# PHYS 250 Midterm 1B

Answer all 4 questions (equal weight). Show your work for partial credit.
You can submit photographs of written pages, print and write on this file and submit a photograph, annotate this file, create answers in some other file(s), etc.

1. Frame S' is moving in frame S with $\beta = \sqrt{3}/2$, and the origins of the frames coincide at t = t' = 0. The following events occur in frame S'

| Event | ct' (meters) | x' (meters) |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 2 | 1 | 0 |
| 3 | 1 | 1 |

A. (2) Which events are simultaneous in S'?

Simultaneous means at the same time regardless of position.
So it's events (0 & 1), and (2 & 3).

B. (2) Which events are at the same point in S' ?

"Same point" means same x-coordinate regardless of time.
So it's events (0 & 2) and (1 & 3).

D. (7) What are the space-time coordinates of the same events in the S frame?

The transformations are

$$ct = \gamma\left(ct' + \beta x'\right) \quad x = \gamma\left(x' + \beta ct'\right)$$

$$\beta = \sqrt{3}/2 = 0.8660$$

$$\gamma = \left[1-\beta^2\right]^{-1/2} = \left[1-\left(\sqrt{3}/2\right)^2\right]^{-1/2} = 2.0$$

| Event | ct (meters) | x (meters) |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 1.732 | 2.000 |
| 2 | 2.000 | 1.732 |
| 3 | 3.732 | 3.732 |

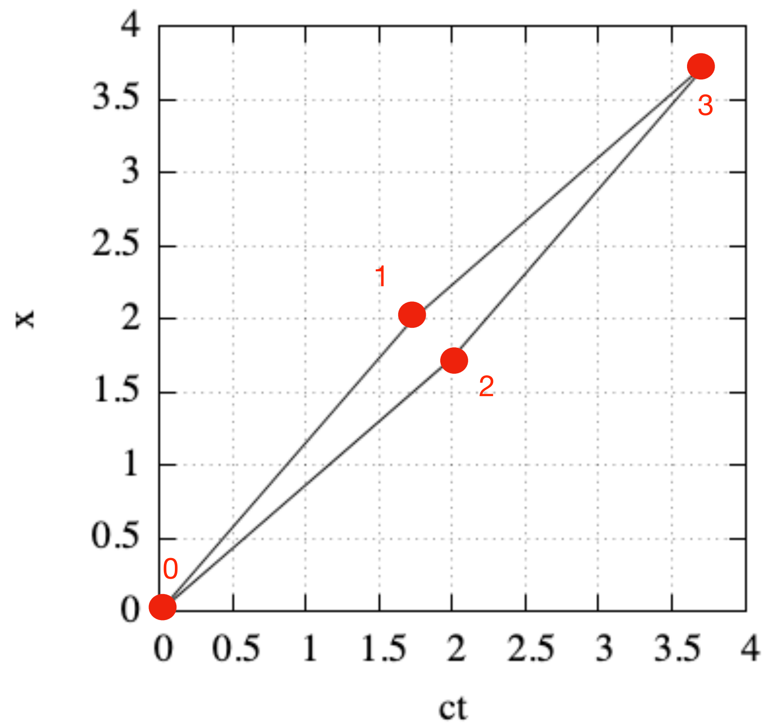
E. (2) Which events are simultaneous in S?

None.

F. (2) Which events are at the same point in S ?

None

H. (6) Draw a roughly-to-scale space-time diagram of the events in the S frame.
Put x on the vertical axis, ct on the horizontal axis, and put ticks at 1 meter on each.
Label the events as 0, 1, 2, and 3.



2. (25) A three-stage starship is launched on a mission to a star that is 10 light-years away from Earth. The first stage accelerates to v = 0.7 c relative to Earth. The second stage accelerates to v = 0.8 c relative to the first stage. The third stage accelerates to v = 0.9 c relative to the second stage. What is the duration of the trip the star to an astronaut in the third stage? Neglect the "burn" time of the stages (and the fact that the acceleration would turn her into jelly).

This is successive velocity addition.

The second stage velocity relative to Earth is $v_2 = \dfrac{v' + u}{1 + \dfrac{v'u}{c^2}} = \dfrac{0.8c + 0.7c}{1 + \dfrac{0.8c \cdot 0.7c}{c^2}} = 0.9615c$ .

