PHYS 250

Lecture 2.1

Photons

Today

Blackbody Radiation Spectrum Mystery

Planck's Constant Fixes It

Photoelectric Effect Mystery

Einstein Fixes It

X-Rays and Bragg Diffraction

Inverse Photoelectric Effect

Compton Scattering: Momentum of the Photon

Photons and Diffraction Patterns

Where Are We?

We're done with relativity. But we will still use $E^2 = \left(pc\right)^2 + \left(mc^2\right)^2$,
which allows particles to have $m = 0$ as long as $E = pc$.

This week is the Photons module.

Electromagnetic waves, which classically could have any amplitude,
in fact can only have certain discrete (quantized) amplitudes.

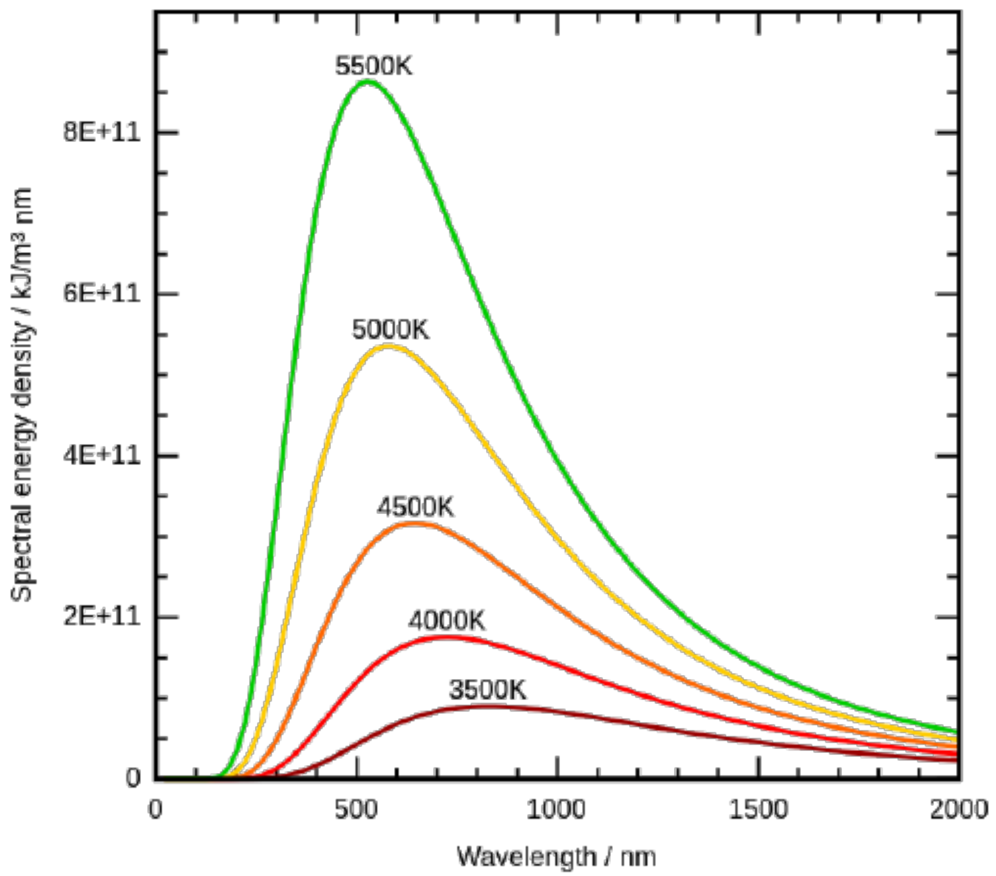
Photons have energy $E = hf$, where h is Planck's constant, and f is the frequency.

Photons also have momentum, satisfying $E = pc$, so $p = E/c = hf/c$.
Photon scattering from charged particles changes the momentum of both.

The next module is Atoms: spectral lines, Bohr model, and X-ray spectra.

Black Body Radiation

Hot objects glow red. Hotter objects glow yellow.
Very hot objects can glow blue, or even ultraviolet.

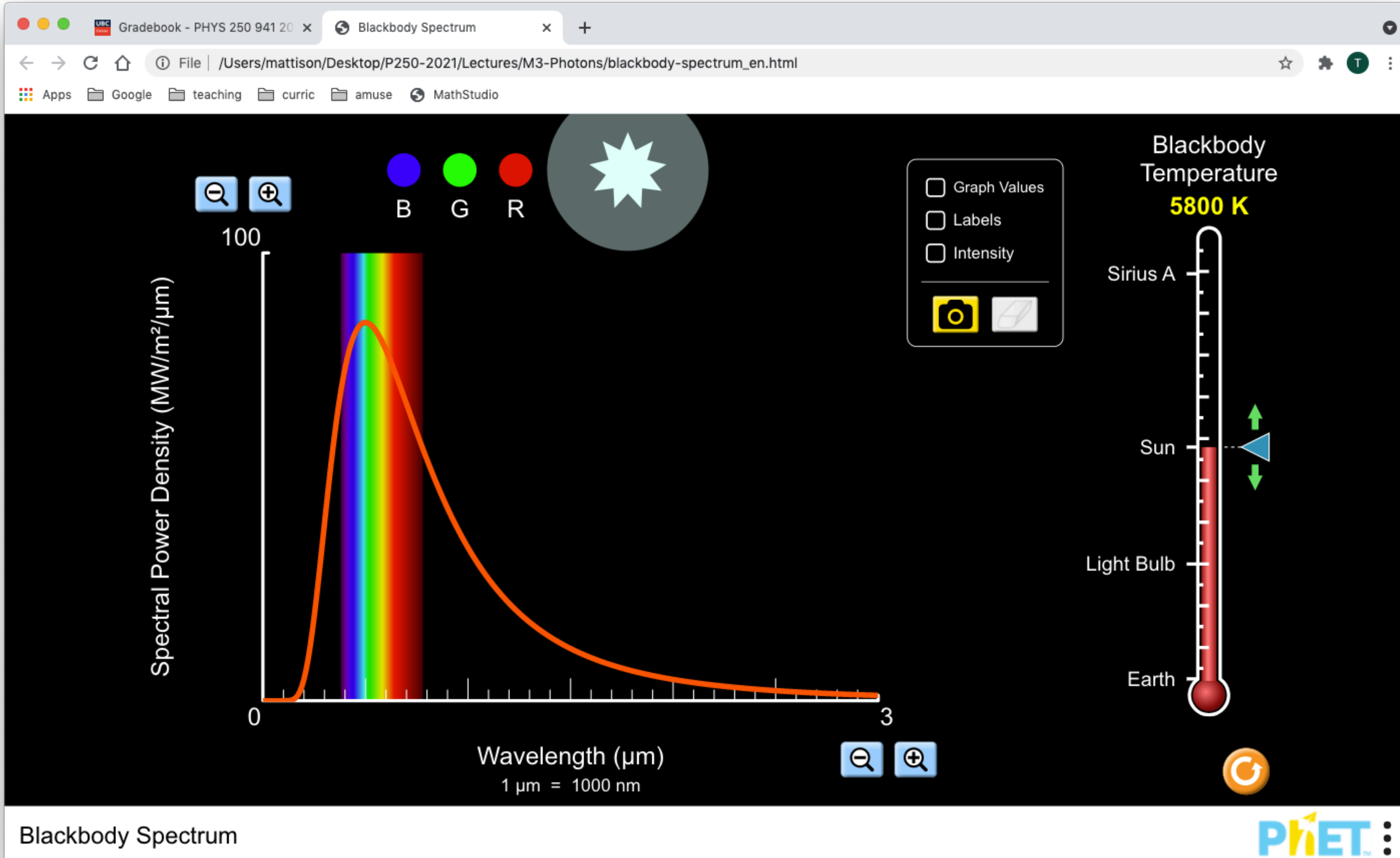


The power increases as we go from long wavelength to shorter wavelength, but there is a peak in the power, and at still shorter wavelength the power goes down.

The peak-power wavelength moves with temperature.

The integrated power is proportional to T^4 (Stephan's Law).

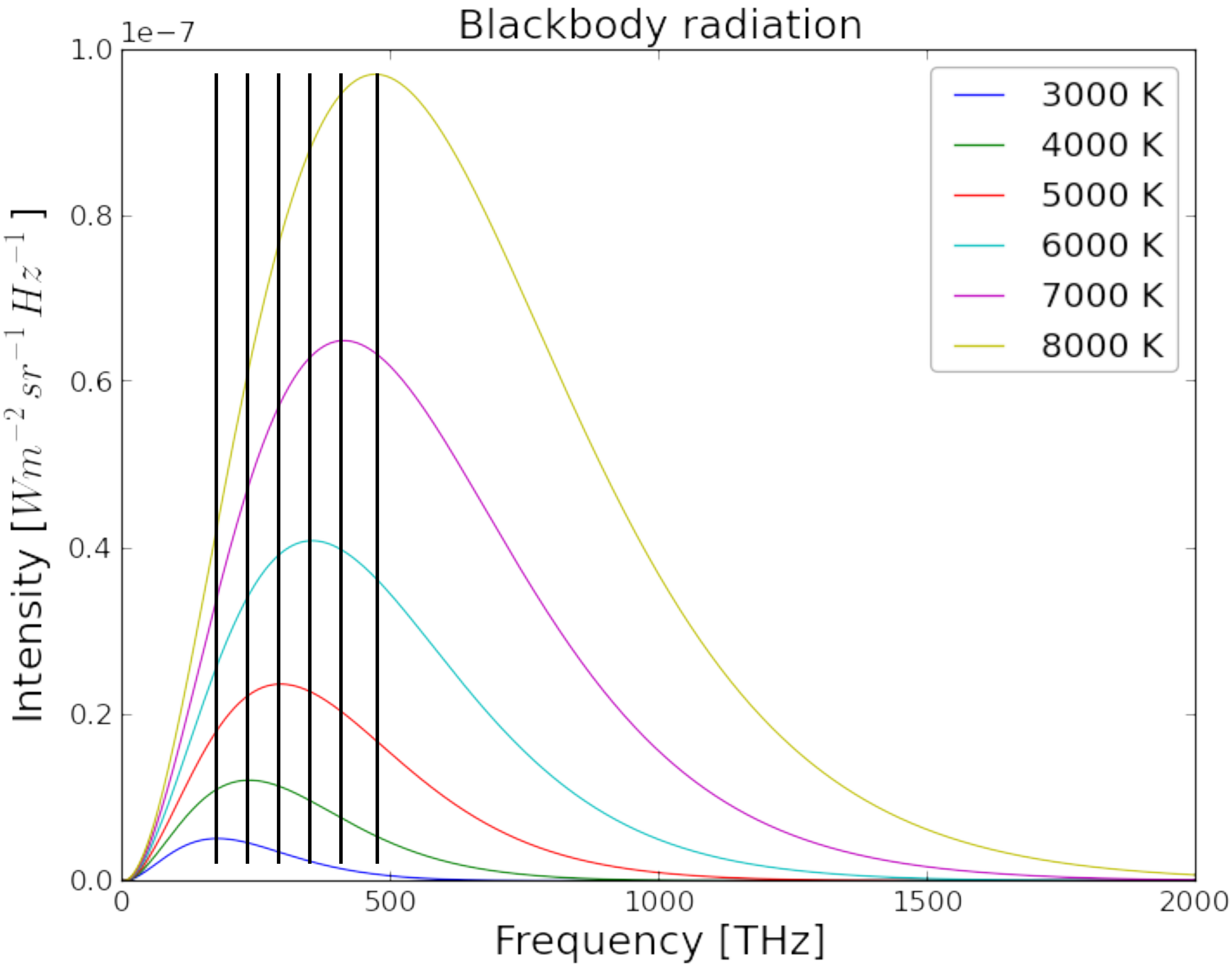
Black Body PhET



Black Body Radiation 2

This is power vs <u>frequency</u>.

