Oversampling Converters

Reference: Chapter 14 of the text
"Analog Integrated Circuit Design"
by
David Johns and Ken Martin

Chapter 18 of the 2nd Edition of the text by Tony Chan Carusone, David Johns, and Ken Martin

The material of this presentation is courtesy of Dr. Ken Martin.

Motivation

- Popular approach for high resolution A/D and D/A (typically low to medium speed)

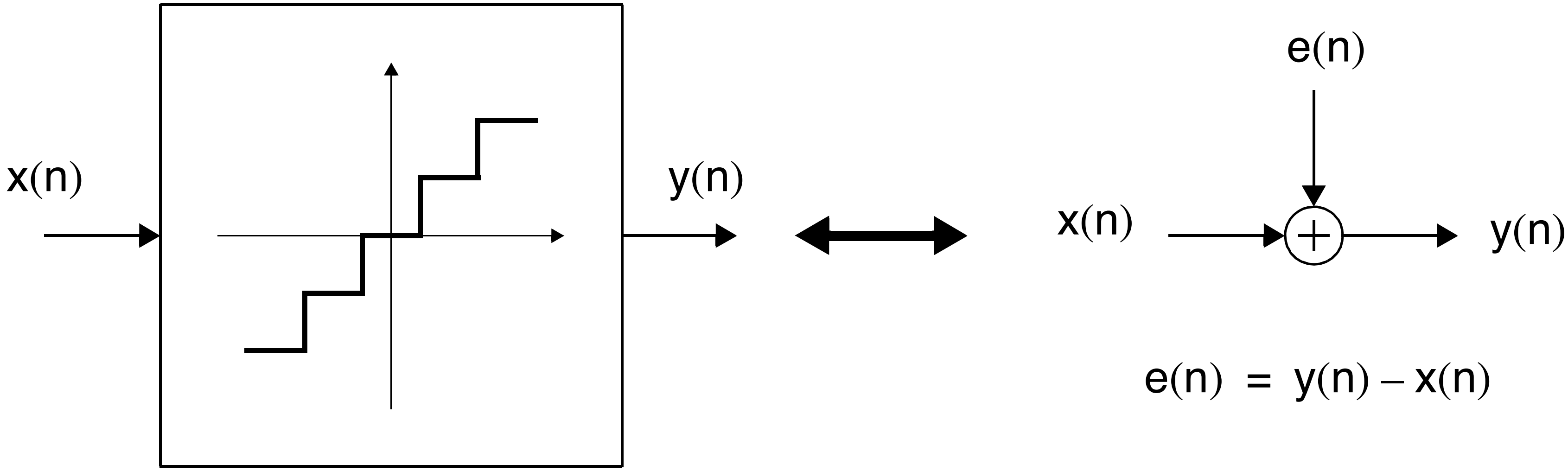
Easier Analog

- reduced requirement on matching tolerances and amplifier gains
- relaxed anti-aliasing specifications
- relaxed smoothing filters

Catch: More complicated digital signal processing

- More suitable for deep submicron technologies

Quantization Noise

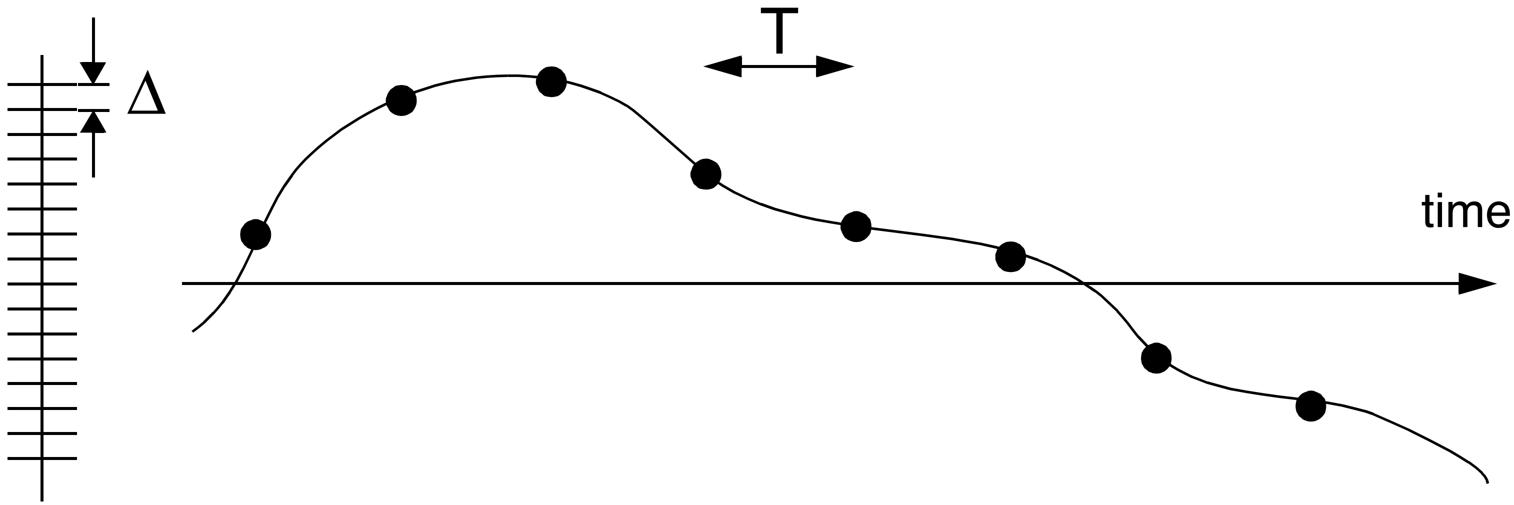


Quantizer

Model

- Above model is exact
  — approximate when assumptions made about $e(n)$
- Often assume $e(n)$ is white, uniformily distributed number between $\pm\Delta/2$
- Δ is difference between two quantization levels

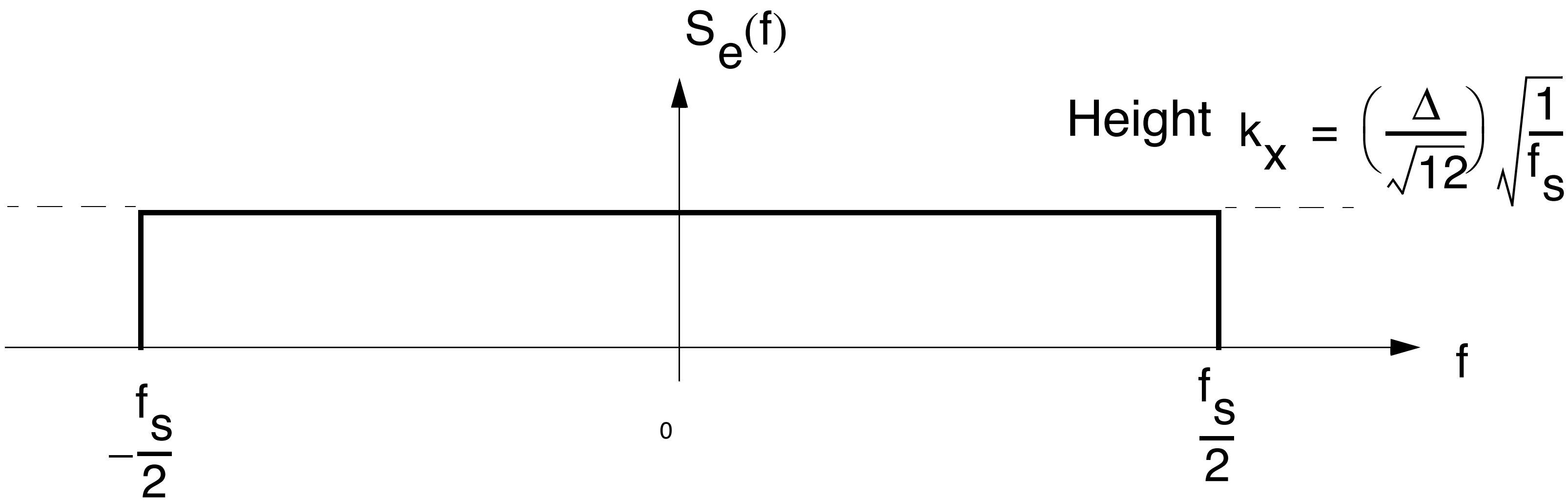
Quantization Noise



- White noise assumption reasonable when:
  — fine quantization levels
  — signal crosses through many levels between samples
  — sampling rate not synchronized to signal frequency
- Sample lands somewhere in quantization interval leading to random error of $\pm\Delta/2$

Quantization Noise

- Quantization noise power can be shown to be $\Delta^2 / 12$ and is ***independent of sampling frequency***
- If white, then spectral density of noise, $S_e(f)$, is constant.

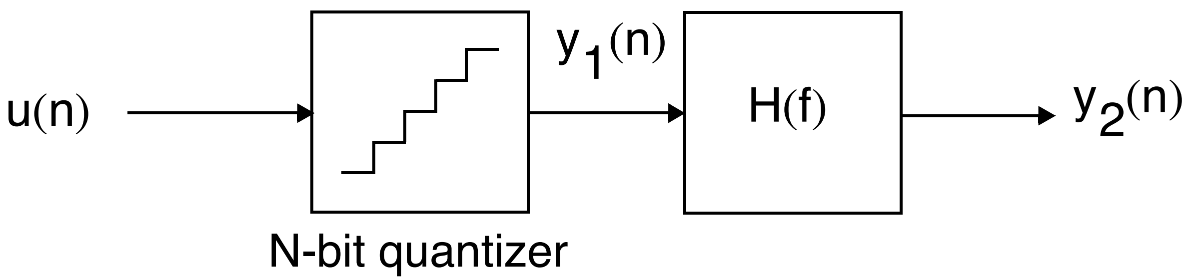


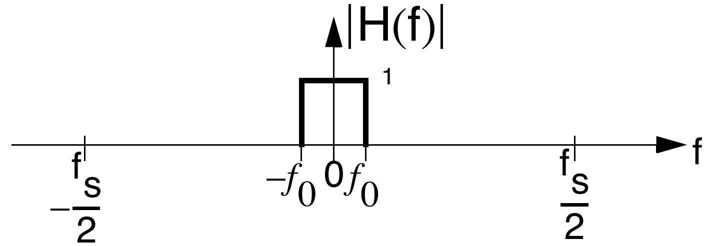
Oversampling Advantage

- Oversampling occurs when signal of interest is bandlimited to f_0 but we sample higher than $2f_0$
- Define oversampling-rate

$$\mathrm{OSR} = f_s/(2f_0) \tag{1}$$

- After quantizing input signal, pass it through a brickwall digital filter with passband up to f_0





