Family Name ____________________ First Name ______________ Student # ______________

PHYS 250 Final Exam
July 27, 2022

Please turn off your cell phones during the exam.

Write on this examination. You can write on the backs of the pages too.
If you use the backs of pages or extra pages, label which problem you are doing.

You may use 2 pages (both sides) of notes, and a calculator, including graphing/programmable.

No laptops, tablets, smartphones, or other wireless devices. No books.

1. (5 points)
The tau lepton has a mass of 1777 MeV/c^2 and a mean lifetime at rest of 0.3 picoseconds.
What total energy in MeV must a tau lepton have in order for its mean lifetime to be one nanosecond?

$$\gamma = \frac{1\times10^{-9}}{0.3\times10^{-12}} = 3333$$

$$E = \gamma mc^2 = 3333\cdot1777 \text{ MeV} = 5.923\times10^{6} \text{ MeV}$$

2. (5 points)
A spaceship departs Earth for Alpha Centauri at 99% of the speed of light. It transmits data at 10 GHz in its own frame while in transit. What frequency should Earth tune its receiver to?

$$f_{Earth} = f_{ship}\cdot\sqrt{\frac{1-\beta}{1+\beta}} = 10 \text{ GHz}\cdot\sqrt{\frac{1-0.99}{1+0.99}}$$

$$= 10 \text{ GHz}\cdot 7.089\times10^{-2} = 7.089\times10^{8} \text{ Hz} = 708.9 \text{ MHz}$$

3. (5 points)
The cathode of an X-ray tube is at –50 kiloVolts relative to the anode.
What is the shortest X-ray wavelength produced?

$$E_{photon} = E_{electron} = 50 \text{ keV} = hf$$

$$\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{1240 \text{ eV-nm}}{50\times10^{3} \text{ eV}} = 2.480\times10^{-2} \text{ nm} = 24.80 \text{ pm}$$

4. What is the energy of a photon from the n = 4 to n = 1 transition of a single electron orbiting an Oxygen nucleus (Z = 8) ?

$$E = 13.6 \text{ eV}\cdot Z^{2}\cdot\left(\frac{1}{n_{final}^{2}} - \frac{1}{n_{initial}^{2}}\right) = 13.6 \text{ eV}\cdot 8^{2}\cdot\left(\frac{1}{1^{2}} - \frac{1}{4^{2}}\right) = 816.0 \text{ eV}$$

5. (5 points) Bohr model
A. Write two correct predictions of the Bohr model

It predicts the correct energy levels for Hydrogen
It predicts that all ground-state Hydrogen atoms are the same size
It predicts the correct size scale for Hydrogen

B. Write two incorrect predictions of the Bohr model

It predicts that Hydrogen atoms are flat
It predicts that the lowest angular momentum is 1 rather than 0

6. An electron has kinetic energy of 20 eV in a region with zero potential

A. What is its wavelength λ in nanometers?

$$E = \frac{p^2}{2m} \rightarrow p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2mc^2}} \cdot \frac{1}{\sqrt{E}} = \frac{1240 \text{ eV-nm}}{\sqrt{2 \cdot 0.511 \times 10^6 \text{ eV}}} \cdot \frac{1}{\sqrt{E}}$$

$$= \frac{1.227 \text{ nm-}\sqrt{\text{eV}}}{\sqrt{E_{\text{eV}}}} = \frac{1.227}{\sqrt{20}} = 0.2743 \text{ nm}$$

B. What is the frequency f in Hertz of the electron's wavefunction?

$$E = hf \rightarrow f = \frac{E}{h} = \frac{20 \text{ eV} \cdot 1.602 \times 10^{-19} \text{ J/eV}}{6.626 \times 10^{-34} \text{ J-s}} = 4.835 \times 10^{15} \text{ Hz}$$

C. Write the time-dependent wavefunction for the electron in 1 dimension in terms of λ and f. (You don't have to write in the above numbers).

$$\psi(x,t) = \exp\left[2\pi i\left(\frac{x}{\lambda} - ft\right)\right]$$

7. (5 points)
What are the boundary conditions at a finite potential step ?

The wavefunction value must be continuous at the step: $\psi_- = \psi_+$

The wavefunction first derivative must be continuous at the step: $\left.\frac{\partial\psi}{\partial x}\right|_- = \left.\frac{\partial\psi}{\partial x}\right|_+$

9. (10 points)
Harmonic oscillator

A. Write the 1-dimensional time-independent Schrodinger Equation
for a particle with mass *m* in the potential corresponding to a spring-constant *k*.

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+\frac{1}{2}kx^2\psi = E\psi$$

B. What is the energy of the ground state, and the first excited state, in terms of *k* and *m*.

$$E_0 = \left(0+\frac{1}{2}\right)\hbar\omega = \frac{1}{2}\hbar\sqrt{\frac{k}{m}} \qquad E_1 = \left(1+\frac{1}{2}\right)\hbar\omega = \frac{3}{2}\hbar\sqrt{\frac{k}{m}}$$

C. Write the ground-state wavefunction in terms of *k* and *m*.
Don't worry about normalization.

$$\psi(x) = \exp\left(\frac{-x^2}{2b^2}\right) \qquad b^2 = \frac{\hbar}{\sqrt{km}} \;\rightarrow\; \psi(x) = \exp\left(\frac{-x^2}{2}\frac{\sqrt{km}}{\hbar}\right)$$

10. (10 points)
An electron is in box that is 1 nm in x, 1 nm in y, and 2 nm in z, with potential zero inside and infinite outside.

