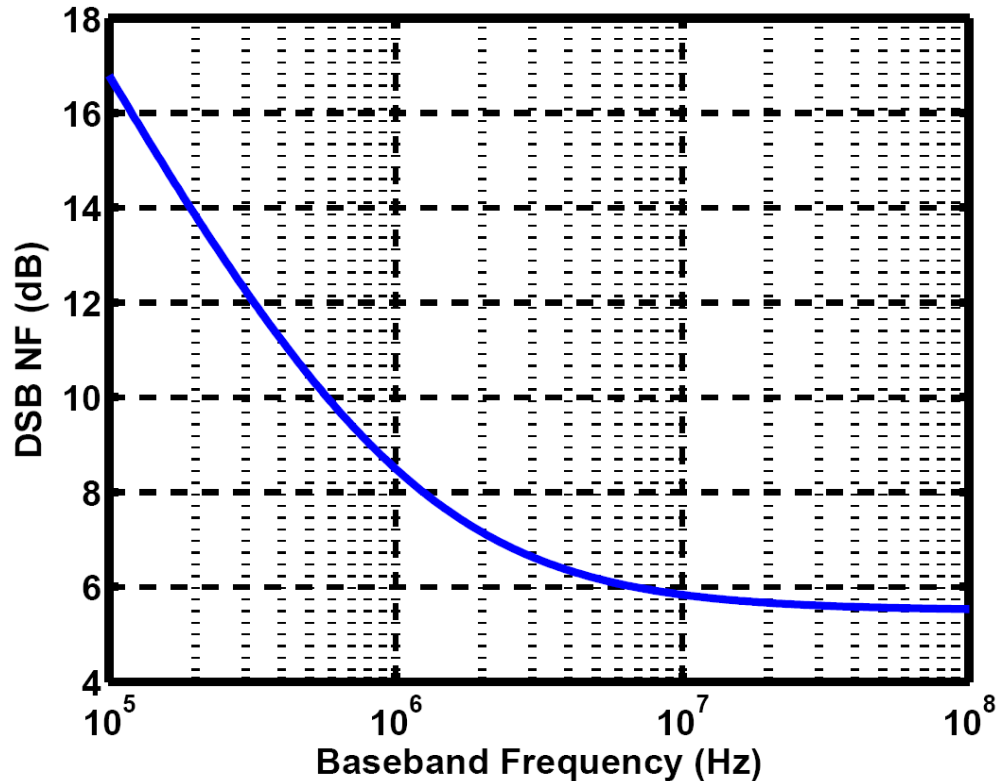
The double-sideband NF is 3 dB less.

# Mixer Design – III

- Simulated  $A_v = 10.3\text{dB}$ , ( $\sim 2\text{dB}$  less than estimated).
- 1dB compression at  $V_{in,p} = 170\text{mV}$  ( $-5.28\text{dBm}$ ).
- Mixer compresses at i/p or o/p first? Reducing  $R_D$  by 5 and scaling the output voltage-swings proportionally, simulation shows that gain drops by 0.5dB only now at  $V_{in,p} = 170\text{mV} \rightarrow$  the output port (switches) reach compression first.
- $IIP_3 = 711\text{mV}_p$  ( $7\text{dBm}$ ).
- $IIP_3 = 12.3\text{dB} + P_{1\text{dB}}$  – perhaps due to higher order non-linearities.