Oversampling Advantage

- Output quantization noise after filtering is:

$$P_e = \int_{-f_s/2}^{f_s/2} S_e^2(f)|H(f)|^2\, df = \int_{-f_0}^{f_0} k_x^2\, df = \frac{\Delta^2}{12}\left(\frac{1}{OSR}\right) \tag{2}$$

- Doubling OSR reduces quantation noise power by 3dB (i.e., 0.5 bits/octave)
- Assuming peak input is a sinusoidal wave with a peak value of $2^N(\Delta/2)$ leading to $P_s = ((\Delta 2^N)/(2\sqrt{2}))^2$
- Can also find peak SNR as:

$$SNR_{max} = 10\ \log\left(\frac{P_s}{P_e}\right) = 10\ \log\left(\frac{3}{2}2^{2N}\right) + 10\log(OSR) \tag{3}$$

Oversampling Advantage

Example

- A dc signal with 1V is combined with a noise signal uniformily distributed between $\pm\sqrt{3}$ giving 0 dB SNR.
  — {0.94, –0.52, –0.73, 2.15, 1.91, 1.33, –0.31, 2.33}.
- Average of 8 samples results in 0.8875
- Signal adds linearly while noise values add in a square-root fashion — noise filtered out.

Example

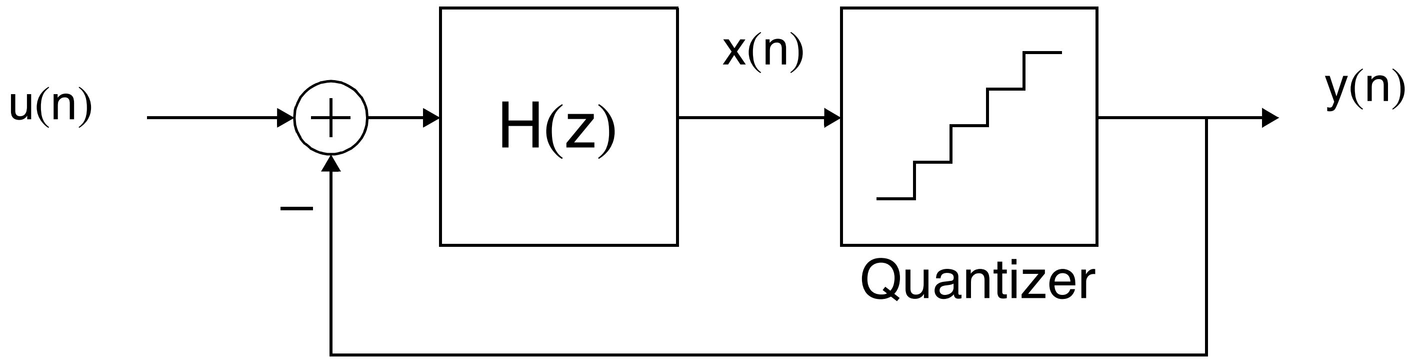
- 1-bit A/D gives 6dB SNR.
- To obtain 96dB SNR requires 30 octaves of oversampling ((96-6)/3 dB/octave)
- If $f_0 = 25\text{ kHz}$, $f_s = 2^{30} \times f_0 = 54,000\text{ GHz}$!

Advantage of 1-bit D/A Converters

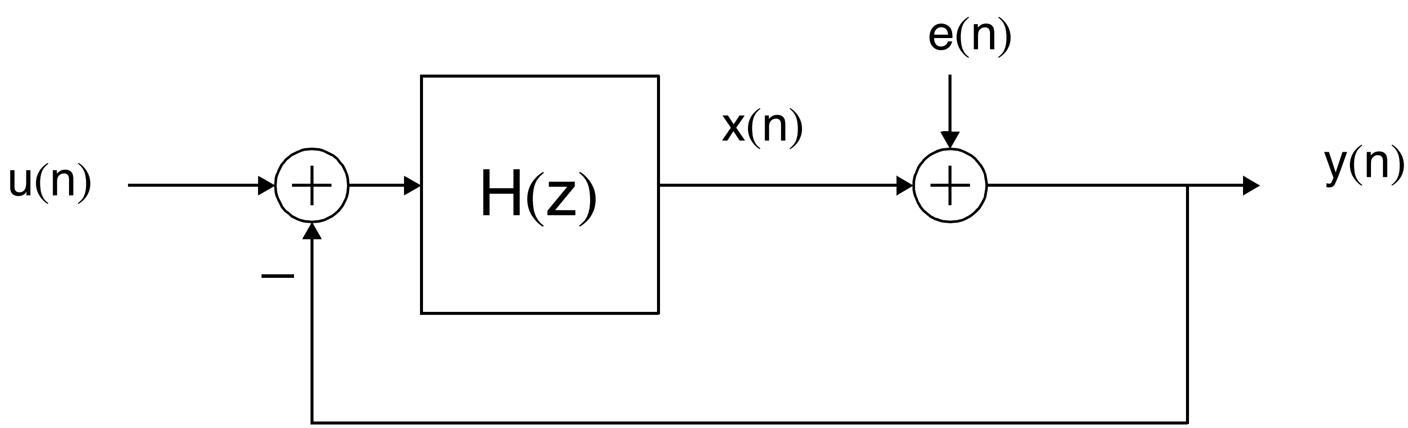
- Oversampling improves SNR but not linearity
- To acheive 16-bit linear converter using a 12-bit converter, 12-bit converter must be linear to 16 bits
  — i.e. integral nonlinearity better than $1/2^4$ LSB
- A 1-bit D/A is ***inherently linear***
  — 1-bit D/A has only 2 output points
  — 2 points always lie on a straight line
- Can acheive better than 20 bits linearity without trimming (will likely have gain and offset error)
- Second-order effects (such as D/A memory or signal-dependent reference voltages) will limit linearity.

Oversampling with Noise Shaping

- Place the quantizer in a feedback loop



Delta-Sigma Modulator



Linear model

Oversampling with Noise Shaping

- Shapes quantization noise away from signal band of interest

Signal and Noise Transfer-Functions

$$S_{TF}(z) \equiv \frac{Y(z)}{U(z)} = \frac{H(z)}{1 + H(z)} \tag{4}$$

$$N_{TF}(z) \equiv \frac{Y(z)}{E(z)} = \frac{1}{1 + H(z)} \tag{5}$$

$$Y(z) = S_{TF}(z)U(z) + N_{TF}(z)E(z) \tag{6}$$

- Choose $H(z)$ to be large over 0 to f_0
- Resulting quantization noise near 0 where $H(z)$ large
- Signal transfer-function near 1 where $H(z)$ large

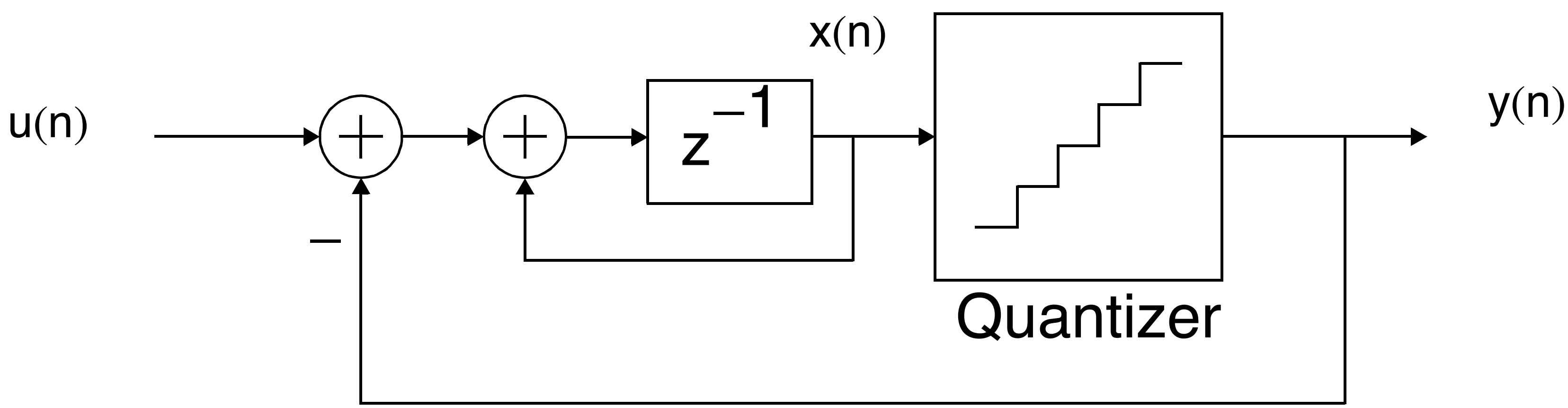
Oversampling with Noise Shaping

- Input signal is limited to range of quantizer output when $H(z)$ large
- For 1-bit quantizers, input often limited to 1/4 quantizer outputs
- Out-of-band signals can be larger when $H(z)$ small
- Stability of modulator can be an issue (particularily for higher-orders of $H(z)$
- Stability defined as when input to quantizer becomes so large that quantization error is greater than $\pm\Delta/2$ — referred to as "overload the quantizer"

First-Order Noise Shaping

- Choose $H(z)$ to be a discrete-time integrator

$$H(z) = \frac{1}{z-1} \tag{7}$$



- If stable, average input of integrator must be zero
- Average value of $u(n)$ must equal average of $y(n)$

Example

- The output sequence and state values when a dc input, $u(n)$, of $1/3$ is applied to a 1'st order modulator with a two-level quantizer of ± 1.0. Initial state for $x(n)$ is 0.1.

n	**x(n)**	**x(n + 1)**	**y(n)**	**e(n)**
0	0.1	–0.5667	1.0	0.9
1	–0.5667	0.7667	–1.0	–0.4333
2	0.7667	0.1	1.0	0.2333
3	0.1	–0.5667	1.0	0.9
4	–0.5667	0.7667	–1.0	–0.4333
5	…	…	…	…