The frequency of the peak is proportional to temperature.



Statistical Mechanics Essentials

We assume all possible ways of arranging the energy are equally likely.

If a continuous degree of freedom gives a quadratic energy contribution, like $1/2\,mv_x^2$, its average energy is $kT/2$.

T is the absolute temperature, with absolute zero of –273.25 C.

k is Boltzmann's constant, although he wasn't sure it would ever be measured!

The probability for a small sub-system to have energy E is proportional to $e^{-\frac{E}{kT}}$, the Boltzmann factor.

If the sub-system has more energy, there are fewer ways of arranging the remaining energy in the full system, so that's less likely.

EM Cavity Modes Essentials

Inside a conducting metal box, there can be oscillating electromagnetic fields.

There are boundary conditions: electric field component parallel to a metal surface must be zero, magnetic field normal to a metal surface must be zero.

If the box is rectangular, the allowed frequencies are $f = \frac{c}{2}\sqrt{\frac{n_x^2}{a_x^2} + \frac{n_y^2}{a_y^2} + \frac{n_z^2}{a_z^2}}$

where a_x, a_y, and a_z are the dimensions of the box, and n_x, n_y, and n_z are the number of half-cycles across each dimension, integers 1, 2, 3, etc.

The possible frequencies are fixed by the dimensions, but classically, there is no restriction on the amplitude at each frequency.

EM fields have energy density, so it is possible to calculate the energy in a given mode from the amplitude of that mode.

Rayleigh-Jeans Prediction

The EM modes form a grid in n_x, n_y, and n_z space, up to infinite n values.

Classically, each mode has average energy $kT/2$.

The number of allowed frequencies less than f is proportional to f^3,
so the number near f is proportional to $f^2\,df$.

Rayleigh and Jeans did the detailed calculation, giving $dI_{RJ}(f) = \frac{2f^2}{c^2} kT\,df$.

I like to write dI instead of I, because it's really the differential power between f and $f+df$. In an experiment, you would integrate over a range of f.

This is the power flow in Joules per second per square meter
that would come out of a small hole cut in a metal box at temperature T.

The finite temperature makes the inside of the box glow.

Changing from Frequency to Wavelength

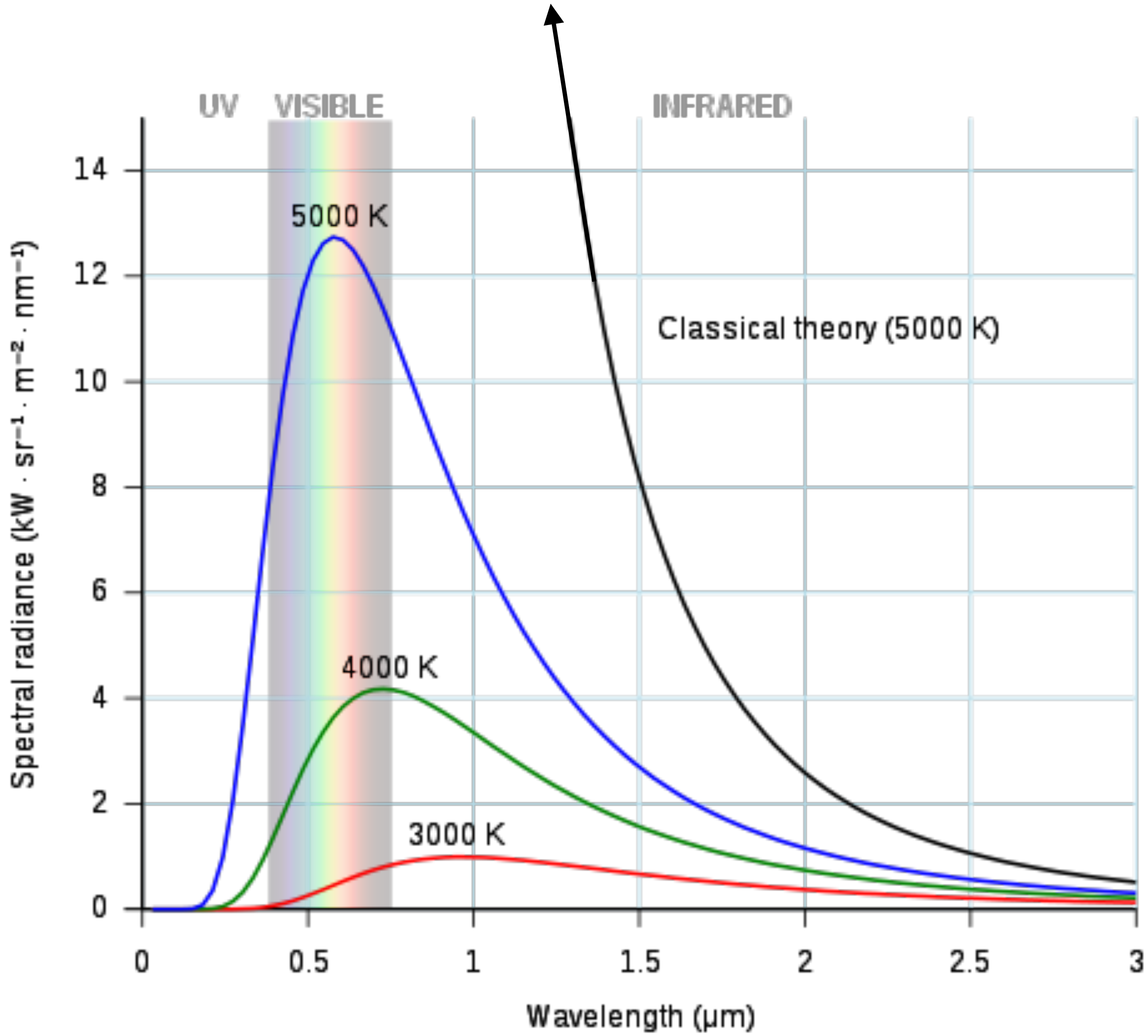
Using $f = \dfrac{c}{\lambda}$ so $df = \dfrac{c}{\lambda^2} d\lambda$ and $f^2 df = \dfrac{c^2}{\lambda^2}\dfrac{c}{\lambda^2} d\lambda$

we can write the Rayleigh-Jeans spectrum using wavelength

$$dI_{RJ}\left(\lambda\right) = \frac{2c}{\lambda^4} kT \, d\lambda$$

Ultraviolet Catastrophe

Rayleigh-Jeans predicts that thermal radiation should be much larger than seen, and go to infinity at short wavelength.   This is the Ultraviolet Catastrophe.



The statistical-mechanics arguments that work for so many other things failed badly for electromagnetic radiation.

Wein Spectrum

In 1893, Wilhelm Wein argued from thermodynamics that the shape of the black body radiation spectrum should be universal and depend only on f/T with f the frequency and T the temperature. But he didn't predict the shape.

In 1896, Wein found that the (available) data on the power vs frequency at different temperatures could be fit by a function with two parameters A and B (which he couldn't predict)

$$dI_W\left(f\right) = A\, f^3 e^{-Bf/T} df$$

The radiated power between f and $f + df$ is $dI_W(f)$.

This can also be written using wavelength

$$dI_W\left(f\right) = A\frac{c^4}{\lambda^5} e^{-Bc/\lambda T} d\lambda$$

Wein Spectrum 2

The Wein shape disagrees with Rayleigh-Jeans at low frequency, where the exponential factor is just 1:

$$dI_W\left(f\right) = A\, f^3 \cancel{e^{-Bf/T}} df \quad \text{vs} \quad dI_{RJ}\left(f\right) = \frac{2 f^2}{c^2} k_B T \, df$$

Wein goes like f^3, and Rayleigh-Jeans goes like f^2.

In 1900, new data became available at low frequency
(infrared is absorbed by normal glass, so special crystal optics was used).

The low-frequency data agreed with Rayleigh-Jeans, and not with Wein.