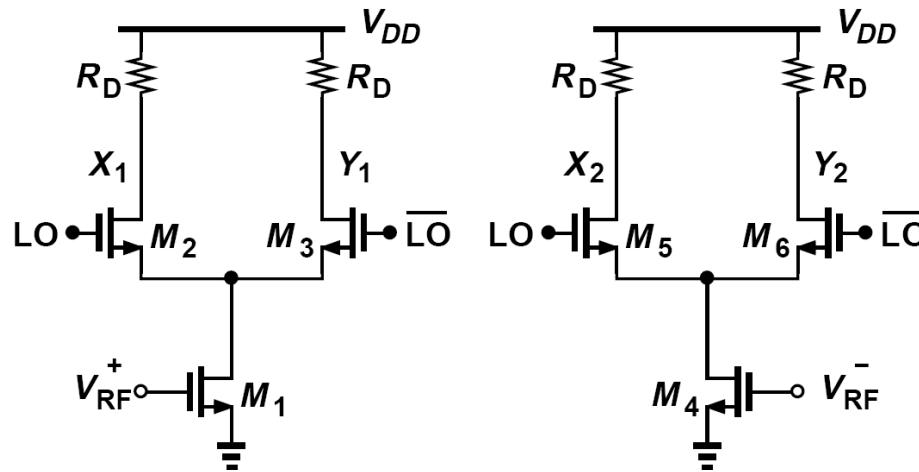


# Mixer Design – IV



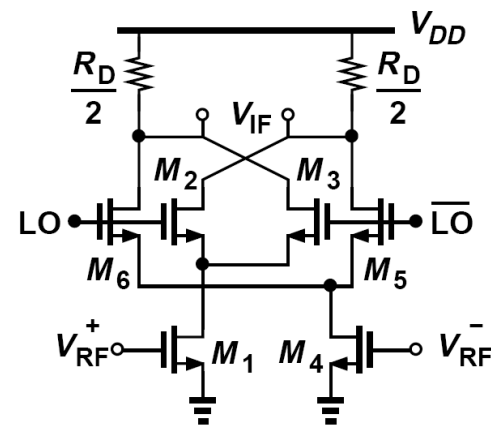
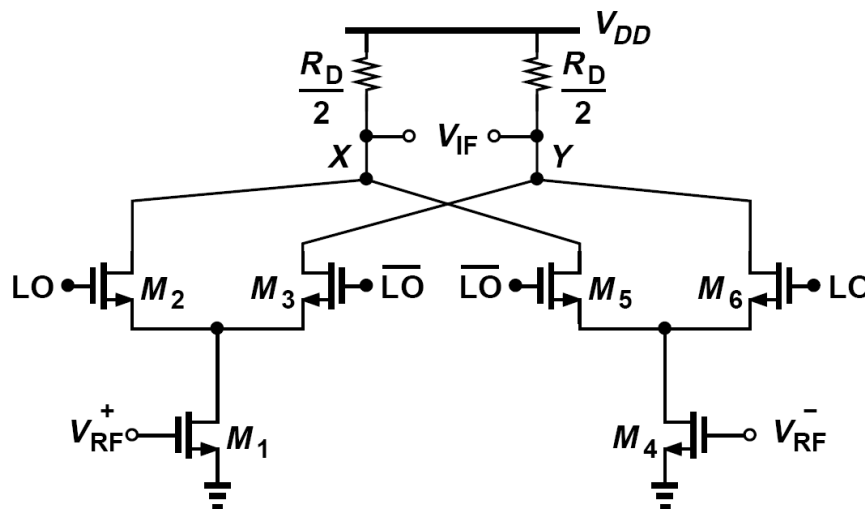
- Flicker noise heavily corrupts the baseband up to several MHz.
- NF @100MHz (thermal) = 5.5dB, ~ 0.7dB higher than predicted.