- Average of $y(n)$ is $1/3$ as expected
- Periodic quantization noise in this case

Transfer-Functions

Signal and Noise Transfer-Functions

$$S_{TF}(z) = \frac{Y(z)}{U(z)} = \frac{1/(z-1)}{1+1/(z-1)} = z^{-1} \tag{8}$$

$$N_{TF}(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+1/(z-1)} = (1-z^{-1}) \tag{9}$$

- Noise transfer-function is a discrete-time differentiator (i.e., a high-pass filter)

$$\begin{aligned} N_{TF}(f) &= 1-e^{-j2\pi f/f_s} = \frac{e^{j\pi f/f_s}-e^{-j\pi f/f_s}}{2j} \times 2j \times e^{-j\pi f/f_s} \\ &= \sin\left(\frac{\pi f}{f_s}\right) \times 2j \times e^{-j\pi f/f_s} \end{aligned} \tag{10}$$

Signal to Noise Ratio

Magnitude of noise transfer-function

$$|N_{TF}(f)| = 2\sin\left(\frac{\pi f}{f_s}\right) \tag{11}$$

Quantization noise power

$$P_e = \int_{-f_0}^{f_0} S_e^2(f)|N_{TF}(f)|^2\,df = \int_{-f_0}^{f_0}\left(\frac{\Delta^2}{12}\right)\frac{1}{f_s}\left[2\sin\left(\frac{\pi f}{f_s}\right)\right]^2 df \tag{12}$$

- Assuming $f_0 << f_s$ (i.e., $OSR >> 1$)

$$P_e \cong \left(\frac{\Delta^2}{12}\right)\left(\frac{\pi^2}{3}\right)\left(\frac{2f_0}{f_s}\right)^3 = \frac{\Delta^2\pi^2}{36}\left(\frac{1}{OSR}\right)^3 \tag{13}$$

Max SNR

- Assuming peak input is a sinusoidal wave with a peak value of $2^N(\Delta/2)$ leading to $P_s = ((\Delta 2^N)/(2\sqrt{2}))^2$
- Can find peak SNR as:

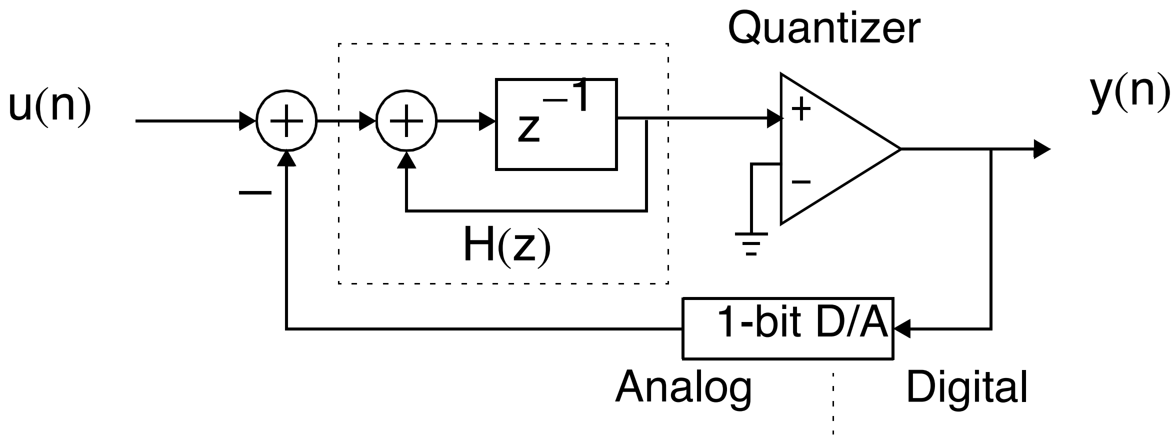
$$\mathrm{SNR}_{max} = 10\log\left(\frac{P_s}{P_e}\right)$$

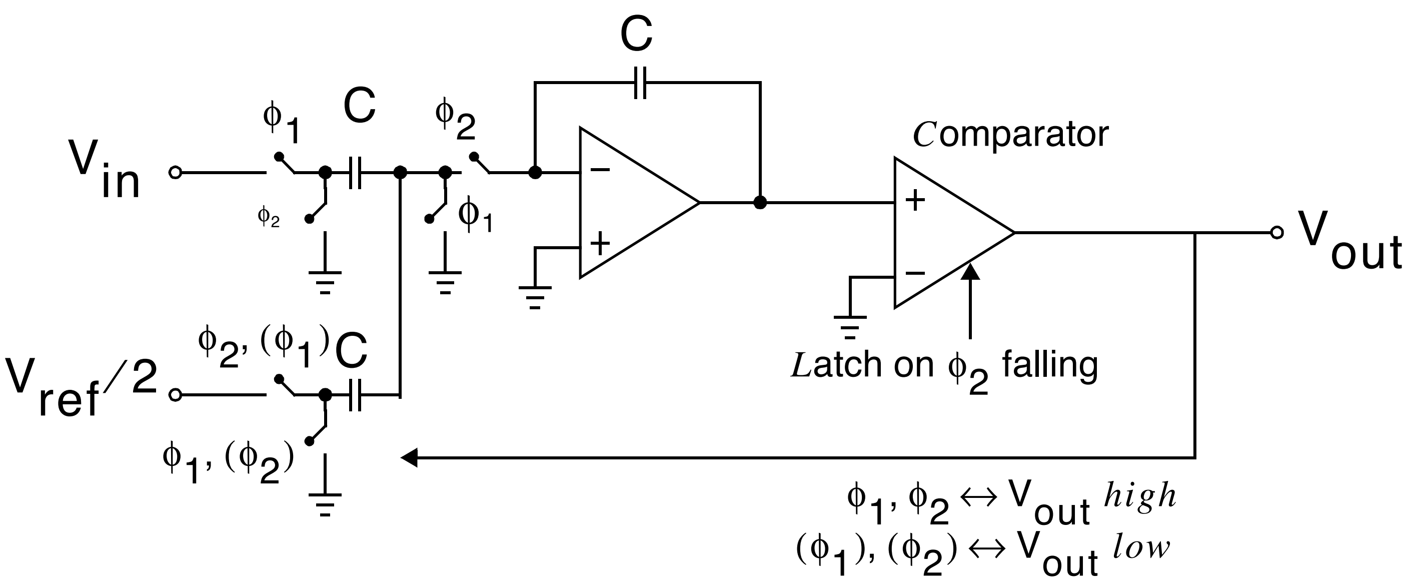
$$= 10\log\left(\frac{3}{2}2^{2N}\right) + 10\log\left[\frac{3}{\pi^2}(OSR)^3\right] \tag{14}$$

$$\mathrm{SNR}_{max} = 6.02N + 1.76 - 5.17 + 30\log(OSR) \tag{15}$$

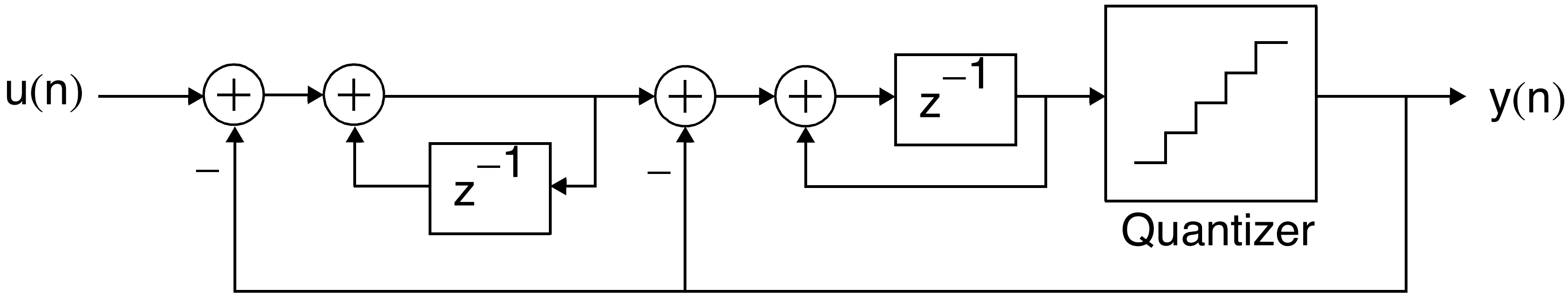
- Doubling OSR gives an SNR improvement $9\ \mathrm{dB}$ or, equivalently, a benefit of 1.5 bits/octave