Planck Spectrum

Max Planck quickly found an empirical formula that fit the data everywhere:

$$dI_P(f) = \frac{A f^3}{\exp(+Bf/T)-1}df = \frac{2h}{c^2}\frac{f^3}{\exp(+hf/k_B T)-1}df$$

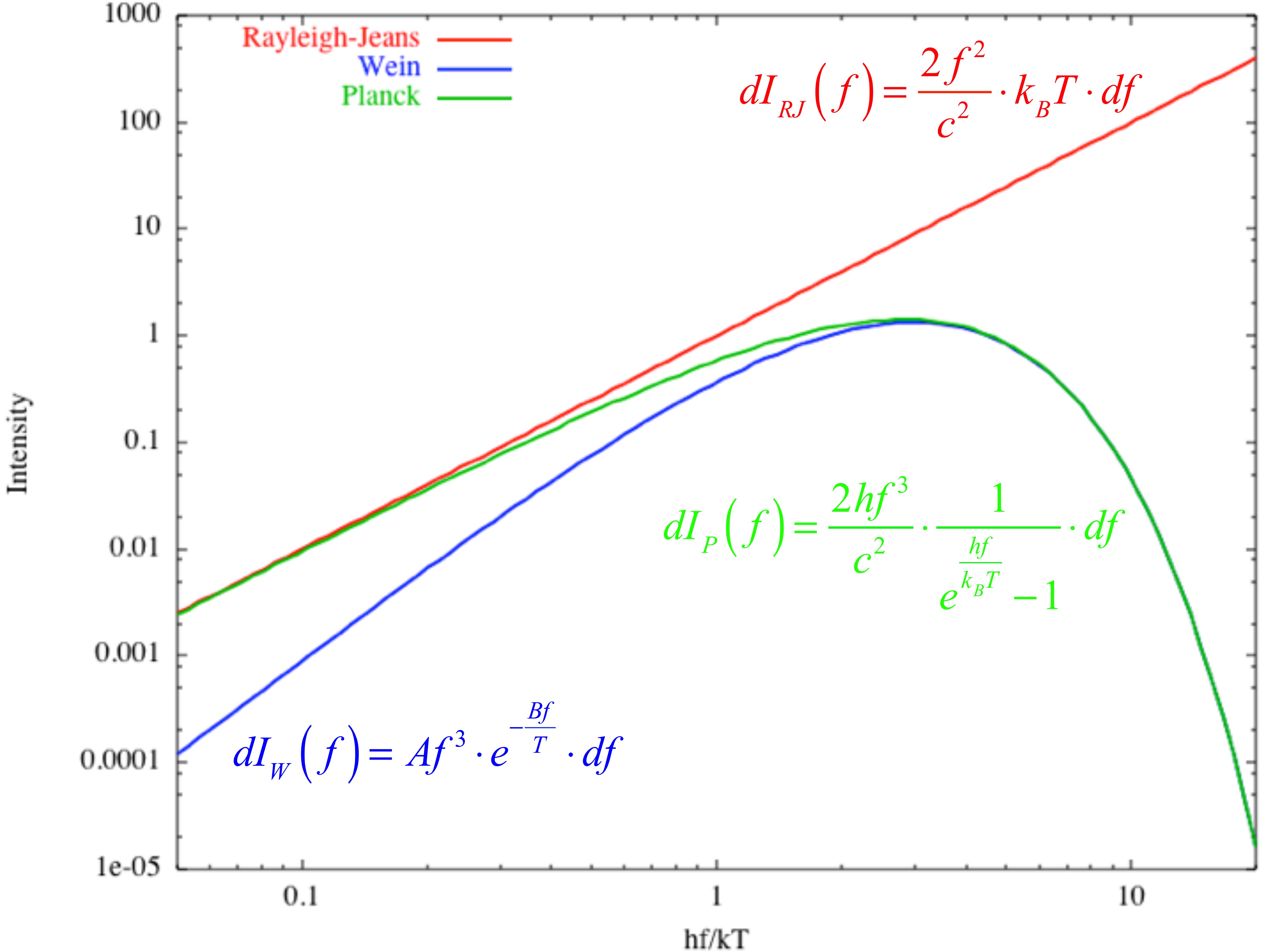
At low frequency, $\exp(+Bf/T)-1 \to \left[1+Bf/T+....\right]-1 \to Bf/T$
so one power of f gets cancelled and we get the Rayleigh-Jeans f^2-dependence.

Then the slope at low frequency of $\frac{dI_{RJ}}{df} = k_B \cdot \frac{2f^2}{c^2}T$ measures Boltzmann's k_B.

At high frequency, the exponential is big, so we neglect the "–1," and dividing by $\exp(+Bf/T)$ is the same as multiplying by $\exp(-Bf/T)$ so we get Wein back.

The exponential behavior at high frequency measures Planck's constant h.

Spectrum Comparison



Quantization of EM Energy

Planck soon found a “derivation” of his formula that used standard statistical mechanics, plus one extra assumption:

The energy of frequency f can only be integer multiples of $E = hf$.

In statistical mechanics, continuous variables have average energy $kT/2$,
but discrete variables are controlled by the Boltzmann factor $\exp\left(-\Delta E/kT\right)$.

This justifies the exponential factor in Wein that the data requires.

At low frequency, $hf << kT$, the energy steps are very small,
so it becomes the continuous case that was assumed for Rayleigh-Jeans.

Planck's Constant

The factor h is Planck's Constant: 6.626×10^{-34} Joule-seconds.

Even for a high frequency like UV light (300 nm, 10^{15} Hz),
$E = hf$ is a very small amount of energy (6.6×10^{-19} Joules).

It's convenient to use eV as our energy unit. This still makes h pretty small.

$$h = \frac{6.626\times10^{-34}\ \text{J-s}}{1.602\times10^{-19}\ \text{J/eV}} = 4.136\times10^{-15}\ \text{eV-s}$$

Using wavelength instead of frequency, $E = hf = h\dfrac{c}{\lambda} = \dfrac{hc}{\lambda}$ and

$$hc = 4.136\times10^{-15}\ \text{eV-s}\cdot 2.998\times10^{8}\ \text{m/s} = 1.240\times10^{-6}\ \text{eV-m}$$

Usually wavelengths are in microns or nanometers, so the most convenient is

$$\boxed{hc = 1.240\ \text{eV-}\mu\text{m} = 1240\ \text{eV-nm}}$$

Example

The wavelength of a red HeNe laser is 632.8 nm. What is the photon energy?

The hard way, using SI units for everything:

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\ \text{m/s}}{632.8\times10^{-9}\ \text{m}} = 4.738\times10^{16}\ \text{s}^{-1}$$

$$E = hf = 6.626\times10^{-34}\ \text{J-s}\cdot4.740\times10^{16}\ \text{s}^{-1} = 3.139\times10^{-17}\ \text{J}$$

$$E_{\text{eV}} = \frac{3.139\times10^{-17}\ \text{J}}{1.602\times10^{-19}\ \text{J/eV}} = 1.960\ \text{eV}$$

The easy way:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV-nm}}{632.8\ \text{nm}} = 1.959\ \text{eV}$$

Quantization is Weird

Classically, EM waves can have any amplitude.

Planck's model says EM waves can only have amplitudes that make their energy $E = nhf$.

But, the energy depends not only on the wave amplitude, but also on the dimensions of the box!

Somehow, the wave "knows" the dimensions of the box, so it "knows" what its possible amplitudes are!

Photoelectric Effect

Heinrich Hertz tried in 1887 to verify the waves in Maxwell’s Equations.

His transmitter was a loop of wire driven by a capacitor switched by a spark gap.

His receiver was a simple loop of wire with another short gap.

The evidence for the waves was a spark jumping across the gap
in the second loop, when the other spark gap in the first loop was fired.

The receiver worked better if the second gap saw light from the first gap.

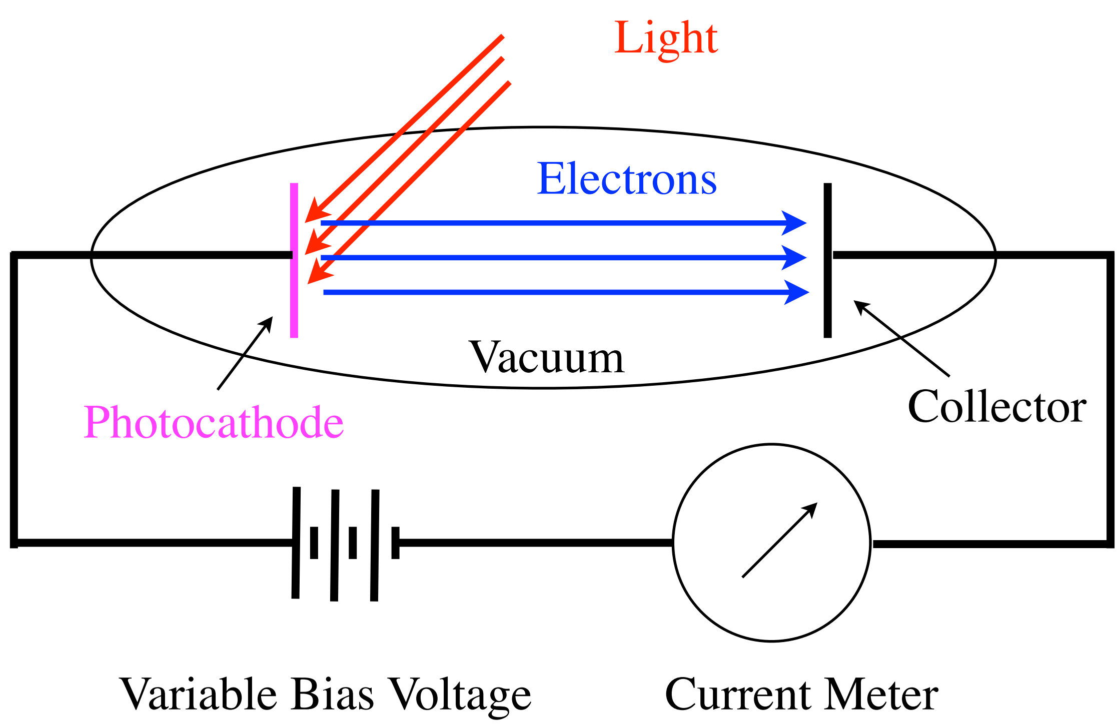
As well as verifying EM waves, Hertz had discovered the photoelectric effect.

Light from the first gap was driving electrons out of the metal of the second gap, which ionized the air in the gap and helped the spark to form.

Photoelectric Effect 2

Shining light on a metal surface (photocathode) ejects electrons.

In a vacuum they can travel to a collector, giving a current.



The current is measured with a (very sensitive) current meter.

It's possible to apply a positive or negative voltage difference between the photocathode and the collector.

Photoelectric Effect 3

Around 1890, Aleksandr Stoletov demonstrated that the current emitted when light hit a metal surface in a vacuum was proportional to the light intensity.

In 1902, Philipp Lenard tried applying positive and negative voltages to the electrode collecting the current. He found that applying a positive voltage (which attracts electrons) to the collector electrode increased the current to a plateau value.

A negative voltage (which repels electrons) decreased the current.

If the voltage was far enough negative, the current stopped completely (the stopping potential).