The third stage velocity relative to Earth is $v_3 = \dfrac{v' + u}{1 + \dfrac{v'u}{c^2}} = \dfrac{0.9c + 0.9615c}{1 + \dfrac{0.9c \cdot 0.9615c}{c^2}} = 0.9979c$

The trip duration observed from Earth is 10 / 0.9979 = 10.02 years

The gamma factor on the ship is $\gamma = \left[1 - \left(v_3/c\right)^2\right]^{-1/2} = \left[1 - 0.9979^2\right]^{-1/2} = 15.58$

So the duration for the astronaut is 10.02 / 15.58 = 0.6432 years.
The distance to the star is contracted to 0.6418 light years in the ship frame.

3. (25) A particle with a rest mass 1.5 GeV/c² has a half-life of 5 nanoseconds in its rest frame. If the particle is accelerated so it has a momentum of p = 5 GeV/c in the lab frame, what is its half-life in the lab frame?

Momentum is $p = \beta\gamma mc$ so $\beta\gamma = pc/mc^2 = 5/1.5 = 3.333$ .

$$\gamma^2 = \frac{1}{1-\beta^2} \to \gamma^2 - \left(\beta\gamma\right)^2 = 1 \to \gamma = \sqrt{1 + \left(\beta\gamma\right)^2} \text{ so } \gamma = \sqrt{1 + 3.333^2} = 3.480,\ \beta = 0.9578$$

So the half-life in the lab frame is $5.0 \cdot 3.480 = 17.40$ ns.

4. Particle X decays into Particle A with rest mass 200 MeV/c²

and momentum $\vec{p}_A = (100,\ -200,\ 300)$ MeV/c , and Particle B with rest mass 300 MeV/c²

and momentum $\vec{p}_B = (200,\ 300,\ 400)$ MeV/c

A. What is the momentum vector of Particle X ?

Just add the momentum 3-vectors: $\vec{p}_X = (300,\ 100,\ 700)$ MeV/c magnitude 768.1 MeV/c

B. What is the energy of Particle X ?

Find the energy of A and B, then add.

$$E_A = \sqrt{m_A^2 + p_A^2} = \sqrt{200^2 + \left(100^2 + 200^2 + 300^2\right)} = \sqrt{200^2 - 374.2^2} = 424.2641 \text{ MeV}$$

$$E_B = \sqrt{m_B^2 + p_B^2} = \sqrt{300^2 + \left(200^2 + 300^2 + 400^2\right)} = \sqrt{300^2 - 538.5^2} = 616.4414 \text{ MeV}$$

$$E_X = 424.3 + 616.4 = 1040.7055 \text{ MeV}$$

C. What is the rest mass of Particle X ?

Use the energy-mass-momentum relation again.

$$m = \sqrt{E^2 - p^2} = \sqrt{1040.7055^2 - \left(300^2 + 100^2 + 700^2\right)} = 702.1879 \text{ MeV/c}^2$$

# PHYS 250 Practice Midterm 2 Problem Solutions

1. An aluminum sphere (work function 4.1 eV) is suspended by an insulating glass fiber inside a metal vacuum chamber. Light with a wavelength of 400 nm shines on the aluminum sphere. What voltage does the aluminum sphere charge up to, including the sign?

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{400 \text{ nm}} = 3.10 \text{ eV}$$

This is less than the work function, so there are no photoelectrons, and the sphere does not charge up at all.

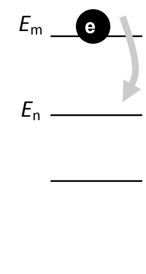
2. X-ray photons with energy of 4 keV shine on a crystal with layer spacing of 200 pm. What is the Bragg scattering angle? (Assume $n = 1$)

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV-nm}}{4000 \text{ eV}} = 0.310 \text{ nm} = 310 \text{ pm}$$

$$2d\sin\theta = n\lambda \rightarrow \sin\theta = \frac{n\lambda}{2d} = \frac{1 \cdot 310 \text{ pm}}{2 \cdot 200 \text{ pm}} = 0.7750 \rightarrow \theta = 50.81° = 0.8867 \text{ rad}$$

This is the angle between the X-ray beam and the surface of the crystal, not the normal.