SC Implementation





Second-Order Noise Shaping



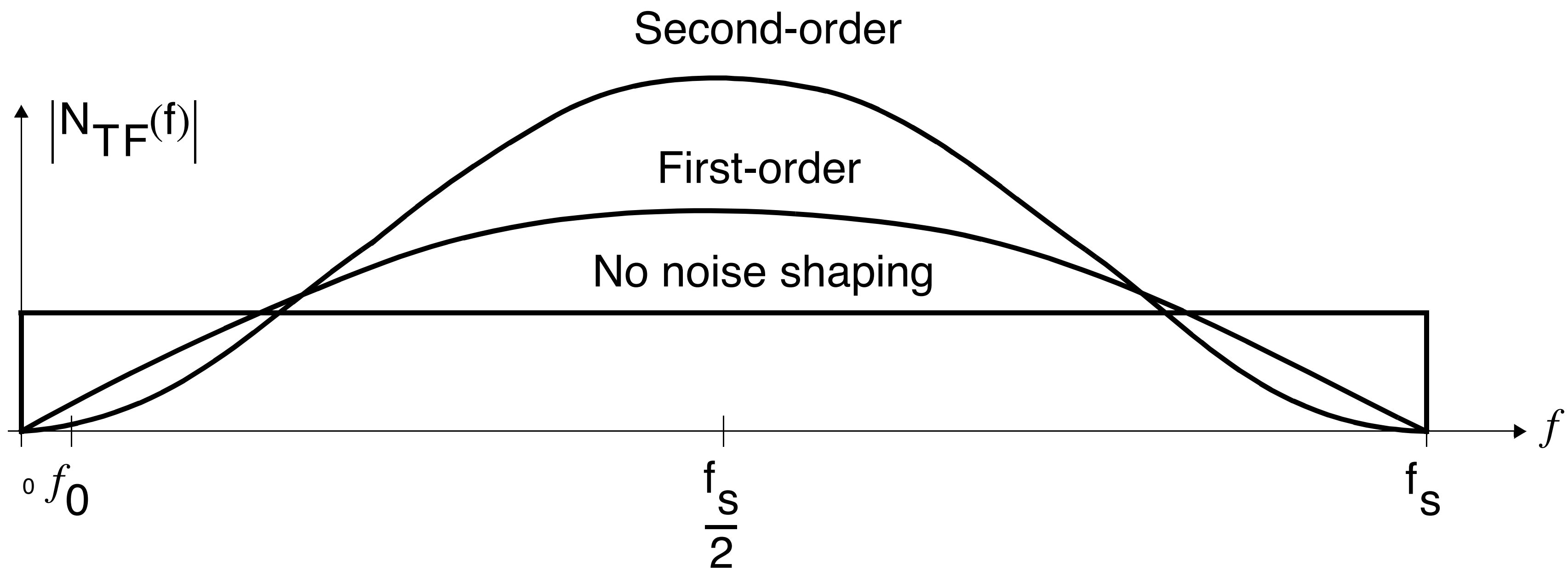
$$S_{TF}(f) = z^{-1} \tag{16}$$

$$N_{TF}(f) = (1 - z^{-1})^2 \tag{17}$$

$$SNR_{max} = 6.02N + 1.76 - 12.9 + 50\log(OSR) \tag{18}$$

- Doubling OSR improves SNR by 15 dB (i.e., a benefit of 2.5 bits/octave)

Noise Transfer-Function Curves



- Out-of-band noise increases for high-order modulators
- Out-of-band noise peak controlled by poles of noise transfer-function
- Can also spread zeros over band-of-interest

Example

- 90 dB SNR improvement from A/D with $f_0 = 25\ \text{kHz}$

Oversampling with no noise shaping

- From before, straight oversampling requires a sampling rate of $54{,}000$ GHz.

First-Order Noise Shaping

- Lose 5 dB (see (15)), require 95 dB divided by 9 dB/ octave, or 10.56 octaves — $f_s = 2^{10.56} \times 2f_0 \cong 75$ MHz

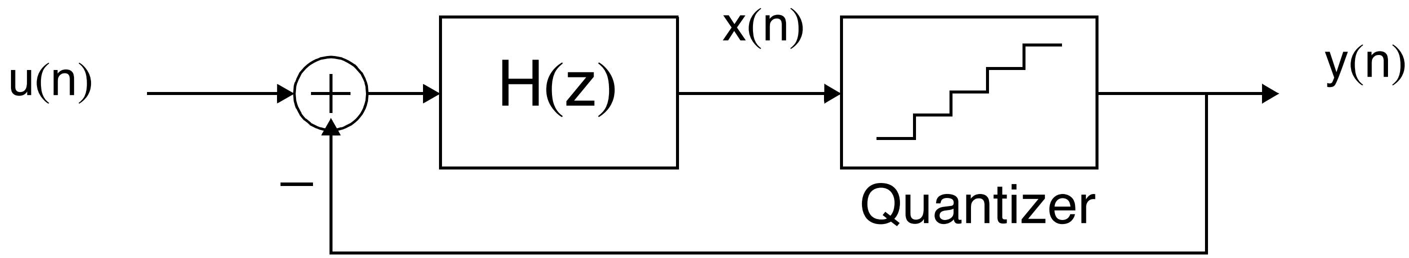
Second-Order Noise Shaping

- Lose 13 dB, required 103 dB divided by 15 dB/ octave, $f_s = 5.8\ \text{MHz}$
- In general, an L'th order 1-bit modulator improves SNR by 6L+3 dB/octave

Quantization Noise Power of 1-bit Modulators

- If output of 1-bit mod is ± 1, total power of output signal, $y(n)$, is normalized power of 1 watt.
- Signal level often limited to well below ± 1 level in higher-order modulators to maintain stability
- For example, if maximum peak level is ± 0.25, max signal power is 62.5 mW.
- Max signal is approx 12 dB below quantization noise (but most of the noise is in different frequency region)
- Quantization filter must have dynamic range capable of handling full power of $y(n)$ at input.
- Easy for A/D — digital filter
- More difficult for D/A — analog filter

Zeros of NTF are poles of H(z)



- Write $H(z)$ as

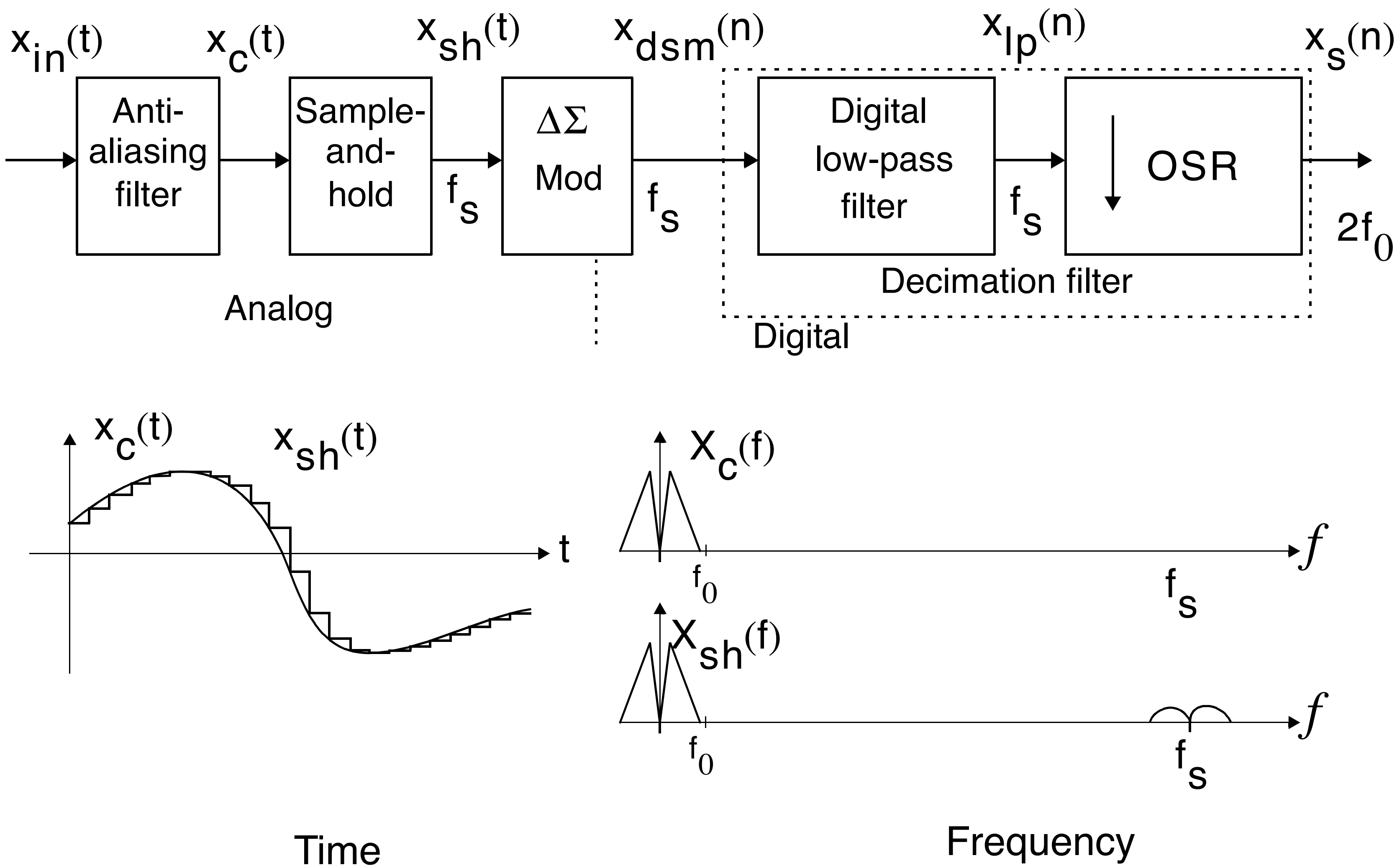
$$H(z) = \frac{N(z)}{D(z)} \tag{19}$$

- NTF is given by:

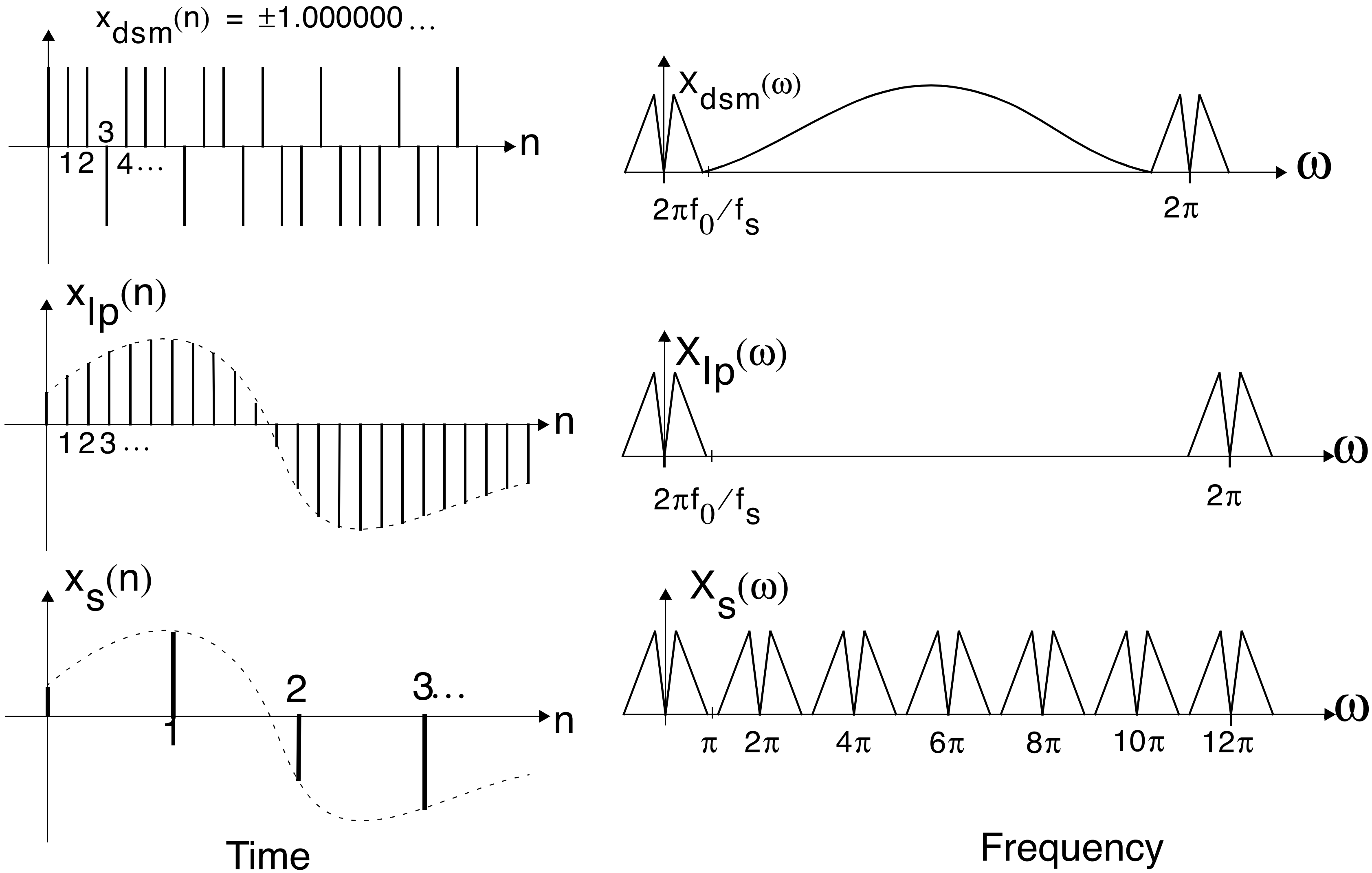
$$\mathrm{NTF}(z) = \frac{1}{1+H(z)} = \frac{D(z)}{D(z)+N(z)} \tag{20}$$