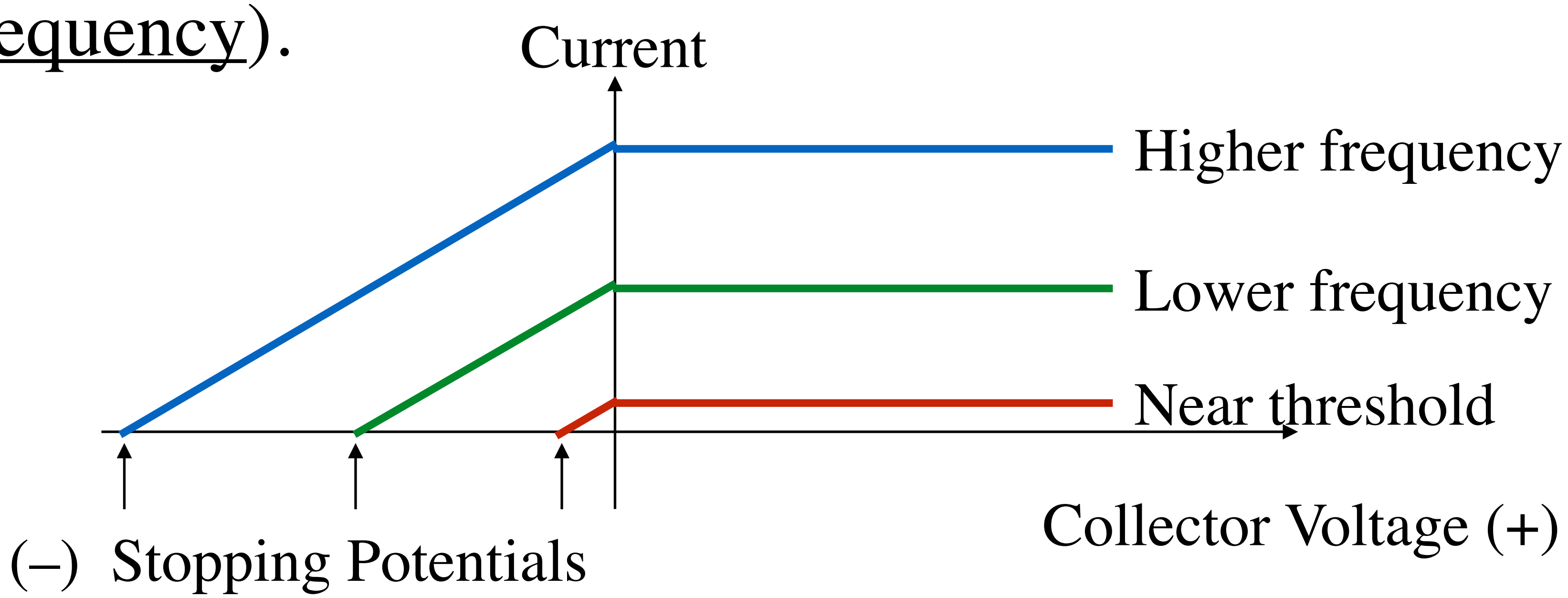
Photoelectric Effect 4

Lenard also tried changing the color of the light.

Making the light more blue (increasing the frequency)
causes the stopping potential to move away from zero (become more negative).

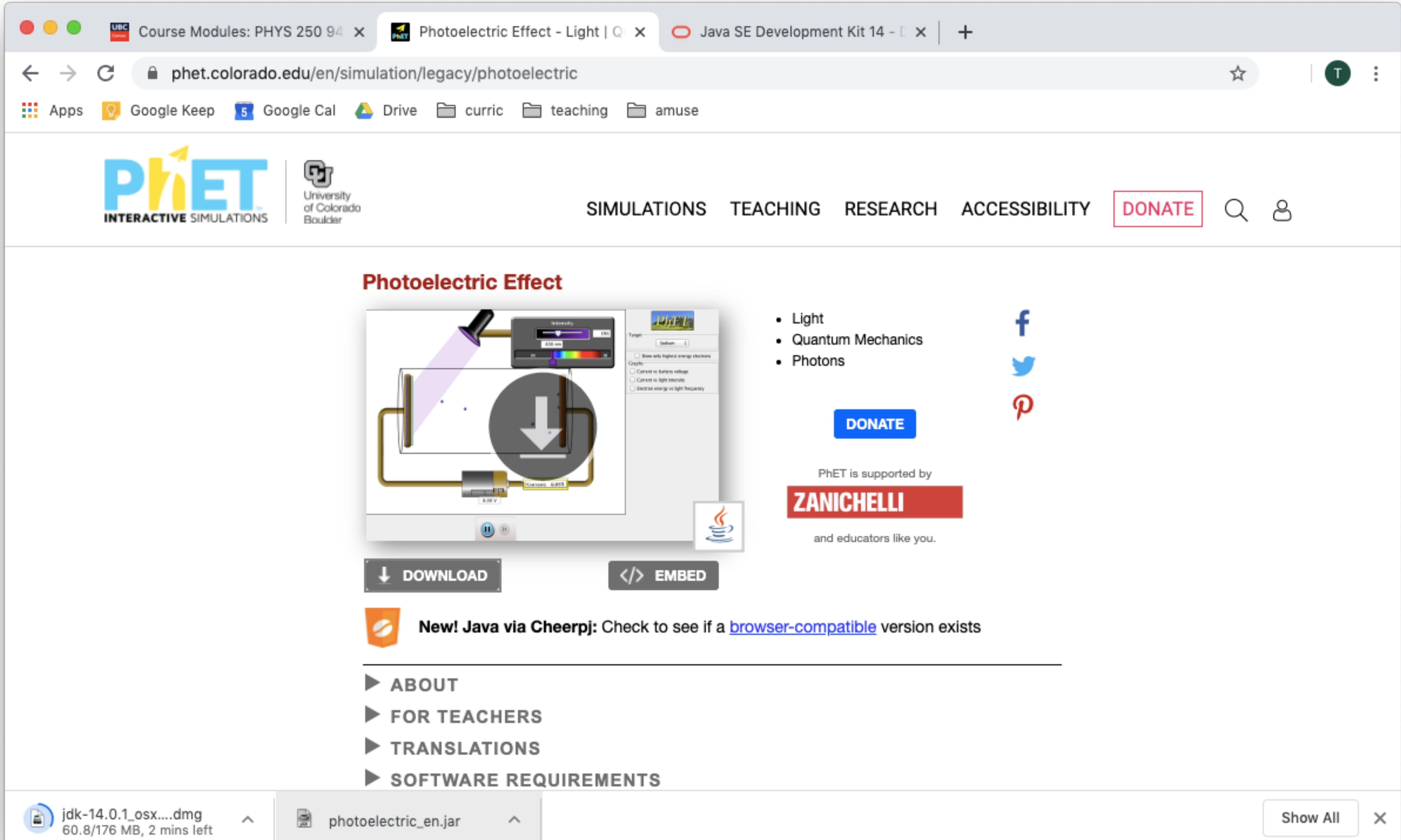
Making the light more reddish (lowering the frequency)
causes the stopping potential to move closer to zero (become less negative).

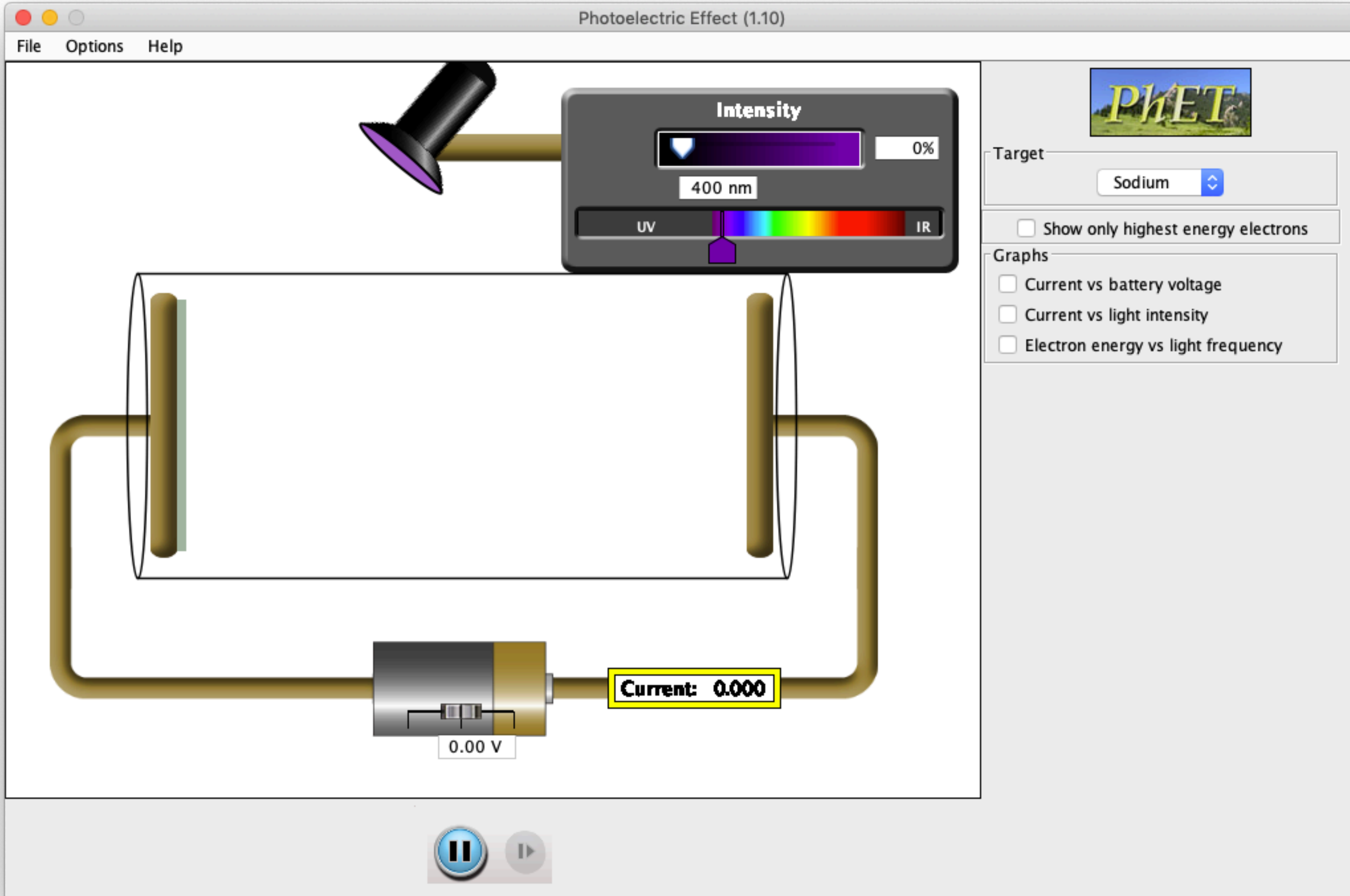
If the frequency is too low, there is no current at all for any collector voltage (threshold frequency).



