PHYS 250 Midterm Solutions
June 5, 2024

1. A starship approaches your starship with $\gamma = 2.0$. (No, γ isn't the Star Trek warp-factor). The approaching starship fires at you a beam of photons that have energy of 10×10^6 electron-Volts per photon.

A. What is the velocity in your frame of the approaching starship in meters/second ? (8 points) (Don't worry about the sign).

$$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/2^2} = 0.8660 \quad v = \beta c = 0.8660 \cdot 2.998 \times 10^8 = 2.596 \times 10^8 \text{ m/s}$$

B. What is the velocity in your frame of the approaching photons in meters/second? (6 points) (Don't worry about the sign).

Photons always move at the speed of light, so $v = 2.998 \times 10^8$ m/s

C. What is the momentum of each photon, in the frame of the approaching starship ? (8 points) (Easier in units based on electron-Volts and speed-of-light c. Don't worry about the sign.)

$E^2 = (pc)^2 + (mc^2)^2$ and photons have $m = 0$, so $E = pc \rightarrow p = \dfrac{E}{c} = 10$ MeV/c

D. Is the photon energy in <u>your</u> frame higher, lower, or equal to 10 MeV ? (6 points)

The starship is approaching, so the photons are blue-shifted to shorter wavelength, so their energy is higher. Or, you could do the calculation below.

E. What is the photon energy in <u>your</u> frame? (8 points) (Easier in units based on electron-Volts and speed-of-light c.)

The Lorentz transform is $\dfrac{E'}{c} = \gamma \cdot \left(\dfrac{E}{c} + \beta p \right)$. The sign is + because the starship is approaching.

Use $E = pc, \gamma = 2, \ \beta = 0.8660$ to get $E' = 2 \cdot (E + 0.8660 \cdot E) = 2 \cdot 1.8660 \cdot 10 \text{ MeV} = 37.32 \text{ MeV}$

2. The visible light spectrum is considered to be $(400-700)\times10^{-9}$ m.
   What is the range of angles in degrees for diffraction of visible light through two slits that are 1.6×10^{-6} m apart? (Assume n = 1, check your calculator mode). (8 points)

For two-slit diffraction, $d\sin\theta = n\lambda \rightarrow \theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$.

Then $\theta_{\text{violet}} = \sin^{-1}\left(\frac{1\cdot 400\times10^{-9}\,m}{1.6\times10^{-6}\text{ m}}\right) = \sin^{-1}0.25 = 14.48°$

and $\theta_{\text{red}} = \sin^{-1}\left(\frac{1\cdot 700\times10^{-9}\,m}{1.6\times10^{-6}\text{ m}}\right) = \sin^{-1}0.4375 = 25.94°$

3. The maximum legal power for a laser pointer is 5 milliWatts.
   If the wavelength is $\lambda = 638$ nanometers $= 639\times10^{-9}$ m,
   how many photons per second are emitted? (8 points)

$$E = hf = \frac{hc}{\lambda} = \frac{1240\text{ eV-nm}}{638\text{ nm}} = 1.944\text{ eV}$$

$$P = 5\times10^{-3}\text{ J/s}\cdot\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}} = 3.121\times10^{16}\text{ eV/s}$$

$$\frac{\text{photons}}{\text{second}} = \frac{3.121\times10^{16}\text{ eV/s}}{1.944\text{ eV}} = 1.606\times10^{16}$$

4. What is the maximum light wavelength to produce photoelectrons from Uranium (work function = 3.6 eV) ? (8 points)

$$\lambda_{\max} = \frac{hc}{E_{\min}} = \frac{1240\text{ eV-nm}}{3.6\text{ eV}} = 344.4\text{ nm}$$

5. What is the minimum voltage for an X-ray tube to produce photons with wavelength of $\lambda = 25$ picometers $= 25\times10^{-12}$ m ? (8 points)

$$E_{\min} = \frac{hc}{\lambda} = \frac{1240\text{ eV-nm}}{25\times10^{-3}\text{nm}} = 4.960\times10^{4}\text{ eV} \rightarrow V_{\min} = 49.60\text{ kV}$$

6. He^{+1} has a single electron orbiting a $Z = 2$ nucleus. What is the wavelength of a photon from the $n = 4$ to $n = 3$ transition of the electron? (8 points)

$$\frac{1}{\lambda} = R \cdot Z^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{1}{91.13 \text{ nm}} \cdot 2^2 \cdot \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = 2.134 \times 10^{-3} \rightarrow \lambda = 486.7 \text{ nm}$$

7. In the Bohr model, what is the energy of the $n = 2$ orbit for the single electron in He^{+1} ? (8 points)

$$E = -E_{\text{Bohr}} \cdot \frac{Z^2}{n^2} = -13.6 \text{ eV} \cdot \frac{2^2}{2^2} = -13.6 \text{ eV}$$

8. A test sample is illuminated by high-energy X-rays. An X-ray spectrometer that is shielded from the incident X-rays registers a prominent (K_α) X-ray signal at 6375 eV. What is the atomic number Z of the test sample? (8 points)

$$E_{K\alpha} = E_{\text{Bohr}} \cdot \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \cdot (Z-1)^2 = 10.2 \text{ eV} \cdot (Z-1)^2 \rightarrow Z = \sqrt{\frac{E_{K\alpha}}{10.2 \text{ eV}}} + 1 = \sqrt{\frac{6375 \text{ eV}}{10.2 \text{ eV}}} + 1 = 26 \,.$$

($Z = 26$ is iron).

9. What is the wavelength of an electron with kinetic energy of $100 \text{ keV} = 10^5 \text{ eV}$? (8 points)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{hc}{\sqrt{2mc^2 E}} = \frac{1.227 \sqrt{\text{eV}}\text{-nm}}{\sqrt{10^5 \text{ eV}}} = 3.880 \times 10^{-3} \text{ nm} = 3.88 \text{ pm}$$