- If poles of $H(z)$ are well-defined then so are zeros of NTF

Architecture of Delta-Sigma A/D Converters



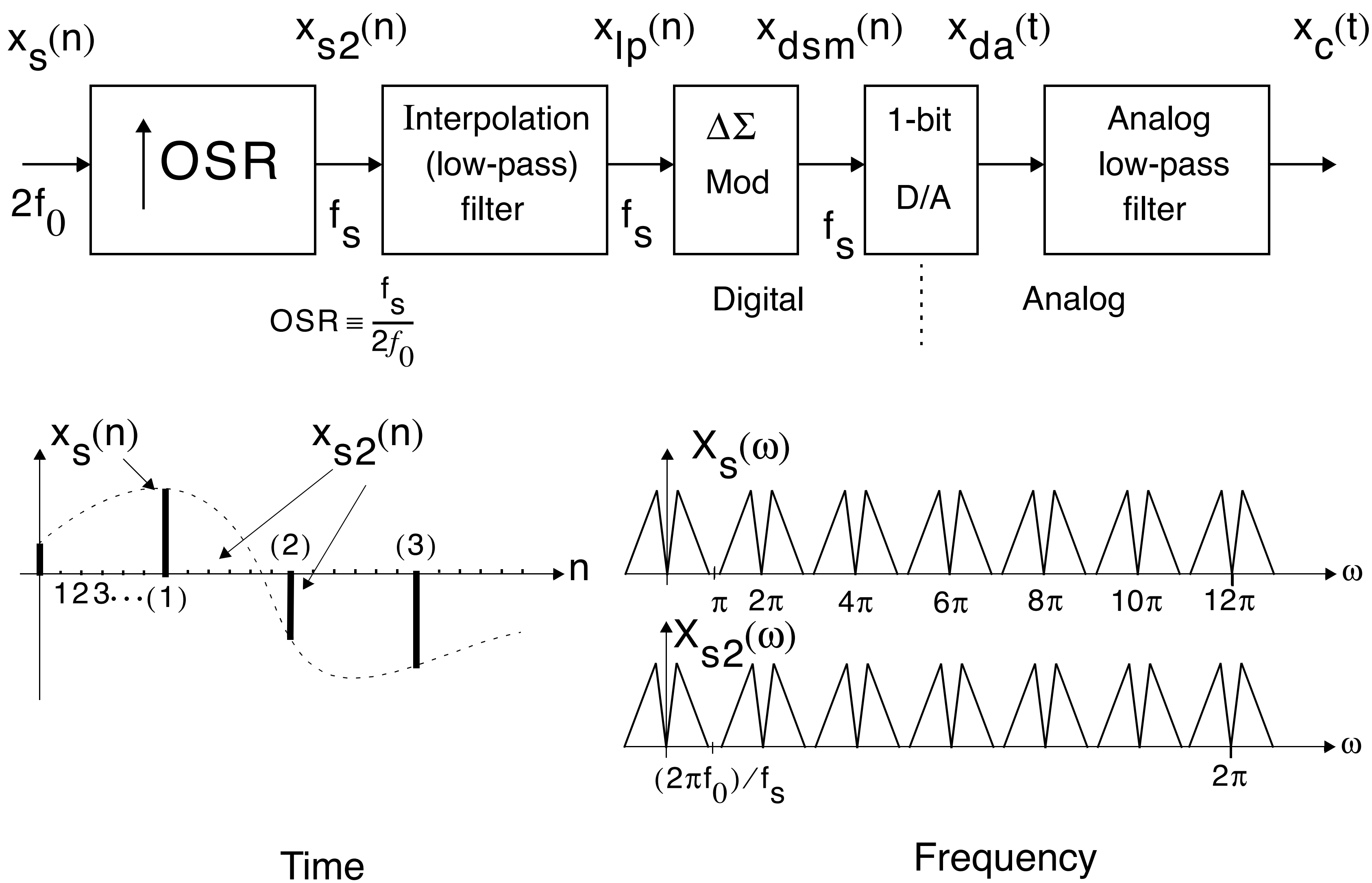
Architecture of Delta-Sigma A/D Converters



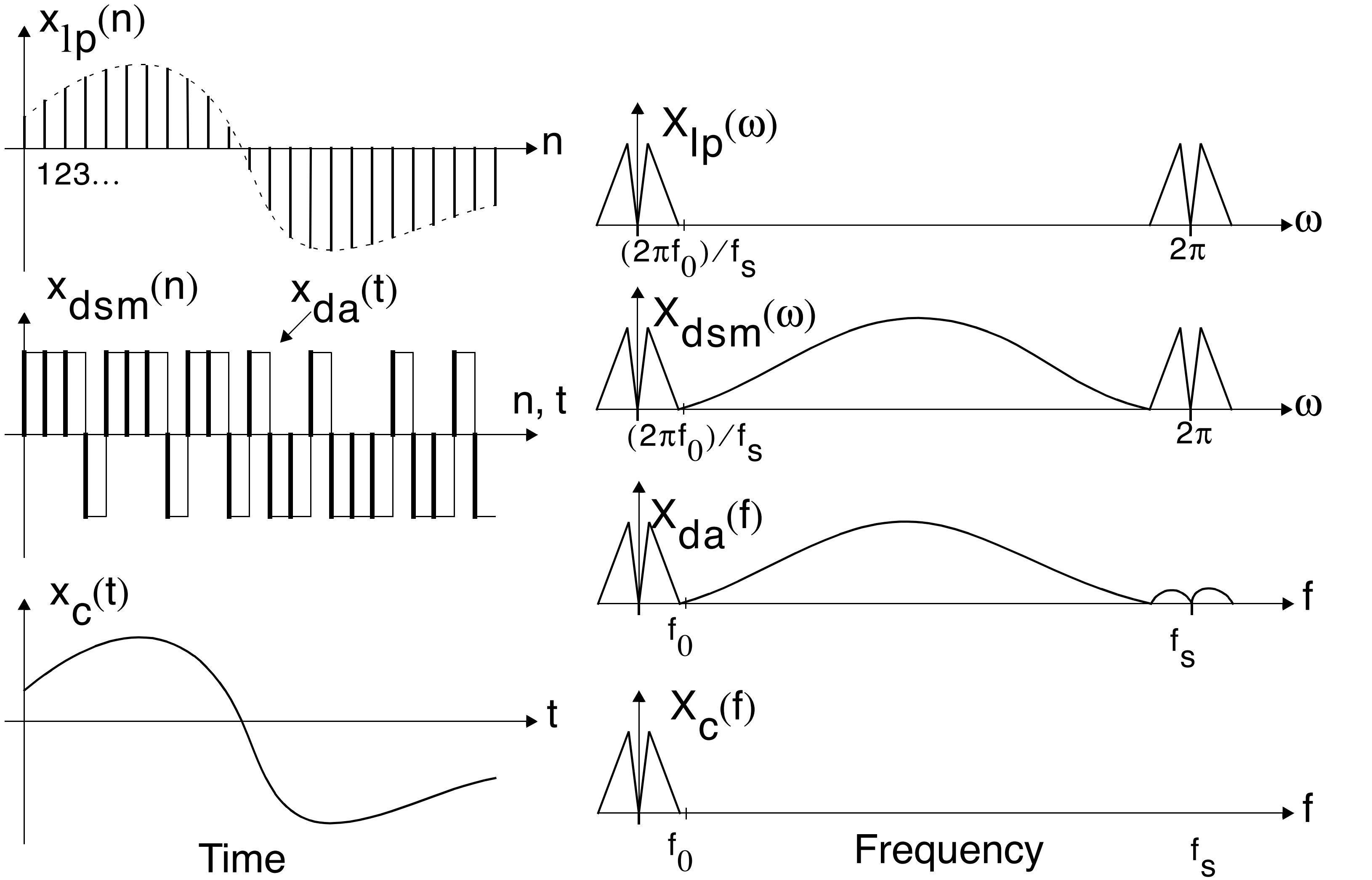
Architecture of Delta-Sigma A/D Converters

- Relaxes analog anti-aliasing filter
- Strict anti-aliasing done in digital domain
- Must also remove quantization noise before downsampling (or aliasing occurs)
- Commonly done with a multi-stage system
- Linearity of D/A in modulator important — results in overall nonlinearity
- Linearity of A/D in modulator unimportant (effects reduced by high gain in feedback of modulator)

