1. Lorentz Transformations

   Event A occurs at $t = 0$ ns and at $x = 0$ m, $y = 0$ m, $z = 0$ m in frame S.
   Event B occurs at $t = 10$ ns and at $x = 7$ m, $y = 6$ m, $z = 5$ m in frame S.

   Frame S' is moving at velocity 2.5×10^{8} m/s <u>in the +z direction</u>,
   with the usual convention that the axes are parallel and the xyz origins overlap at $t = t' = 0$

   A. What are t' in ns and x', y', and z' in meters for event A in the S' frame?

With the usual convention, $t' = 0$ ns, $x' = y' = z' = 0$ m.

   B. What are t' in ns and x', y', and z' in meters for event B in the S' frame?

Since the velocity is in the z direction, the x and y coordinates don't change:
$x' = x = 7\text{ m},\quad y' = y = 6\text{ m}$

But the t' and z' coordinates get transformed, so we need

$$\beta = \frac{v}{c} = \frac{2.5\times10^{8}\text{ m/s}}{2.998\times10^{8}\text{ m/s}} = 0.8339 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = 1.812\,.$$

The transformations are

$$ct' = \gamma\left(ct - \beta z\right) = 1.812\cdot\left(2.998\times10^{8}\cdot10\times10^{-9} - 0.8339\cdot5\right)$$

$$= 1.812\cdot\left(2.998 - 4.170\right) = -2.124\text{ m, so}$$

$$t' = \frac{ct'}{c} = \frac{-2.124\text{ m}}{2.998\times10^{8}\text{ m/s}} = -7.084\times10^{-9}\text{ s} = -7.084\text{ ns}$$

$$z' = \gamma\left(z - \beta ct\right) = 1.812\cdot\left(5 - 0.8339\cdot2.998\times10^{8}\cdot10\times10^{-9}\right)$$

$$= 1.812\cdot\left(5 - 2.500\right) = 4.530\text{ m}$$

   C. In the S' frame, did event A happen before event B?

No, event B occurs before event A in the S' frame.
Events A and B are separated far enough in space that the time-order is frame-dependent.

2. Relativistic Particles

Particle A is a photon with energy 250 MeV, moving in the $+z$ direction.
Particle B has rest mass 100 MeV/c² and momentum 300 MeV/c in the $-y$ direction.

A. What are the components of the 4-momentum of particle A ?

For a photon, $E = pc$, so $p = \dfrac{E}{c}$.

$$\underset{\rightarrow}{A} = \left(\frac{E}{c},\, p_x,\, p_y,\, p_z\right) = \left(\frac{250\text{ MeV}}{c},\, 0,\, 0,\, \frac{250\text{ MeV}}{c}\right) = \left(250,\, 0,\, 0,\, 250\right)\text{ MeV/c}$$

B. What are the components of the 4-momentum of particle B ?

$$E = \sqrt{\left(pc\right)^2 + \left(mc^2\right)^2} = \sqrt{\left(300\text{ MeV/c}\cdot\text{c}\right)^2 + \left(100\text{ MeV/c}^2\cdot\text{c}^2\right)^2} = 316.2\text{ MeV}$$

$$\underset{\rightarrow}{B} = \left(\frac{316.2\text{ MeV}}{c},\, 0,\, -300\text{ MeV/c},\, 0\right) = \left(316.2,\, 0,\, -300,\, 0\right)\text{ MeV/c}$$

C. If A collides with B, what is the total centre of mass energy ?

$$\left(\frac{E_{CM}}{c}\right)^2 = \left(\underset{\rightarrow}{A} + \underset{\rightarrow}{B}\right)^2 \text{ so } E_{CM} = \sqrt{c^2\cdot\left(\underset{\rightarrow}{A} + \underset{\rightarrow}{B}\right)^2}$$

$$\underset{\rightarrow}{A} + \underset{\rightarrow}{B} = \left(\left[250 + 316.2\right],\, 0,\, -300,\, 250\right) = \left(566.2,\, 0,\, -300,\, 250\right)\text{ MeV/c}$$

$$\left(\underset{\rightarrow}{A} + \underset{\rightarrow}{B}\right)^2 = \frac{E}{c}^2 - \vec{p}^2 = 566.2^2 - \left[0^2 + \left(-300\right)^2 + 250^2\right] = 1.681\times10^5\ \left(\text{MeV/c}\right)^2$$