# Double-Balanced Mixer (Gilbert)



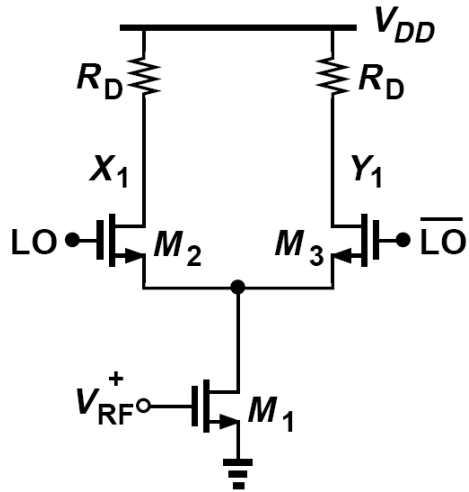
$$V_{X1} = -V_{Y1} = -V_{X2} = V_{Y2}$$

$$\rightarrow \text{Short } X1\text{-}Y2 \text{ and } X2\text{-}Y1$$

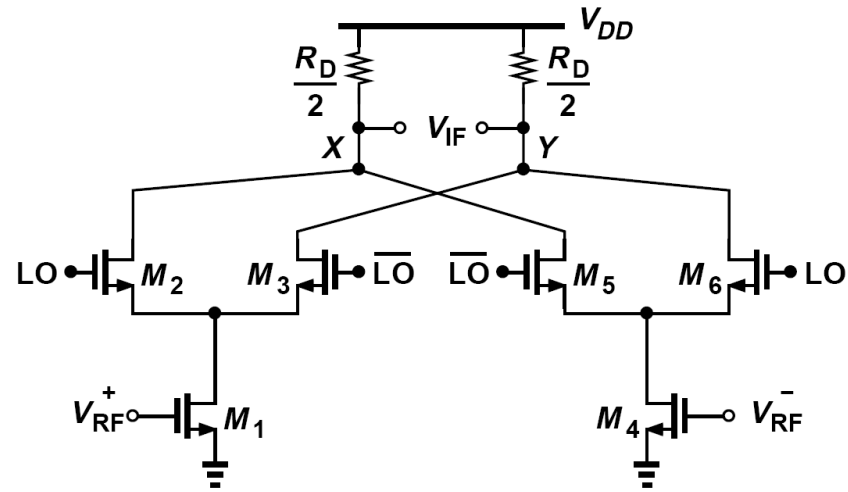


$I_{\text{bias}}$  of DBM = 2x  $I_{\text{bias}}$  of SBM

# LO-IF Feedthrough: DBM vs. SBM



$$I_{DC} + IRF \cos \omega_{RF} t$$



$$I_{DC} + IRF \cos \omega_{RF} t \quad I_{DC} - IRF \cos \omega_{RF} t$$

- SBM has LO (and its harmonics) feedthrough to IF because  $I_{out,diff}$  has  $I_{DC}$  component mixing with LO (and its harmonics).
- DBM has no LO-IF feedthrough (ideally) because  $I_{out,diff}$  has no  $I_{DC}$  component mixing with LO (and its harmonics).

# Conversion Gain: SBM vs. DBM

Voltage conversion gain of SBM =  $(V_{X1} - V_{Y1})/V_{RF}^+$ .

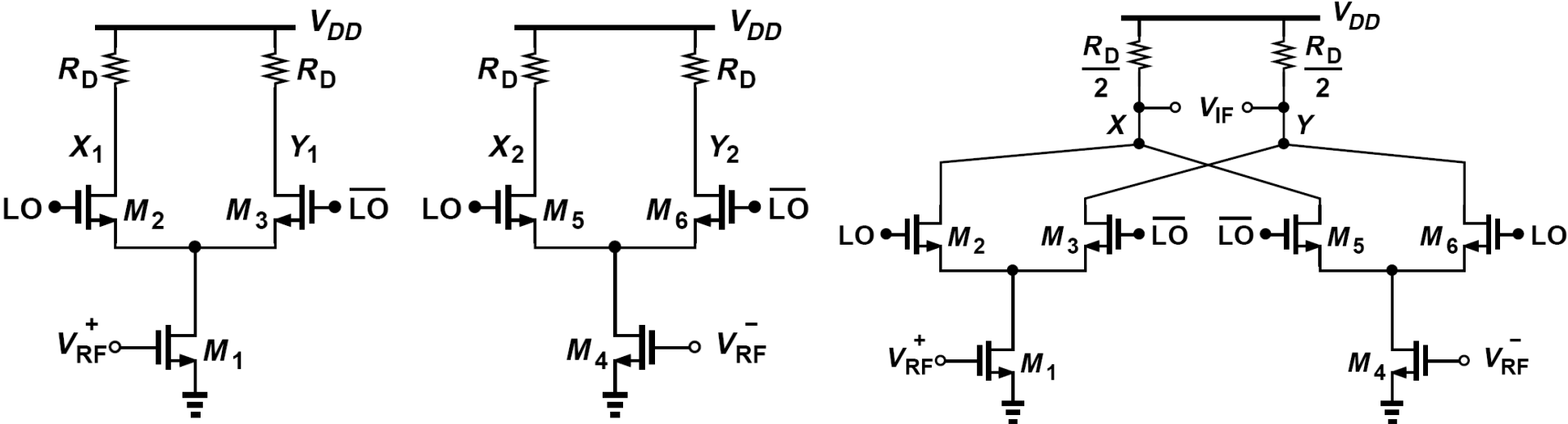
But, shorting  $Y_2$  to  $X_1$ , and  $X_2$  to  $Y_1$ , these node voltages remain unchanged.