A. Write the formula to calculate the energy in eV from the quantum numbers n_x, n_y, and n_z.

$$E = \frac{(\hbar\pi)^2}{2m}\left[\left(\frac{n_x}{1\text{ nm}}\right)^2 + \left(\frac{n_y}{1\text{ nm}}\right)^2 + \left(\frac{n_2}{2\text{ nm}}\right)^2\right]$$

$$= \frac{(\hbar c\pi)^2}{2mc^2}\left[\left(\frac{n_x}{1\text{ nm}}\right)^2 + \left(\frac{n_y}{1\text{ nm}}\right)^2 + \left(\frac{n_2}{2\text{ nm}}\right)^2\right]$$

$$= \frac{\pi^2(197.3\text{ eV-nm})^2}{2\cdot 0.511\times 10^6\text{eV}}\left[\left(\frac{n_x}{1\text{ nm}}\right)^2 + \left(\frac{n_y}{1\text{ nm}}\right)^2 + \left(\frac{n_2}{2\text{ nm}}\right)^2\right]$$

$$= 0.3759\text{ eV}\left[n_x^2 + n_y^2 + \frac{n_z^2}{4}\right]$$

B. Fill in the following table with energies in eV

nx	ny	nz	Energy	n2sum
1	1	1	0.8458	2.25
1	1	2	1.128	3
1	1	3	1.598	4.25
1	1	4	2.255	6
2	1	1	1.973	5.25
2	1	2	2.255	6
2	1	3	2.725	7.25
2	1	4	3.383	9

11. (10 points)
Spherical Schrodinger Equation

A. Write the time-independent Schrodinger Equation in spherical coordinates with a potential $V(r)$.

$$E\psi = \frac{-\hbar^2}{2m}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial\psi}{\partial r}\right]+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial\psi}{\partial\theta}\right]+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}\right\}+V(r)\psi$$

B. Write the equation that must be solved to determine the energies.

$$\frac{-\hbar^2}{2m}\frac{\partial^2 U(r)}{\partial r^2}+V(r)\cdot U(r)+\frac{\hbar^2\ell(\ell+1)}{2mr^2}=EU(r)$$

C. Write the general form of the solution. Explain any functions and indices and what values they can take.

$$\psi(r,\theta,\phi)=\frac{U_{k\ell}(r)}{r}Y_\ell^m(\theta,\phi)$$

$U_{k\ell}(r)$ is a solution to part B

$Y_\ell^m(\theta,\phi)$ are the spherical harmonics

k is radial excitation index, integers starting with 0 or 1

ℓ is angular momentum, integers starting with 0

m is the other index of the spherical harmonic, $-\ell \le m \le +\ell$

12. (5 points)

What are ℓ and m for the un-normalized Spherical Harmonic $\sin^5\theta\cdot\left(13\cos^2\theta-1\right)\cdot\exp(5i\phi)$ and how do you know?

$\ell = 7$ from power of sine plus highest power of cosine

$m = +5$ from ϕ exponential

13. (5 points)

What are n, ℓ, and m for this un-normalized Hydrogen wavefunction, and how do you know?

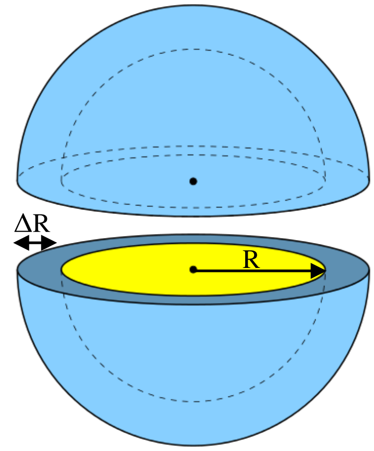
$$\left(\frac{r}{a_0}\right)^3\exp\left(\frac{-r}{4a_0}\right)\cdot\left(5\cos^3\theta-3\cos\theta\right)$$

n = 4 from the radial exponential denominator
$\ell = 3$ from the highest power of cosine plus sine
m = 0 due to lack of ϕ dependence

14. (5 points)
Fill in the table with the <u>numbers</u> of Hydrogen states with the given n and ℓ. Some cells will be blank.

n=5	1	3	5	7	9
n=4	1	3	5	7	
n=3	1	3	5		
n=2	1	3			
n=1	1				
	L=0	L=1	L=2	L=3	L=4

16. (10 points)
A particle with mass m is in a spherically symmetric shell potential that is infinite for $r < R$, and for $r > R+\Delta R$, and zero for $R < r < R+\Delta R$.



A. What are the boundary conditions on the wavefunction?

zero at R and R+ΔR

B. What is the wavefunction for the ground state? Don't worry about normalization.

$$\psi(r) = \frac{1}{r}\cdot\sin\frac{\pi(r-R)}{\Delta R}$$

C. What is the energy of the ground state?

$$E_1 = \frac{(\hbar\pi)^2}{2m(\Delta R)^2}$$

D. What is the energy of the first radial excitation, with $\ell = 0$?

$$E_2 = 4\cdot\frac{(\hbar\pi)^2}{2m(\Delta R)^2}$$

E. If $\Delta R \ll R$, what can you say qualitatively about the energy of the first angular excitation with $\ell = 1$ compared to the energy of the first radial excitation with with $\ell = 0$?
Hint: approximate the centrifugal term as being independent of r.

The centrifugal term is $\dfrac{\hbar^2\ell(\ell+1)}{2m\left(R+\frac{\Delta R}{2}\right)^2}$

which because $R \gg \Delta R$ is much smaller than the difference between the ground state and the first radial excitation. So the first angular excitation is only slightly higher than the ground state and much lower than first radial excitation.