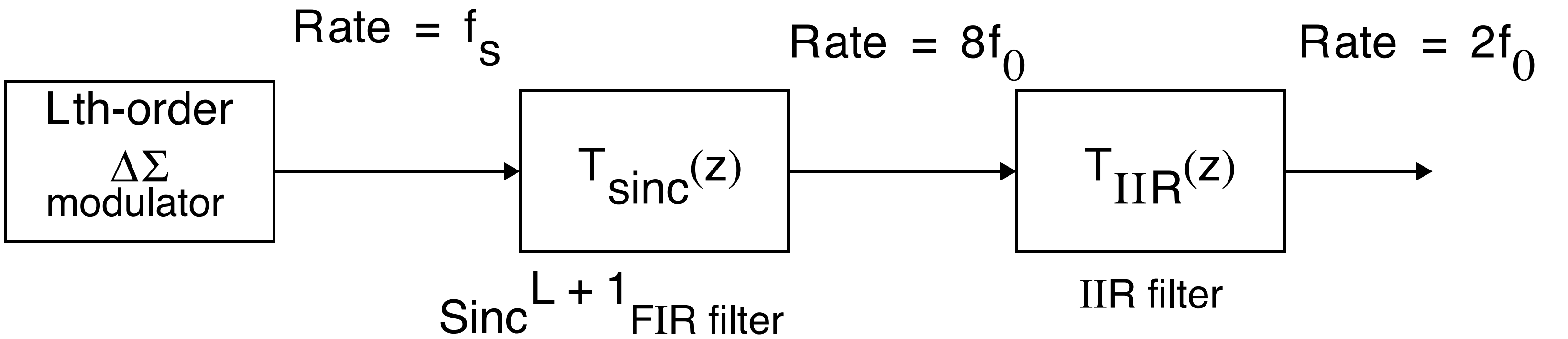
Architecture of Delta-Sigma D/A Converters

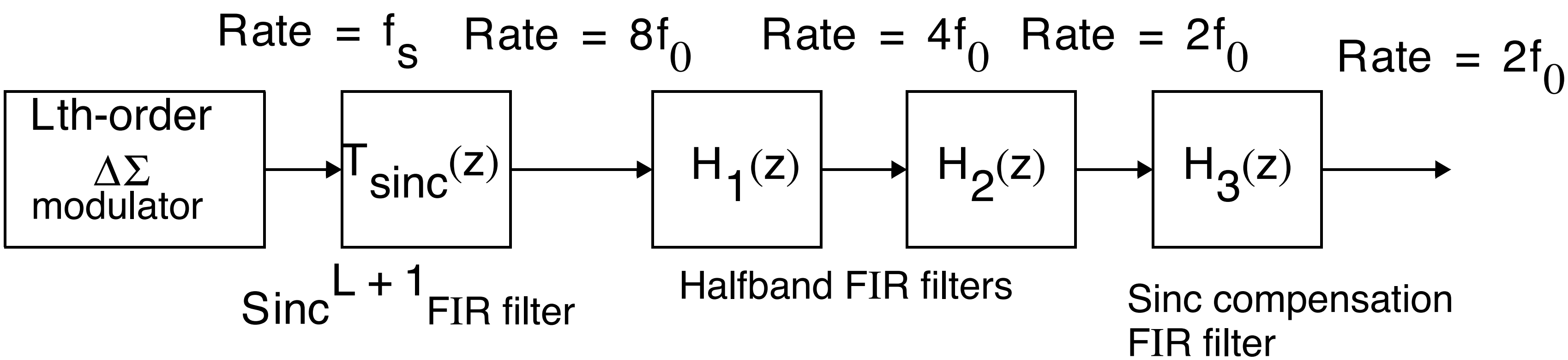


Architecture of Delta-Sigma D/A Converters



Multi-Stage Digital Decimation





- Sinc filter removes much of quantization noise
- Following filter(s) — anti-aliasing filter and noise

Sinc Filter

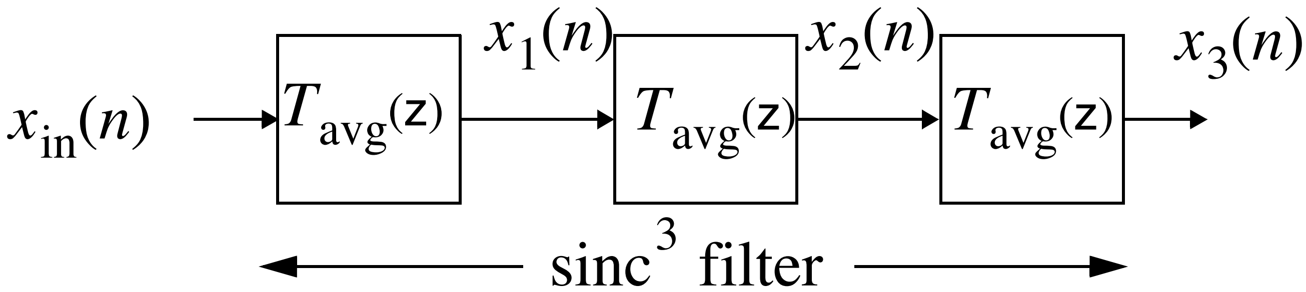
- $\mathrm{sinc}^{\mathrm{L+1}}$ is a cascade of $\mathrm{L+1}$ averaging filters

$$T_{avg}(z) = \frac{Y(z)}{U(z)} = \frac{1}{M}\sum_{i=0}^{M-1} z^{-i} \tag{21}$$

- M is integer ratio of $f_s/(8f_0)$
- It is a linear-phase filter (symmetric coefficients)
- If M is power of 2, easy division (shift left)
- Can not do all decimation filtering here since not sharp enough cutoff

Sinc Filter

- Consider $x_{in}(n) = \{1, 1, -1, 1, 1, -1, \ldots\}$ applied to $M = 4$ averaging filters in cascade



- $x_1(n) = \{0.5, 0.5, 0.0, 0.5, 0.5, 0.0, \ldots\}$
- $x_2(n) = \{0.38, 0.38, 0.25, 0.38, 0.38, 0.25, \ldots\}$
- $x_3(n) = \{0.34, 0.34, 0.31, 0.34, 0.34, 0.31, \ldots\}$
- Converging to sequence of all 1/3 as expected

Sinc Filter Response

- Can rewrite averaging filter in recursive form as

$$T_{avg}(z) = \frac{Y(z)}{U(z)} = \frac{1}{M}\left(\frac{1 - z^{-M}}{1 - z^{-1}}\right) \tag{22}$$

and a cascade of $L + 1$ averaging filters results in

$$T_{\mathrm{sinc}}(z) = \frac{1}{M^{L+1}}\left(\frac{1 - z^{-M}}{1 - z^{-1}}\right)^{L+1} \tag{23}$$

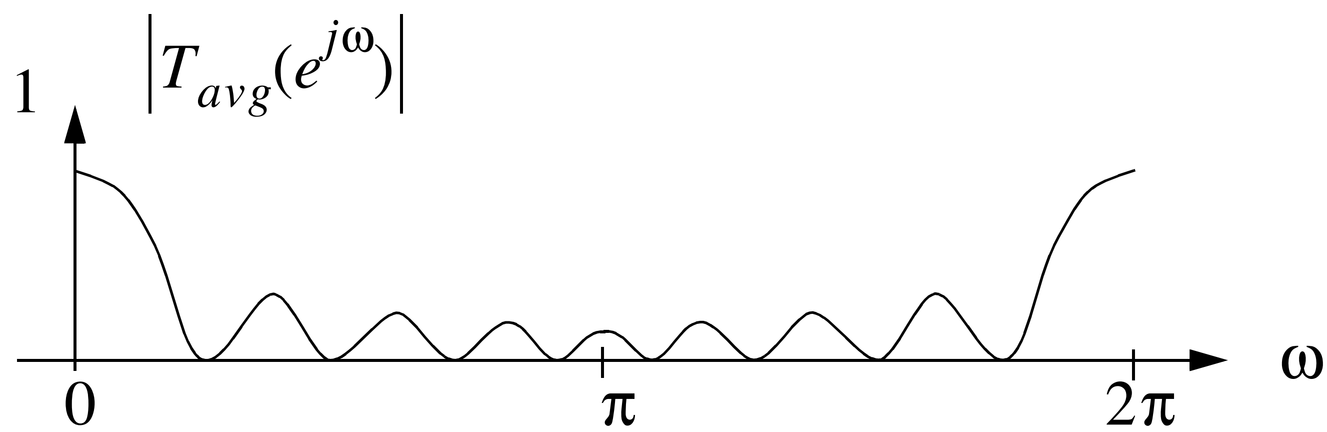
- Use $L + 1$ cascade to roll off quantization noise faster than it rises in L'th order modulator

Sinc Filter Frequency Response

- Let $z = e^{j\omega}$

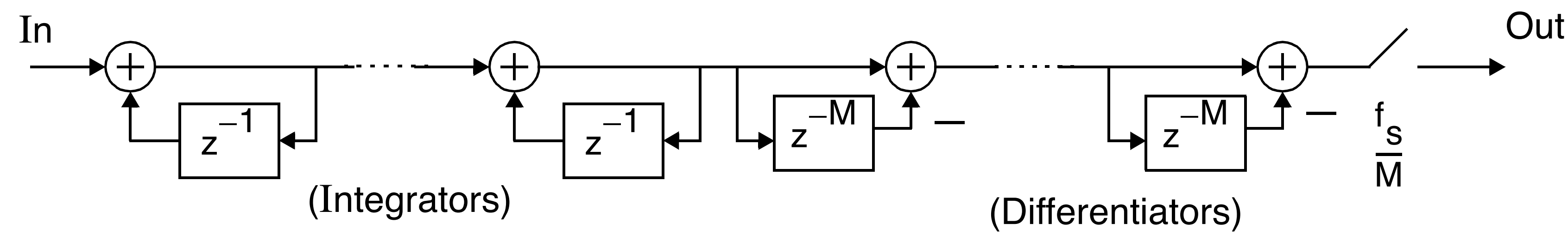
$$T_{avg}(e^{j\omega}) = \frac{\mathrm{sinc}\left(\frac{\omega M}{2}\right)}{\mathrm{sinc}\left(\frac{\omega}{2}\right)} \tag{24}$$