Differential voltage conversion gain of DBM is given by

$$\frac{V_X - V_Y}{V_{RF}^+ - V_{RF}^-} = \frac{V_{X1} - V_{Y1}}{2V_{RF}^+}$$

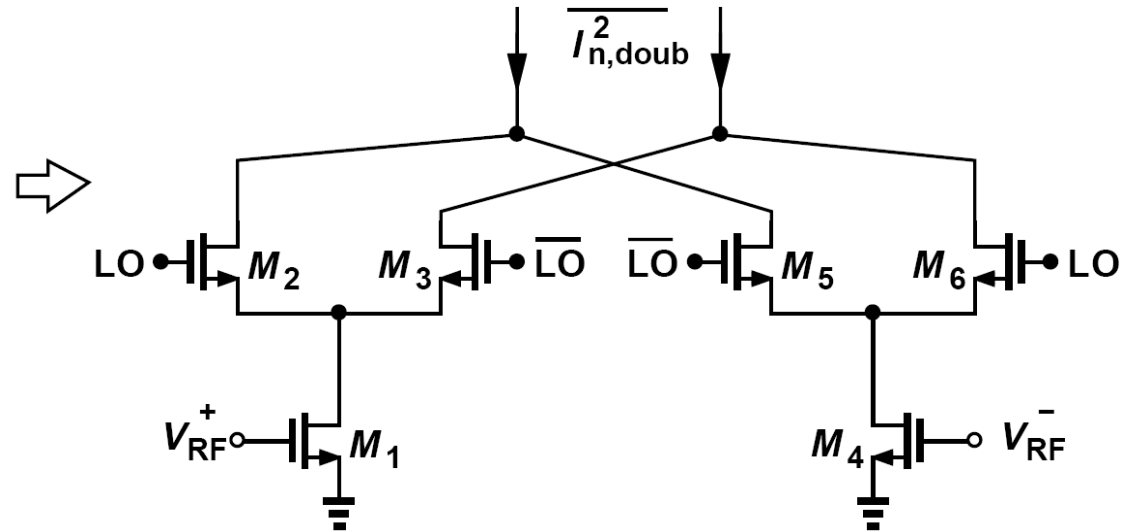
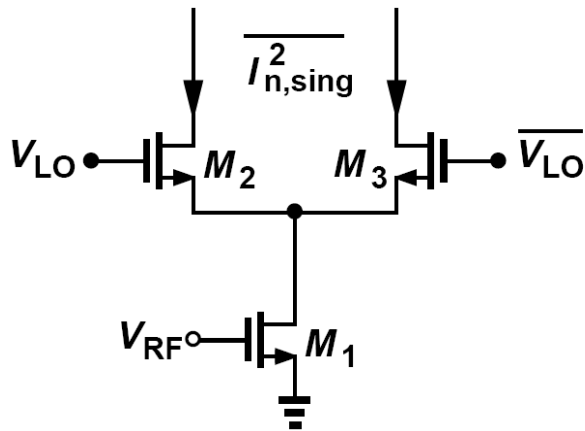
= 1/2 of SBM counterpart.