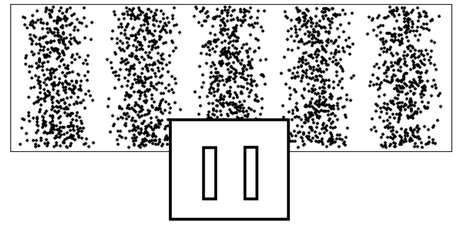
3. The diagram at right shows the electronic energy levels in an atom with an electron at energy level $E_m$. When this electron moves from energy level $E_m$ to $E_n$, light is emitted. The greater the energy difference between the electronic energy levels $E_m$ and $E_n$ …




A. …the more photons emitted.
B. …the brighter (higher intensity) the light emitted.
C. …the longer the wavelength (the more red) of the light emitted.
D. …the shorter the wavelength (the more blue) of the light emitted.
E. More than one of the above answers is correct.

The energy difference has no effect on the number of photons, or the light intensity, so A and B are not true. $E = hc/\lambda$, so greater energy difference makes the wavelength shorter not longer, so D is true and C is false.

4. You shoot a beam of photons through a pair of slits at a screen. The beam is so weak that the photons arrive at the screen one at a time, but eventually they build up an interference pattern, as shown in the picture at right. What can you say about which slit any particular photon went through?



A. Each photon went through either the left slit or the right slit. If we had a good enough detector, we could determine which one without changing the interference pattern.
B. Each photon went through either the left slit or the right slit, but it is fundamentally impossible to determine which one.
C. Each photon went through both slits. If we had a good enough detector, we could measure a photon in both places at once.
D. Each photon went through both slits. If we had a good enough detector, we could measure a photon going through one slit or the other, but this would destroy the interference pattern.
E. It is impossible to determine whether the photon went through one slit or both.

D is correct. Even if there is only a single photon in the apparatus, there is still an interference pattern. So the single photon must go through both slits and be capable of interfering with itself, and A, B, and E are incorrect. C is not correct, because if we try to observe the location of a photon, we force the wavefunction to collapse to one point or another, and we never observe it in two places at the same time. Instead, reliable detection of where the photon in will destroy the interference pattern.

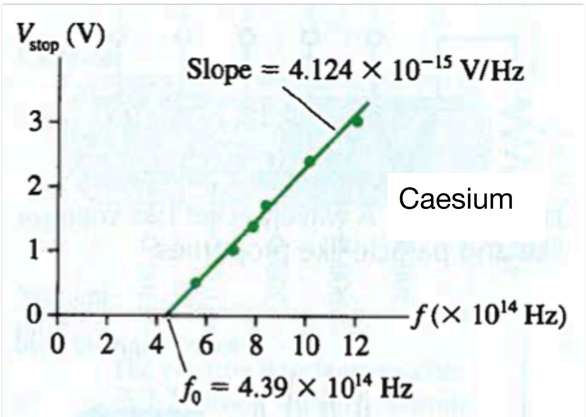
5. Metal 1 has a larger work function than metal 2. Both are illuminated with the same short-wavelength ultraviolet light. Do photoelectrons from metal 1 have a

A. higher, B. same, C. lower

speed as photoelectrons from metal 2?

Larger work function means the photoelectrons have lower maximum energy. So photoelectrons from metal 1 have lower energy, and thus lower speed, so the answer is C.

Use Milikan's photoelectric-effect data in the Figure below to determine (refers to question 6, 7 )




6. The work function, in eV, of caesium is

A. 2.8 eV B. 1.8 eV C. 2.0 eV D. 3.6 eV E. not enough information

The work function is the energy of a photon that can just barely create a photoelectron.

In this case that is at $4.39\times10^{14}$ Hz, where $E = hf = \frac{hcf}{c} = \frac{1240 \text{ eV-nm}\cdot 4.39\times10^{14} \text{ Hz}}{2.998\times10^{8}\times10^{9} \text{ nm/s}} = 1.816 \text{ eV}$

So the answer is B.

7. An experimental value of Planck's constant (using the given data)

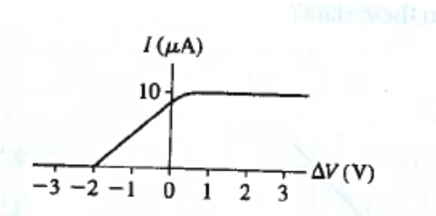
A. 5.59 x $10^{-34}$ Js B. 6.60 x $10^{-34}$ Js C. 6.61 x $10^{-34}$ Js D. 6.62 x $10^{-34}$ Js E. 6.63 x $10^{-34}$ Js

The slope should be Planck's constant, but here it is in eV-s instead of J-s.