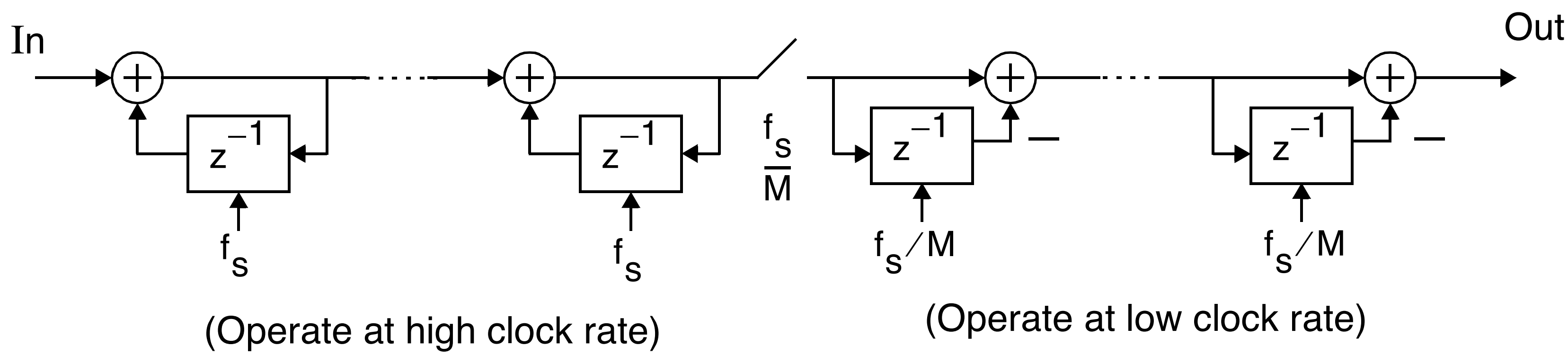
where $\mathrm{sinc}(x) \equiv \sin(x)/x$



Sinc Implementation

$$T_{\mathrm{sinc}}(z) = \left(\frac{1}{1-z^{-1}}\right)^{L+1}(1-z^{M})^{L+1}\frac{1}{M^{L+1}} \tag{25}$$

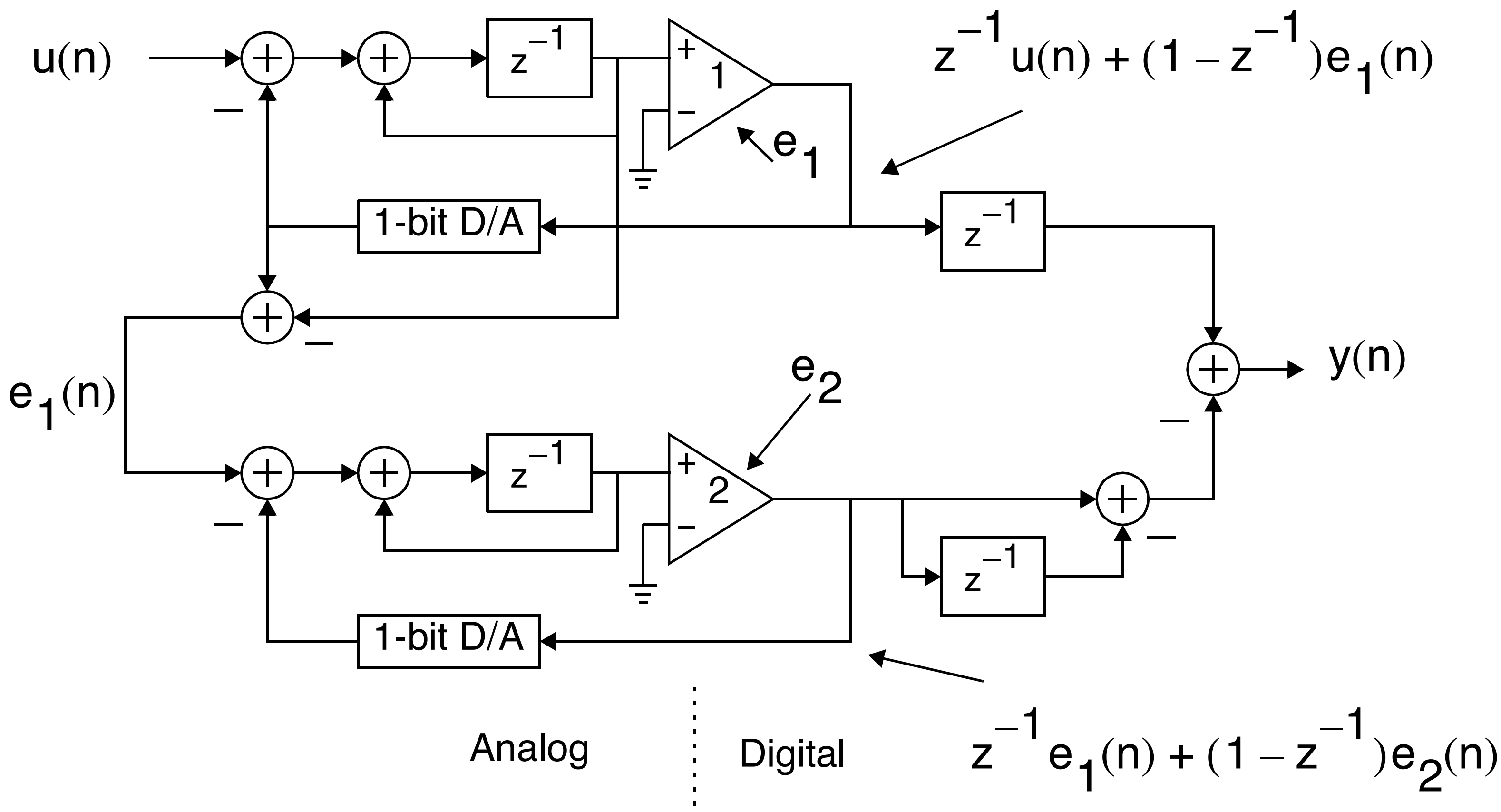




- If 2's complement arithmetic used, wrap-around okay since followed by differentiators

MASH Architecture

- Multi-stAge noise SHaping - MASH
- Use multiple lower order modulators and combine outputs to cancel noise of first stages



MASH Architecture

- Output found to be:

$$Y(z) = z^{-2}U(z) - (1 - z^{-1})^2 E_2(z) \tag{26}$$

Multibit Output

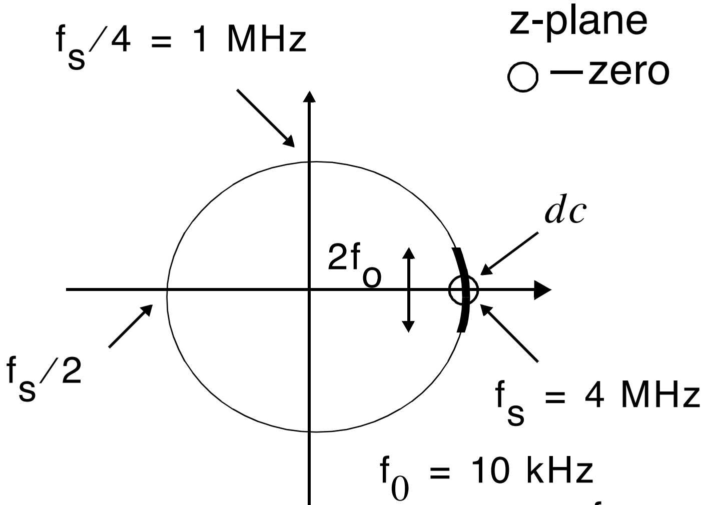
- Output is a 4-level signal though only single-bit D/A's
  — if D/A application, then linear 4-level D/A needed
  — if A/D, slightly more complex decimation

A/D Application

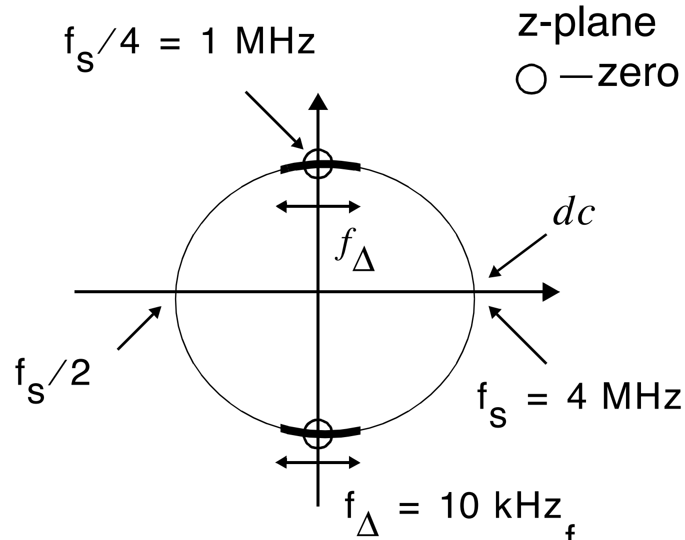
- Mismatch between analog and digital can cause first-order noise, e_1, to leak through to output
- Choose first stage as higher-order (say 2'nd order)

Bandpass Oversampling Converters

- Choose $H(z)$ to have high gain near freq f_c
- NTF shapes quantization noise to be small near f_c
- OSR is ratio of sampling-rate to twice bandwidth — not related to center frequency

