Relativity Formulas

$c = 2.998 \times 10^8$ m/s $= 29.98$ cm/ns is the same in all frames

$u =$ velocity of the origin of frame S' as observed from frame S, normally in $+x$ direction

$$\beta = \frac{u}{c} \quad -1 \le \beta \le +1 \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \left(1-\beta^2\right)^{-1/2} \quad \gamma \ge 1$$

$$\beta = \sqrt{1-\gamma^{-2}} \quad \gamma^2 - \left(\beta\gamma\right)^2 = 1$$

$$\gamma \approx 1 + \frac{1}{2}\beta^2 \text{ for } \beta \ll 1 \qquad \beta \approx 1 - \frac{1}{2}\gamma^{-2} \text{ for } \gamma \gg 1$$

Lorentz Transformations	Inverse	Energy-Momentum
$ct' = \gamma\left(ct - \beta x\right)$	$ct = \gamma\left(ct' + \beta x'\right)$	$\frac{E'}{c} = \gamma\left(\frac{E}{c} - \beta p_X\right)$
$x' = \gamma\left(x - \beta ct\right)$	$x = \gamma\left(x' + \beta ct'\right)$	$p'_X = \gamma\left(p_X - \beta\frac{E}{c}\right)$
$y' = y \quad z' = z$		$p'_Y = p_Y \quad p'_Z = p_Z$

A moving clock ticks slower: $\Delta t_{\text{moving}} \cdot \gamma = \Delta t_{\text{yours}}$

A moving ruler looks shorter: $\Delta x_{\text{moving}} = \frac{\Delta x_{\text{yours}}}{\gamma}$

Velocity Addition: $\beta_{1+2} = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$

Doppler Effect: $f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-\beta}{1+\beta}}$ $\quad \lambda_{\text{obs}} = \lambda_{\text{source}}\sqrt{\frac{1+\beta}{1-\beta}}$ $\quad \beta > 0$ means distance is increasing

Space-Time 4-Vector: $\underset{\rightarrow}{X} = \left(ct, x, y, z\right) = \left(ct, \vec{x}\right)$

4-Vector Dot Product: $\underset{\rightarrow}{X}_1 \cdot \underset{\rightarrow}{X}_2 = ct_1 \cdot ct_2 - \vec{x}_1 \cdot \vec{x}_2$ is Lorentz-invariant

Relativistic Momentum: $p_{\text{rel}} = \gamma m v = \beta\gamma m c$

Relativistic Energy: $E_{\text{rel}} = \gamma m c^2$ $\quad$ For small β, $E \approx mc^2 + \frac{1}{2}mv^2 \approx mc^2\left(1 + \frac{1}{2}\beta^2\right)$

Energy-Momentum 4-Vector: $\underset{\rightarrow}{P} = \left(\frac{E_{\text{rel}}}{c}, \vec{p}_{\text{rel}}\right)$

$\underset{\rightarrow}{P}^2 = \frac{E_{\text{rel}}^2}{c^2} - \vec{p}_{\text{rel}}^{\,2} = \left(m_0 c\right)^2$ is Lorentz-invariant

Energy-momentum-mass relation: $E^2 = \left(pc\right)^2 + \left(m_0 c^2\right)^2 \rightarrow E^2 = p^2 + m_0^2$ in $c = 1$ units

Electron-Volt Units: 1 eV $= 1.602 \times 10^{-19}$ Joule $= q_e$ in Coulombs

$m_e = 0.511$ MeV/c^2 $\quad m_p = 938.3$ MeV/c^2 $\quad m_n = 939.6$ MeV/c^2