MIDTERM #2, MATH 300

Wednesday, March 22, 2006

Student No: ____________________ Name (Print): ____

1. (12 marks) Answer true or false to the following statements by putting either true or false in the boxes. Give valid reasons for all your answers.

   (a) If $f(z)$ is analytic on a simple closed smooth curve C then $\oint_C f(z)dz = 0$.

   (b) The function $f(z) = ze^{1/z}$ has a pole at $z = 0$.

   (c) The power series $\sum_{n=0}^{\infty}(-1)^n \frac{z^n}{(2n)!}$ converges to the function $\cos\sqrt{z}$ for all z.

   (d) If the power series $f(z) = \sum_{n=0}^{\infty} a_n z^n$ converges for $z = 2 + \imath$ then it converges for $z = \imath$.

2. (12 marks) The following questions require little or no computation.

(a) Suppose $f(z)$ and $g(z)$ are analytic for $|z| \leq 1$ and $f(z) + g(z) = 0$ for all z such that $|z| = 1$. Show that $f(z) + g(z) = 0$ for all z such that $|z| \leq 1$.

(b) Find the Laurent series for $f(z) = \dfrac{1}{z^2(z-1)}$ valid for $|z| > 1$.

(c) Find the radius of convergence R of the power series $\displaystyle\sum_{j=0}^{\infty} \frac{z^{2j}}{3^j}$.

3. (12 marks) Compute $\displaystyle\int_C \frac{\sin \pi z}{z^2(z-2)} dz$, where C is the circle $|z| = 1$ with the positive orientation.

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4. (12 marks) Suppose $P(z) = (z - r_1)^{s_1}(z - r_2)^{s_2}$ is a polynomial with distinct roots ($r_1 \neq r_2$). Show that $\displaystyle\oint_{C_R} \frac{zP'(z)}{P(z)} dz = 2\pi\imath(r_1 s_1 + r_2 s_2)$ for all R sufficiently large, where C_R is the positively oriented circle $|z| = R$.