Cartoon: not linear on left, or flat on the right. Intensities are unequal for clarity.

Photoelectric Effect PhET

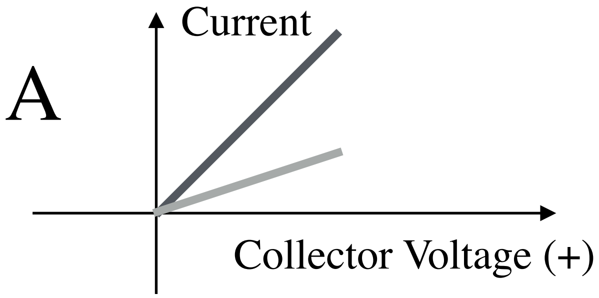



Photoelectric Effect (1.10)
File
Options
Help
Intensity
0%
400 nm
UV
IR
PhET
Target
Sodium
Show only highest energy electrons
Graphs
Current vs battery voltage
Current vs light intensity
Electron energy vs light frequency
Current: 0.000
0.00 V

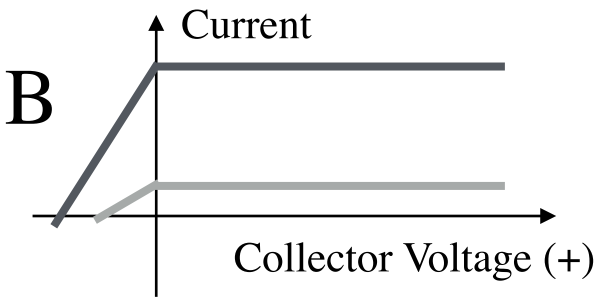
Clicker Question

Which graph shows two different intensities with the same color ?

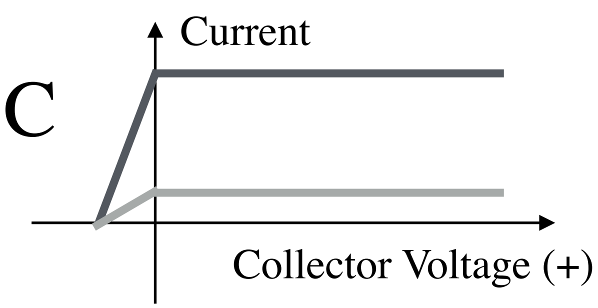
A



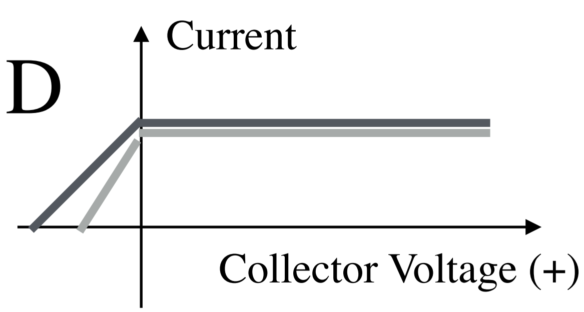
B



C



D

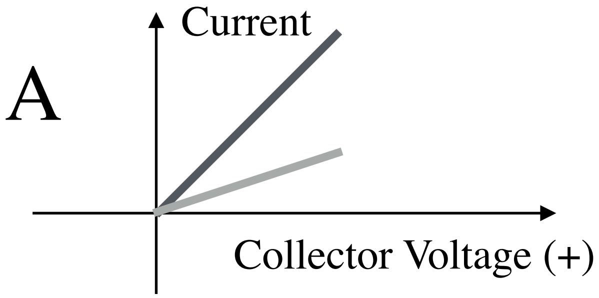


E Something Else

Clicker Answer

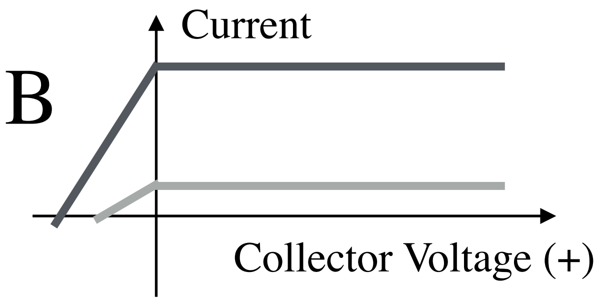
Which graph shows two different intensities with the same color ?

A



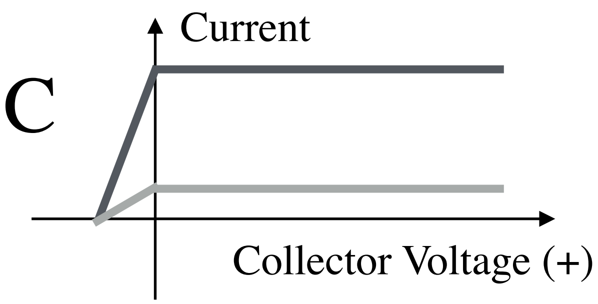
No plateau, and no current for negative voltage

B



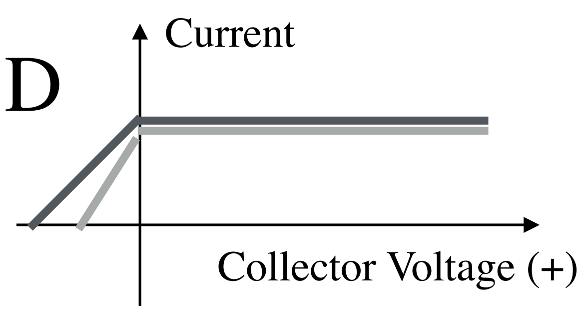
Stopping voltages should be the same

C



Stopping voltages same, plateaus different

D



Plateau currents should be different

E Something Else

Photoelectric Effect is Weird

Electrons are bound in metal, so it will take some energy to get them out.

If you heat up metals, electrons will come out without any light shining on them.
That's how old-fashioned vacuum tubes work.

But you would expect only the <u>total light intensity</u> to matter.

You could get the required energy by shining <u>any</u> color of light,
with enough intensity.

You would not expect that the <u>color</u> of the light would matter.

But the data says that if the frequency is too low, there is zero current,
no matter what the light intensity is.

Einstein’s Photoelectric Explanation

In 1905, Einstein noted that the stopping potential implied that there was a maximum ejected electron kinetic energy: $E_{\text{max}} = qV_{\text{stop}}$.

The dependence of the stopping voltage on frequency implied that the maximum electron energy increased with light frequency.

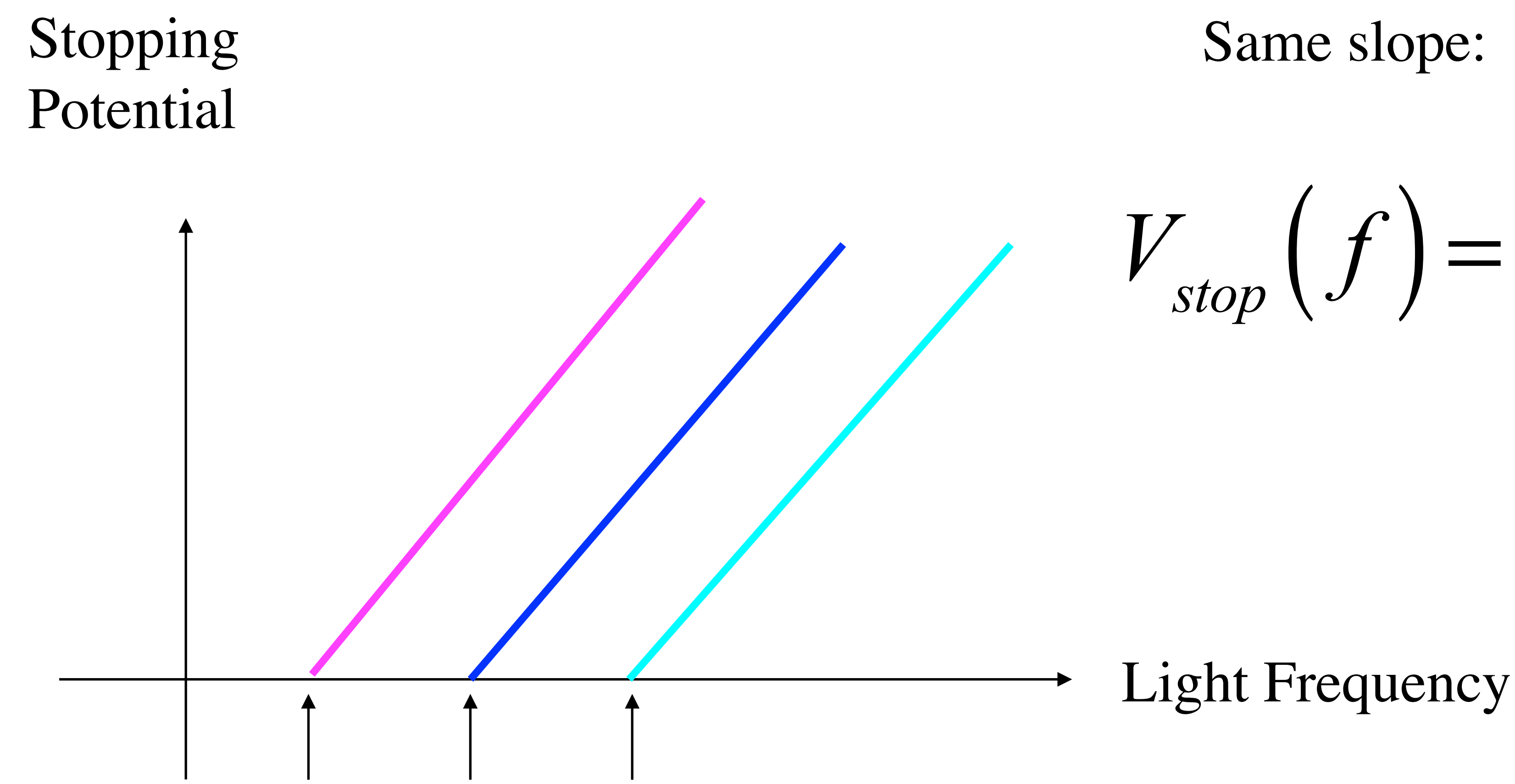
Einstein knew about Planck’s work that implied $E = hf$, and made the bold guess that

$$qV_{\text{stop}} = hf - q\phi_{\text{work}}$$

where $q\phi_{\text{work}}$ is the energy required to remove an electron from the metal surface, called the “work function.” We would expect it to depend on the metal.