Cause: limited voltage headroom disallows a load resistance of  $R_D$



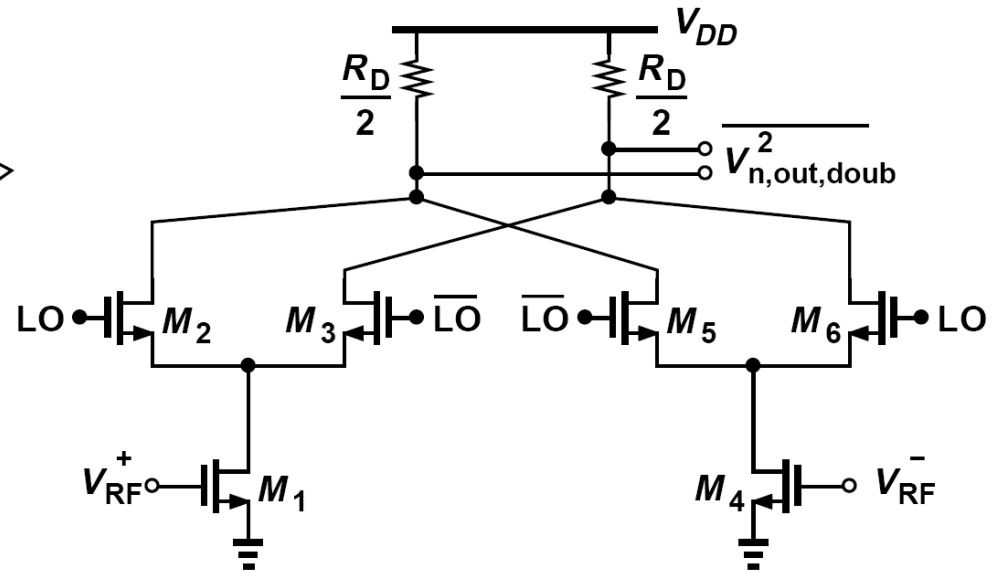
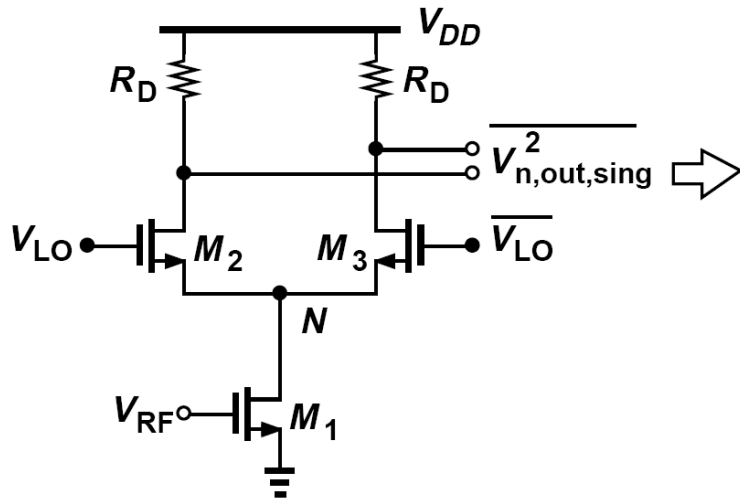
# Noise: SBM vs. DBM

Assuming  $I_{\text{bias}}$  of DBM =  $2 \times I_{\text{bias}}$  of SBM



$$\overline{I_{n,doub}^2} = 2 \overline{I_{n,sing}^2}$$

# Noise: SBM vs. DBM



O/P noise voltages, with load resistors differing by 2X

$$\overline{V_{n,out,sing}^2} = \overline{I_{n,sing}^2} (R_D)^2$$

$$\overline{V_{n,out,doub}^2} = \overline{I_{n,doub}^2} \left(\frac{R_D}{2}\right)^2$$

As voltage conversion gain of DBM =  $\frac{1}{2}$  of SBM, input-referred noise voltages are

$$\overline{V_{n,in,sing}^2} = \frac{1}{2} \overline{V_{n,in,doub}^2}$$