Alternatively, $\left(\underset{\rightarrow}{A} + \underset{\rightarrow}{B}\right)^2 = \underset{\rightarrow}{A}^2 + 2\underset{\rightarrow}{A}\cdot\underset{\rightarrow}{B} + \underset{\rightarrow}{B}^2 = m_A^2 + 2\underset{\rightarrow}{A}\cdot\underset{\rightarrow}{B} + m_B^2$

$m_A^2 = 0 \quad m_B^2 = 100^2 = 1.000\times10^4$

$$\underset{\rightarrow}{A}\cdot\underset{\rightarrow}{B} = \left(250,\, 0,\, 0,\, 250\right)\cdot\left(316.2,\, 0,\, -300,\, 0\right) = 250\cdot316.2 - \left[0\cdot0 + 0\cdot\left(-300\right) + 250\cdot0\right] = 7.905\times10^4$$

$$\left(\underset{\rightarrow}{A} + \underset{\rightarrow}{B}\right)^2 = 0 + 2\cdot7.905\times10^4 + 1.000\times10^4 = 1.681\times10^5$$

Either way, $E_{CM} = \sqrt{c^2\cdot1.681\times10^5\ \left(\text{MeV/c}\right)^2} = 410.0\text{ MeV}$

3. Photoelectric Effect

It is observed that applying –2.1 Volts to the collector relative to the photocathode completely stops the current produced by 250 nm wavelength light on a photocathode.

What is the work function of the material?

$$qV_{\text{stop}} = \frac{hc}{\lambda} - q\phi_{\text{work}} \rightarrow \phi_{\text{work}} = \frac{1}{q}\frac{hc}{\lambda} - V_{\text{stop}}$$

If we use eV units, $hc = 1240$ eV-nm and $\frac{1}{q}$ converts eV into Volts.

The convention is that V_{stop} is a positive number.

$$\phi_{\text{work}} = \frac{1}{e}\frac{1240 \text{ eV-nm}}{250 \text{ nm}} - 2.1 \text{ V} = 4.960 \text{ V} - 2.1 \text{ V} = 2.860 \text{ V}$$

4. X-rays

What is the minimum incidence angle for Bragg scattering of X-rays from a 15 kV tube from planes of a crystal with plane spacing of 150 picometers?

$$E = 15 \text{ keV} = 15 \times 10^3 \text{ eV}$$

Planck: $E = hf = \dfrac{hc}{\lambda} \rightarrow \lambda = \dfrac{hc}{E} = \dfrac{1240 \text{ eV-nm}}{15 \times 10^3 \text{eV}} = 8.267 \times 10^{-2} \text{nm} = 82.67 \text{ pm}$

Bragg: $2d \sin\theta = n\lambda \rightarrow \sin\theta = \dfrac{n\lambda}{2d}$ minimum is $n = 1$

$$d = 150 \text{ pm} = 0.150 \text{ nm}$$

$$\sin\theta = \frac{hc}{2dE} = \frac{1240 \text{ eV-nm}}{2 \cdot 0.150 \text{ nm} \cdot 15 \times 10^3 \text{ eV}} = 0.2756$$

$$\theta = 0.2792 \text{ radians} = 16.00°$$

1. When an electron is knocked out of the innermost shell of an Iron atom ($Z = 26$), and an electron from the next higher shell drops into that state, what is the energy in eV of the photon that is produced?

Moseley's Law for such a transition is $E = 13.6\text{ eV} \cdot (Z-1)^2 \cdot \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

For $Z = 26$, this is $E = 13.6\text{ eV} \cdot (26-1)^2 \cdot \left(1 - \frac{1}{4}\right) = 6375\text{ eV}$

2. What is the Bohr Model radius in picometers (1 pm = 10^{-12} m) for one electron in the $n = 3$ orbit around a bare Carbon nucleus ($Z = 6$)

The Bohr Model radius is $r = 52.97\text{ pm} \cdot \frac{n^2}{Z}$.

For the stated case, $r = 52.97\text{ pm} \cdot \frac{3^2}{6} = 79.46\text{ pm}$