The threshold frequency is then $hf_{\text{thresh}} = q\phi_{\text{work}}$.

Einstein's Prediction



Different Work Functions

Same slope:

$$V_{stop}(f) = \frac{hf}{q} - \phi_{work}$$

The work function can't be predicted, and varies with material and surface properties, and the experiments require very good vacuum to keep it constant.

But eventually, Einstein's guess was proven to be precisely correct. Einstein received the Nobel Prize in 1921, for the photoelectric effect (not relativity!)

Work Functions

Work functions of metals (in eV):

Aluminum	4.08 eV	Cesium	2.1	Lead	4.14	Potassium	2.3
Beryllium	5.0 eV	Cobalt	5.0	Magnesium	3.68	Platinum	6.35
Cadmium	4.07 eV	Copper	4.7	Mercury	4.5	Selenium	5.11
Calcium	2.9	Gold	5.1	Nickel	5.01	Silver	4.73
Carbon	4.81	Iron	4.5	Niobium	4.3	Sodium	2.28
				Uranium	3.6		
				Zinc	4.3		

Alkali metals with a single weakly bound outer electron are around 2.2 V.

Common engineering metals are around 4-5 V.

Platinum is very high at 6.35 V.

Clicker Question

A photon with wavelength 600 nm produces a photoelectron with kinetic energy E. What photoelectron energy would be produced by a photon of wavelength 300 nm ?

A. Maybe no photoelectron

B. 0.5 E

C. A bit less than 0.5 E

D. 2 E

E. A bit more than 2 E

Clicker Question

A photon with wavelength 600 nm produces a photoelectron with kinetic energy *E*. What photoelectron energy would be produced by a photon of wavelength 300 nm ?

A. Maybe no photoelectron. 300 nm is higher energy, there should be some.

B. 0.5 *E* It's twice the energy, not half

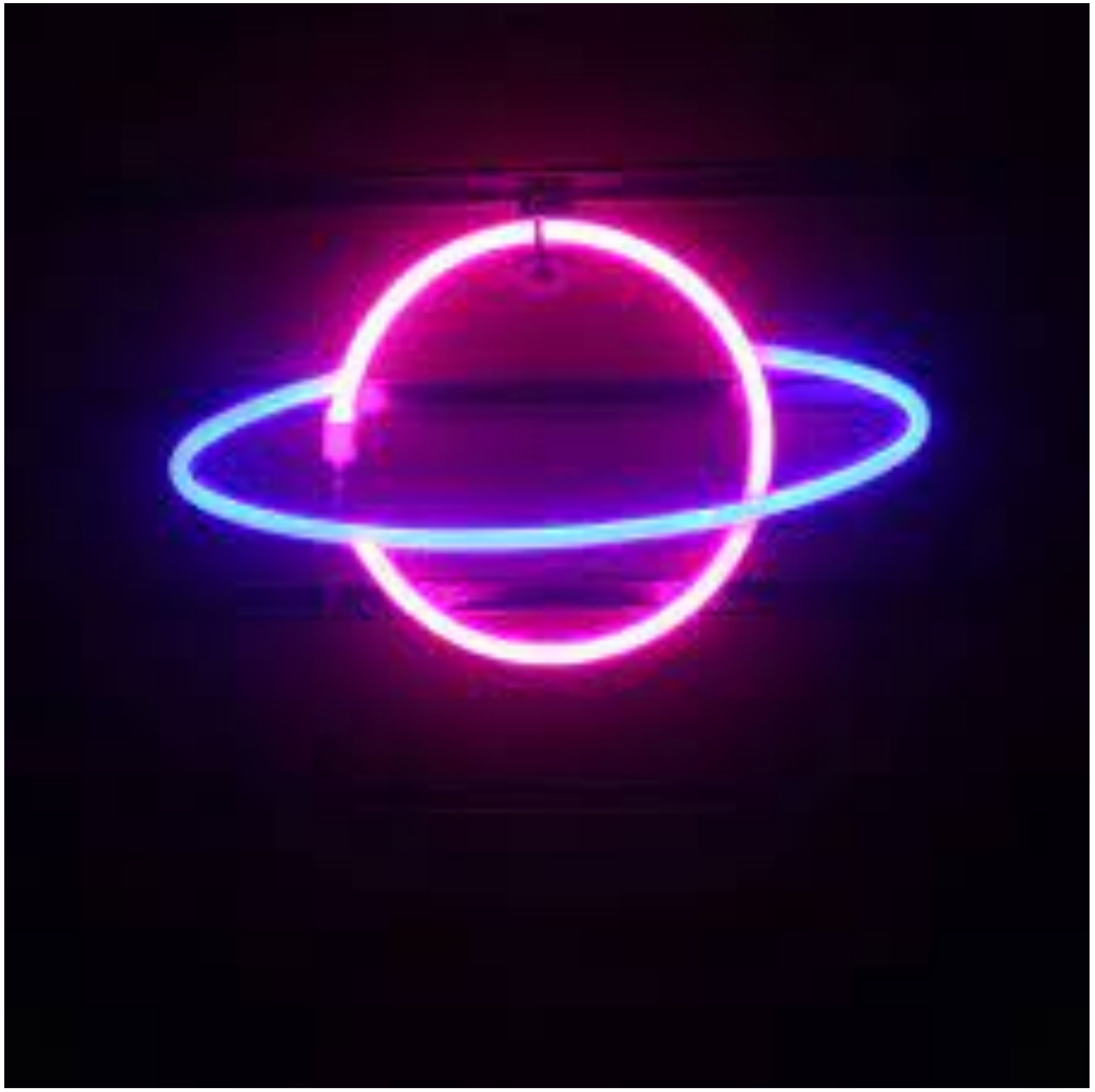
C. A bit less than 0.5 *E* Ditto

D. 2 *E* Getting warmer, but

E. A bit more than 2 *E* The fixed work function subtracts a bigger fraction of the photon energy at 600 vs 300 nm

Gas Discharge Tubes

If you pump out almost all of the air inside a glass tube,
and apply high voltage to electrodes at the end,
you get a glowing discharge.



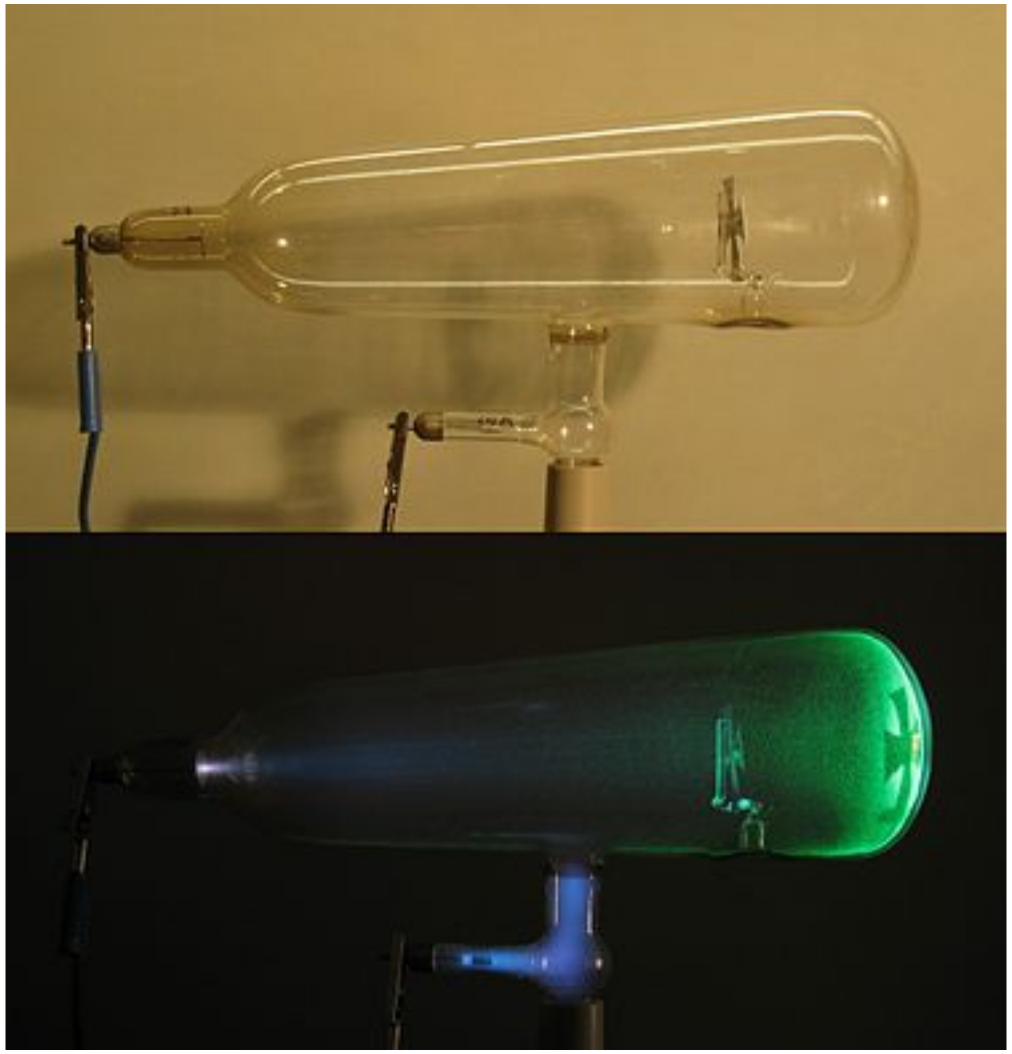
The residual gas gets ionized and can conduct electricity.

The light is produced by electrons recombining with ionized atoms. Different gases give different colors.

We'll talk lots more about that next week.

Crookes Tube

Much higher vacuum,
and a U-shaped discharge path,
with a “floating” obstacle.



“Cathode rays” come from the negative electrode.