So we multiply by $1.602\times10^{-19}$ J/eV to get $6.607\times10^{-34}$ J-s

So the answer is C.

8. How many photoelectrons are ejected per second in the experiment represented by the graph below.




A. 10 electrons, B. $10^{10}$ electrons, C. $10^{12}$ electrons, D. $10^{11}$ electrons, E. $10^{13}$ electrons

For positive bias voltage, we get $10\mu$A, which we convert into electrons per second

by dividing by the electron charge, $\dfrac{10\times10^{-6}\ \text{C/s}}{1.602\times10^{-19}\ \text{C/electron}} = 6.242\times10^{13}\ \text{electrons/s}$

So E is the closest. (The phrasing of this question isn't very good, because any of the answers would be true at SOME bias voltage).

9. A 100W lightbulb emits about 5W of visible light (the other 95W are emitted as infrared radiation, or lost as heat). The average wavelength of the visible light is about 600nm, so make that simplifying assumption that all the light has this wavelength. How many visible light photons does the bulb emit per second?

A. 10 photons, B. $10^{19}$ photons, C. $10^{18}$ photons, D. $10^{17}$ photons, E. $10^{15}$ photons

The energy of a 600 nm photon is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV-nm}}{600\ \text{nm}} = 2.067\ \text{eV}\cdot\frac{1.602\times10^{-19}\ \text{Joule}}{1\ \text{eV}} = 3.311\times10^{-19}\ \text{J}$$

$$5\ \text{W} = \frac{5\ \text{J}}{\text{s}}\cdot\frac{1\ \text{photon}}{3.311\times10^{-19}\text{J}} = 1.510\times10^{19}\ \text{photons/second}$$

So the answer is B.

# PHYS 250 Midterm 2

Answer all 4 questions (equal weight). Show your work for partial credit.
You can submit photographs of written pages, print and write on this file and submit a photograph, annotate this file, create answers in some other file(s), etc.

1. (15 points)
   An aluminum sphere (work function 4.1 eV) is suspended by an insulating glass fiber inside a metal vacuum chamber. Light from a hydrogen discharge tube shines on the aluminum sphere through a window that is transparent to all wavelengths. What voltage does the aluminum ball charge up to, including the sign?

   The hydrogen spectrum has photons with energies of up to 13.6 eV (transitions from continuum states down to the ground state. The maximum kinetic energy of a photo-electron is 13.6 – 4.1 = 9.5 eV. Electrons will leave the aluminum ball until it charges up to +9.5 Volts. At that point, even the highest energy photoelectrons will be attracted back.

2. (15 points)
   What is the diameter in the Bohr model of an atom of $Li^{+2}$ in the $n$ = 10 state?

   The Bohr model radius is $r = 52.97 \text{ pm} \cdot \frac{n^2}{Z}$, and Lithium is $Z = 3$, so the diameter is

   $$d = 52.97 \cdot \frac{10^2}{3} \cdot 2 = 3531 \text{ pm}$$

3. (15 points)
   Electrons from a source at –100 V shine on a metallic crystal with layer spacing of 200 pm. What is the Bragg scattering angle? (Assume $n$ = 1)

   Bragg's Law is $2d\sin\theta = n\lambda$. The de Broglie wavelength is $\lambda = h/p$.

   We get the momentum from the kinetic energy: $E = \frac{p^2}{2m} \rightarrow p = \sqrt{2mE}$.

   Combining: $\sin\theta = \frac{n}{2d}\lambda = \frac{n}{2d}\frac{h}{p} = \frac{n}{2d}\frac{h}{\sqrt{2mE}} = \frac{n}{2d}\frac{hc}{\sqrt{2mc^2E}}$

   $E = 100$ eV, $d = 200 \times 10^{-3}$ nm, h$c = 1240$ eV-nm and $mc^2 = 511.1 \times 10^3$ eV,

   $$\sin\theta = \frac{1}{2 \cdot 200 \times 10^{-3}} \cdot \frac{1240}{\sqrt{2 \cdot 511.1 \times 10^3 \cdot 100}} = 0.3066 \rightarrow \theta = 17.86°$$