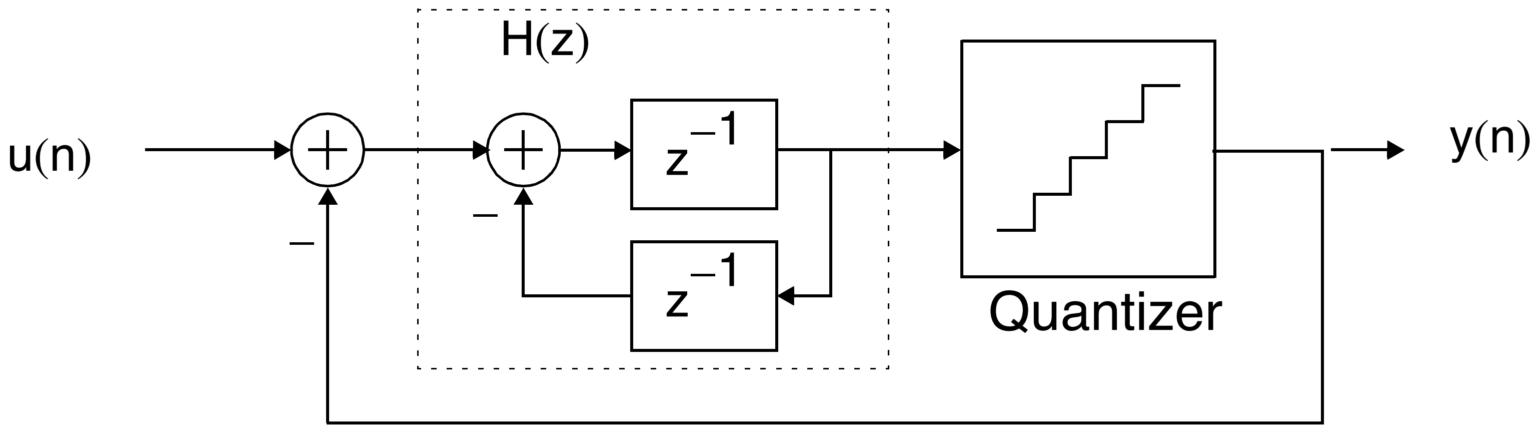


Lowpass $OSR = \frac{f_s}{2f_0} = 200$



Bandpass $OSR = \frac{f_s}{2f_\Delta} = 200$

Bandpass Oversampling Converters



- Above $H(z)$ has poles at $\pm j$ (which are zeros of NTF)
  - $H(z)$ is a resonator with infinite gain at $f_s/4$
  - $H(z) = z/(z^2+1)$
- Note one zero at +j and one zero at -j
  - similar to lowpass first-order modulator
  - only 9 dB/octave
- For 15 dB/octave, need 4’th order BP modulator

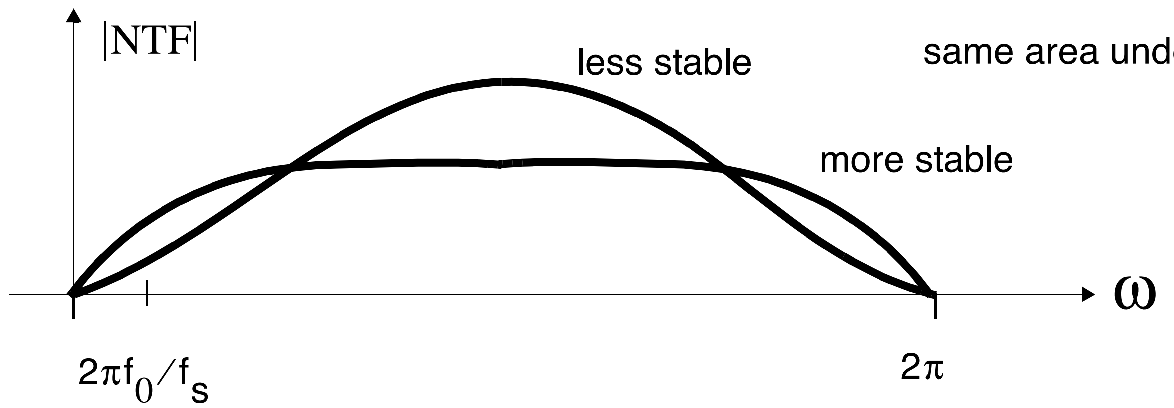
Modulator Stability

- Since feedback involved, stability is an issue
- Considered stable if quantizer input does not ***overload*** quantizer
- Non-trivial to analyze due to quantizer nonlinearity
- There are rigorous tests to guarantee stability but they are too conservative
- For a 1-bit quantizer, heuristic test is:

$$\left|N_{TF}(e^{j\omega})\right| \le 1.5 \qquad \text{for } 0 \le \omega \le \pi \tag{27}$$

- Peak of NTF should be less than 1.5

Modulator Stability



Stability Detection

- Might look at input to quantizer
- Might look for long strings of 1s or 0s at comp output

When instability detected ...

- reset integrators
- Damp some integrators to improve stable

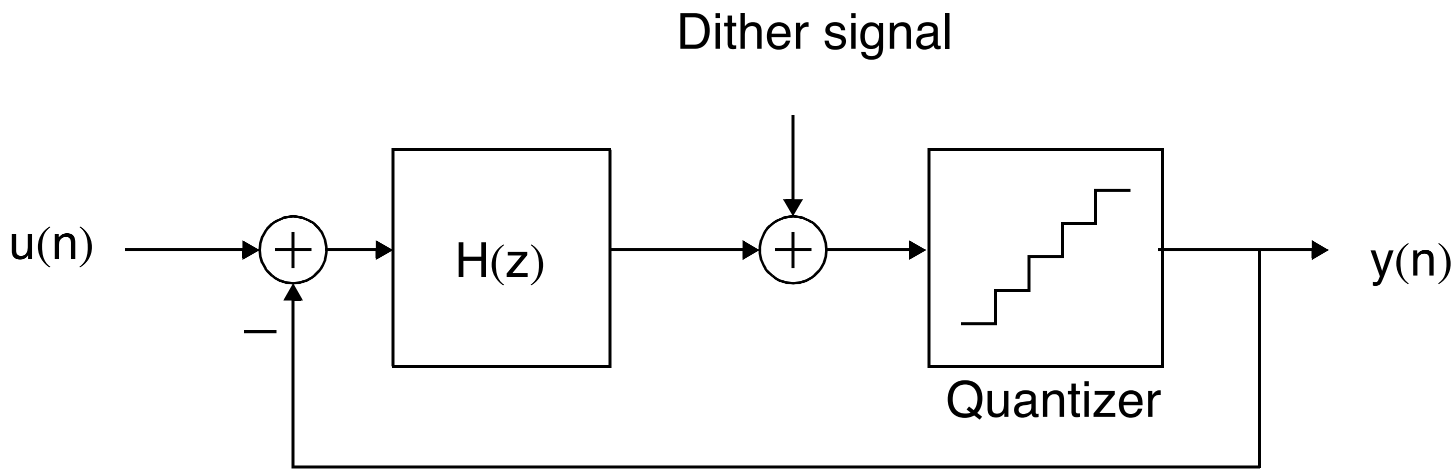
Idle Tones

- 1/3 into 1'st order modulator results in output

$$y(n) = \{1, 1, -1, 1, 1, -1, 1, 1, \ldots\} \quad (28)$$

- Fortunately, tone is out-of-band at $f_s / 3$
- $(1/3 + 1/24) = 3/8$ into modulator has tone at $f_s / 16$
- Similar examples can cause tones in band-of-interest and are not filtered out — say $f_s / 256$
- Also true for higher-order modulators
- Tones might not lie at single frequency but be short term periodic patterns.

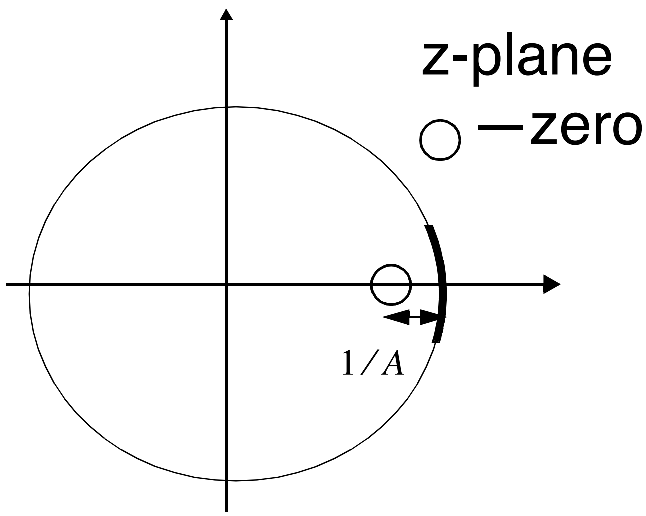
Dithering

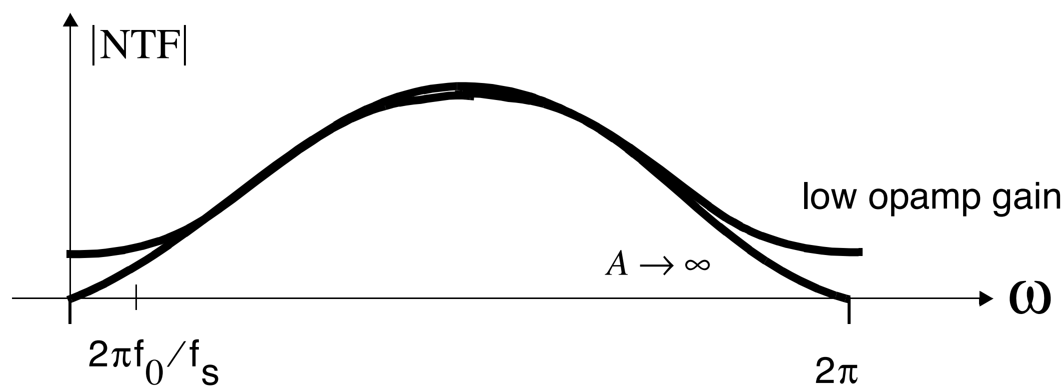


- Add pseudo-random signal into modulator to break up idle tones (not just mask them)
- If added before quantizer, it is noise shaped and large dither can be added.
  - A/D: few bit D/A converter needed
  - D/A: a few bit adder needed
- Might affect modulator stability

Opamp Gain

- Finite opamp gain, A, moves pole at $z = 1$ left by $1/A$





- Flattens out noise at low frequency
— only 3 dB/octave for high OSR
- Typically, require

$$A > OSR/\pi \tag{29}$$

Multi-bit Oversampled Converters

- A multi-bit DAC has many advantages
  - more stable - higher peak $|\mathrm{NTF}|$
  - higher input range
  - less quantization noise introduced
  - less idle tones (perhaps no dithering needed)
- Need highly linear multi-bit D/A converters

Example

- A 4-bit DAC has 18 dB less quantization noise, up to 12 dB higher input range — perhaps 30 dB improved SNR over 1-bit