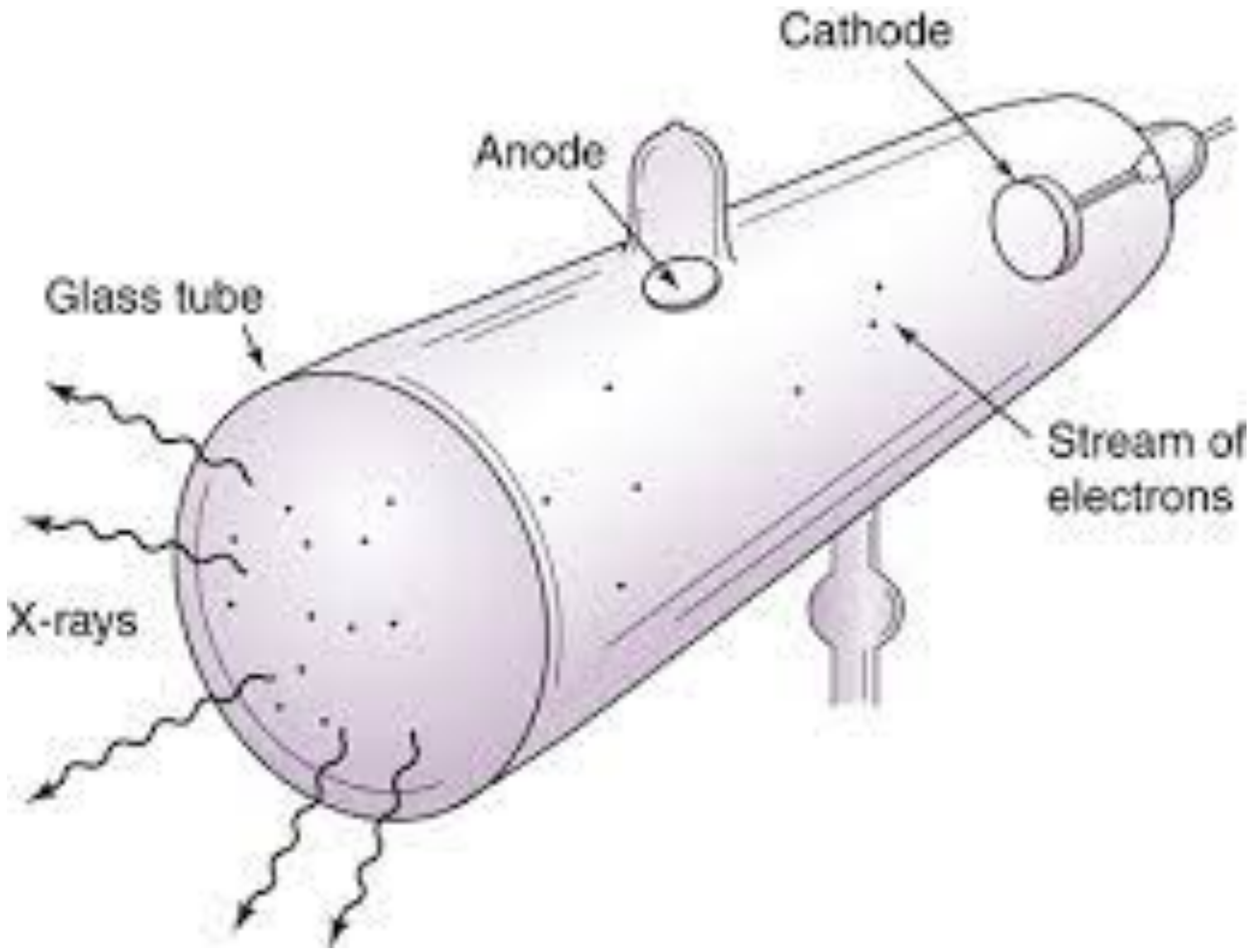
When they hit the far wall,
the glass glows (fluorescence).

The obstacle makes a shadow
(it stops the cathode rays, which travel in straight lines).

Roentgen and X-Rays

In 1895, Wilhelm Roentgen noticed that if you raise the voltage high enough, the cathode rays hitting the glass wall produce a new kind of "rays" that escape through the glass, and cause some "fluorescent" materials to glow.

He called them X-rays.



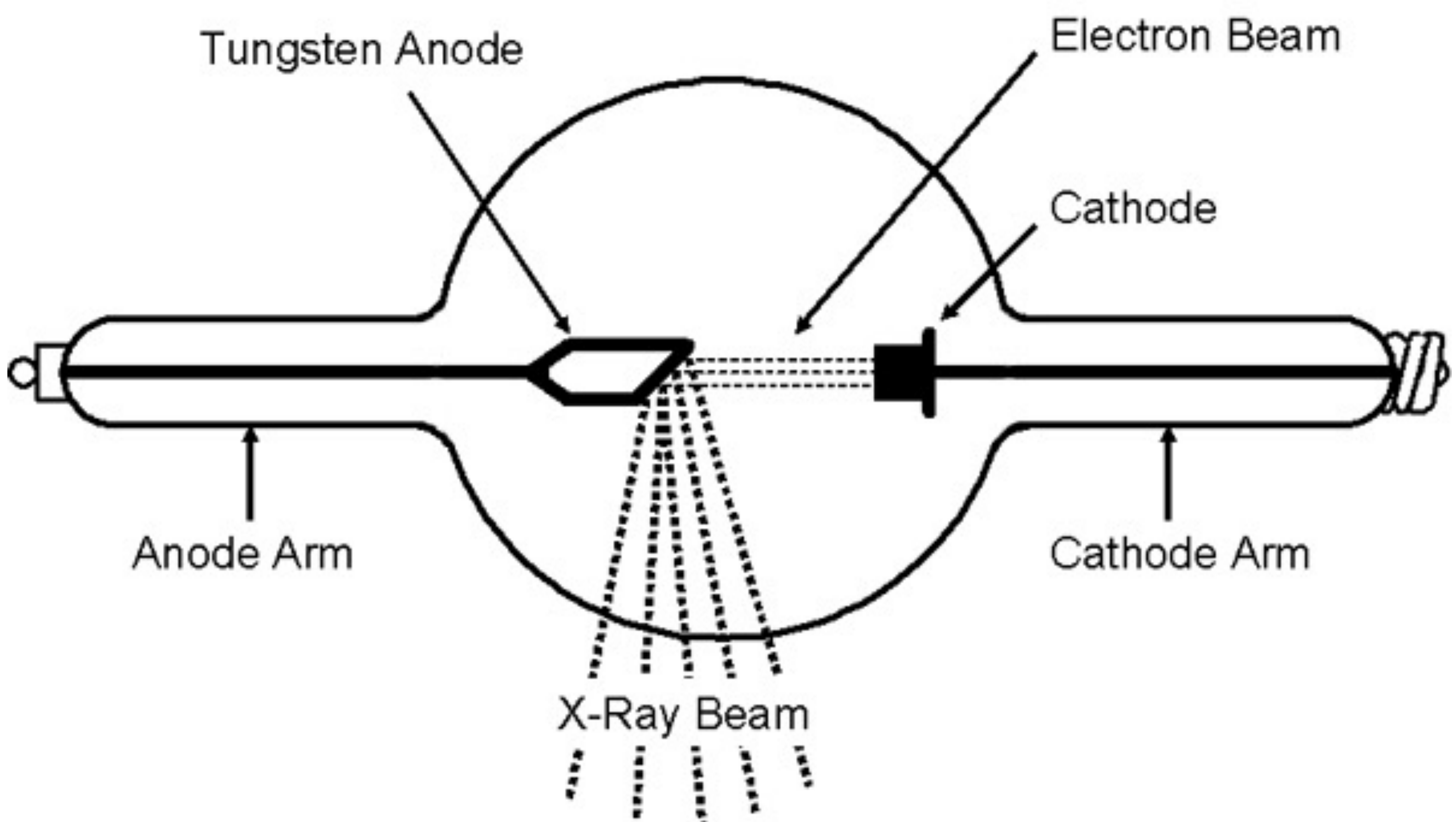
He also discovered that (with enough tube voltage) the rays will penetrate flesh, but were stopped by bone, or his wedding ring.

Better X-Ray Tube

Replace the cathode by a hot "thermionic" cathode.

Apply several thousand volts to attract the electrons to the anode.

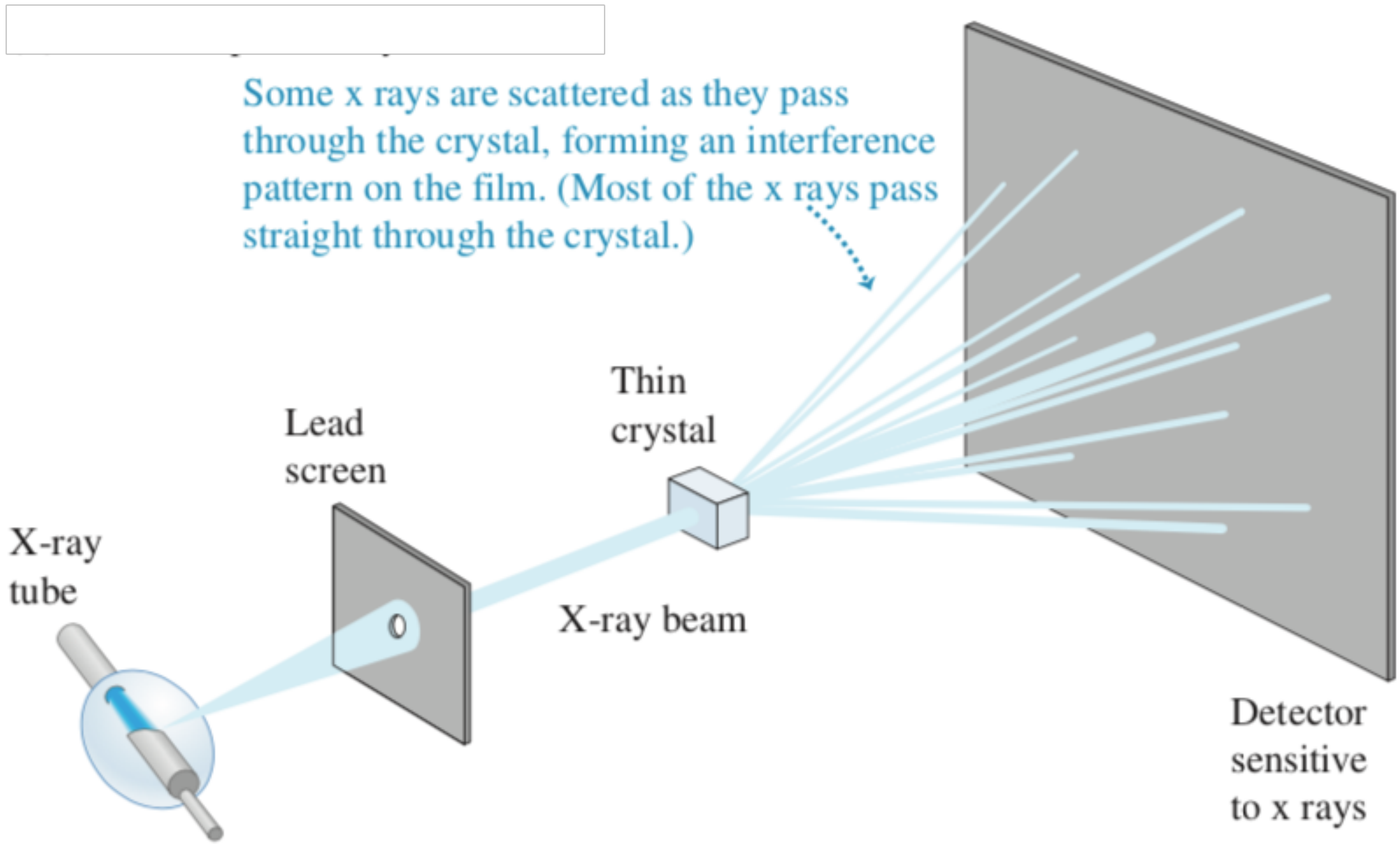
Make the anode out of tungsten so it doesn't melt.



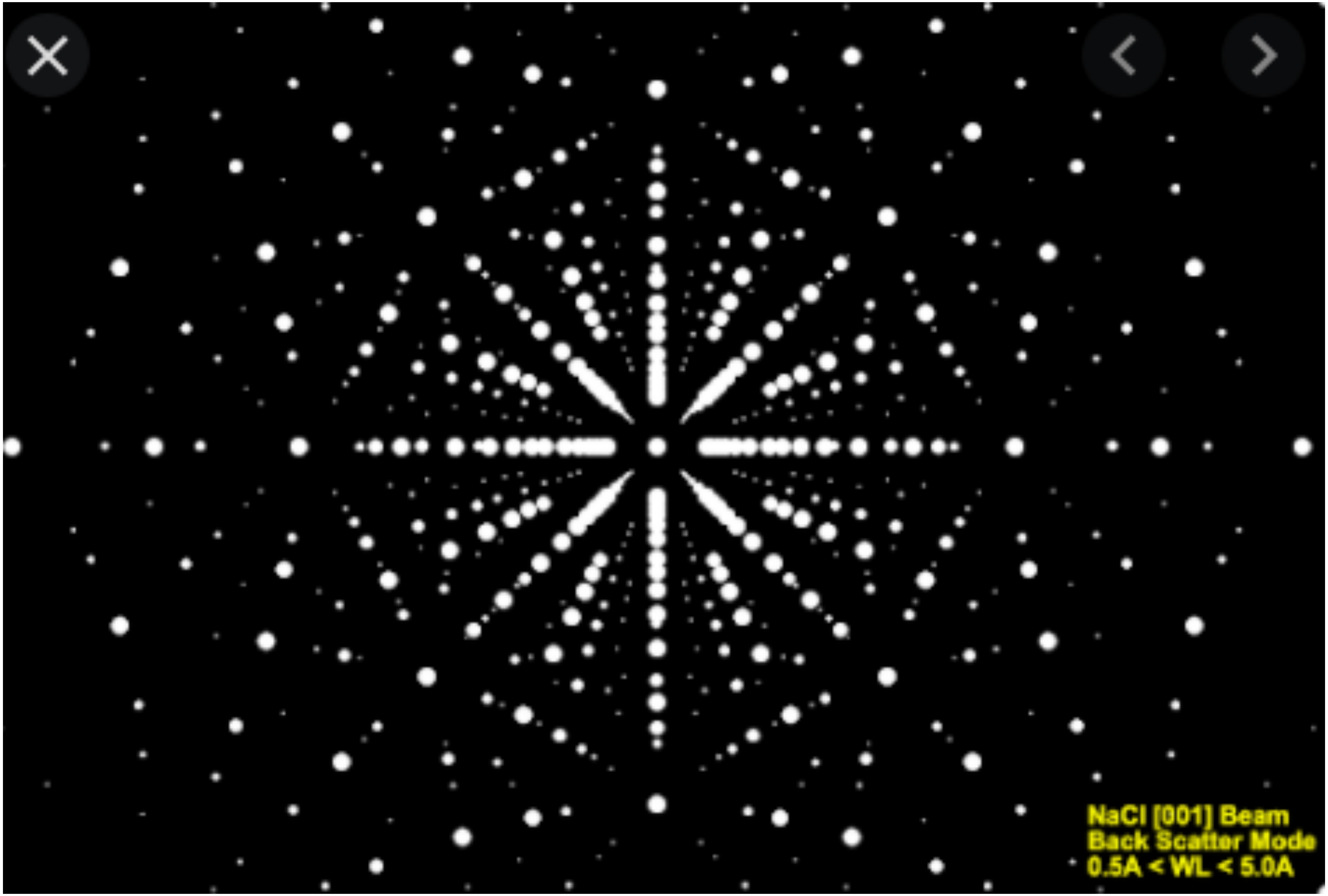
X-Rays and Crystals

In 1912, Max von Laue speculated that X-rays might be EM waves, with wavelengths comparable to atomic spacing.

He passed a collimated X-ray beam through a crystal, and found that a pattern of spots was formed on the film!



Salt Crystal Diffraction Pattern



X-Ray Diffraction

The size of an atom is ~ 100 pm, so it's comparable to the X-ray wavelength. So X-rays scatter in all directions from each atom.

The scattered X-rays from one atom tend to be out of phase with another atom.

But in a crystal, the atoms are regularly spaced. So there are some directions where the separation of atoms is a multiple of the X-ray wavelength, and the scattered X-rays are in phase. That makes the spots in the pattern.

You get this kind of picture when the X-ray beam contains a broad range of wavelengths, but X-ray tubes do that.

It's possible to analyze the spot pattern to determine the structure of the crystal.

Diffraction from Stacked Planes

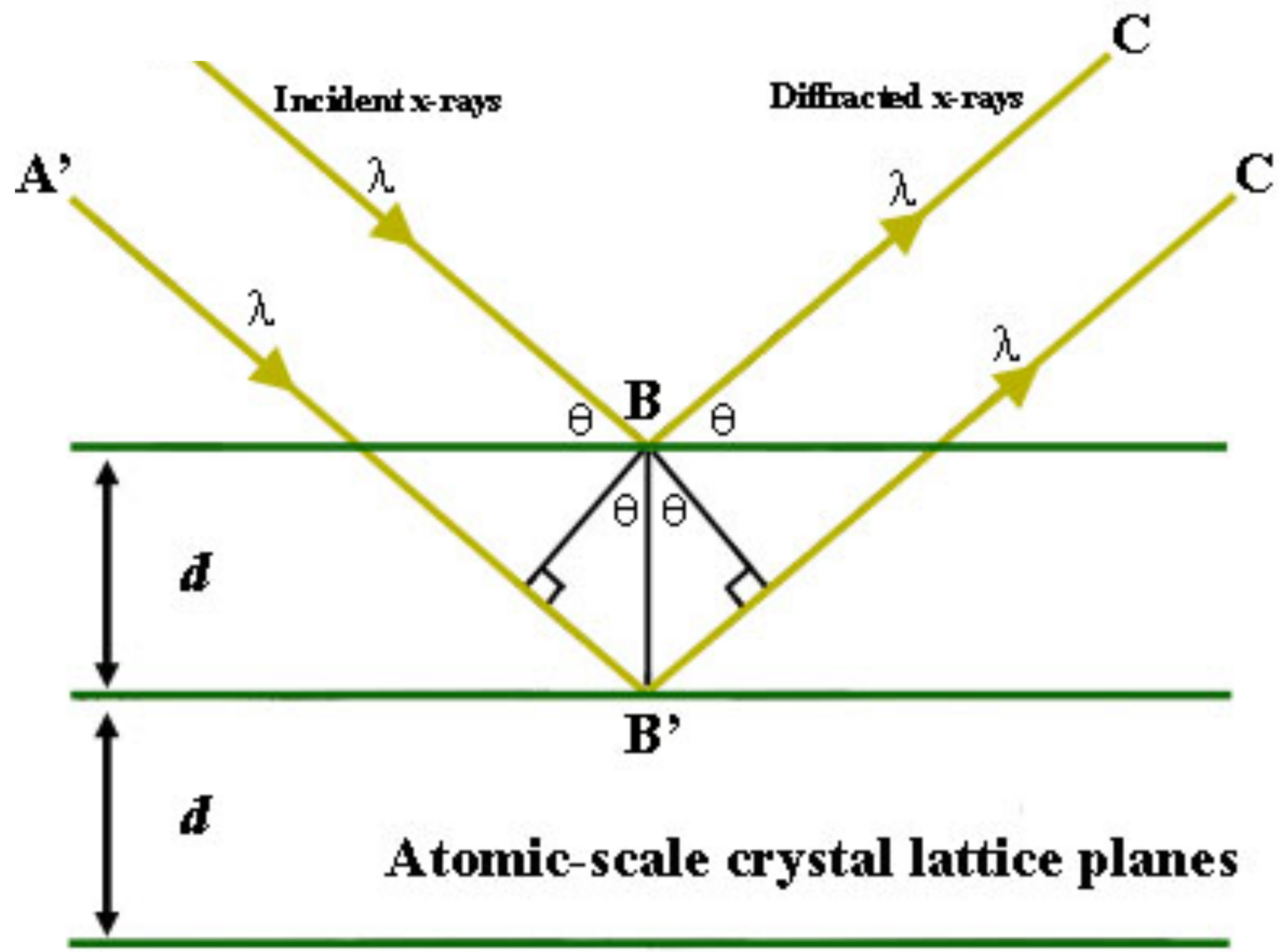
For many weakly-reflecting planes, you get reflections from all of them, at reflection-angle = incidence-angle. The reflections interfere with each other.

For plane spacing d, and angle θ from the surface (not the normal) the extra path length for the reflection from the second plane is $2d\sin\theta$.

The reflections are in phase if

$$2d\sin\theta = n\lambda$$

This is called Bragg's Law.

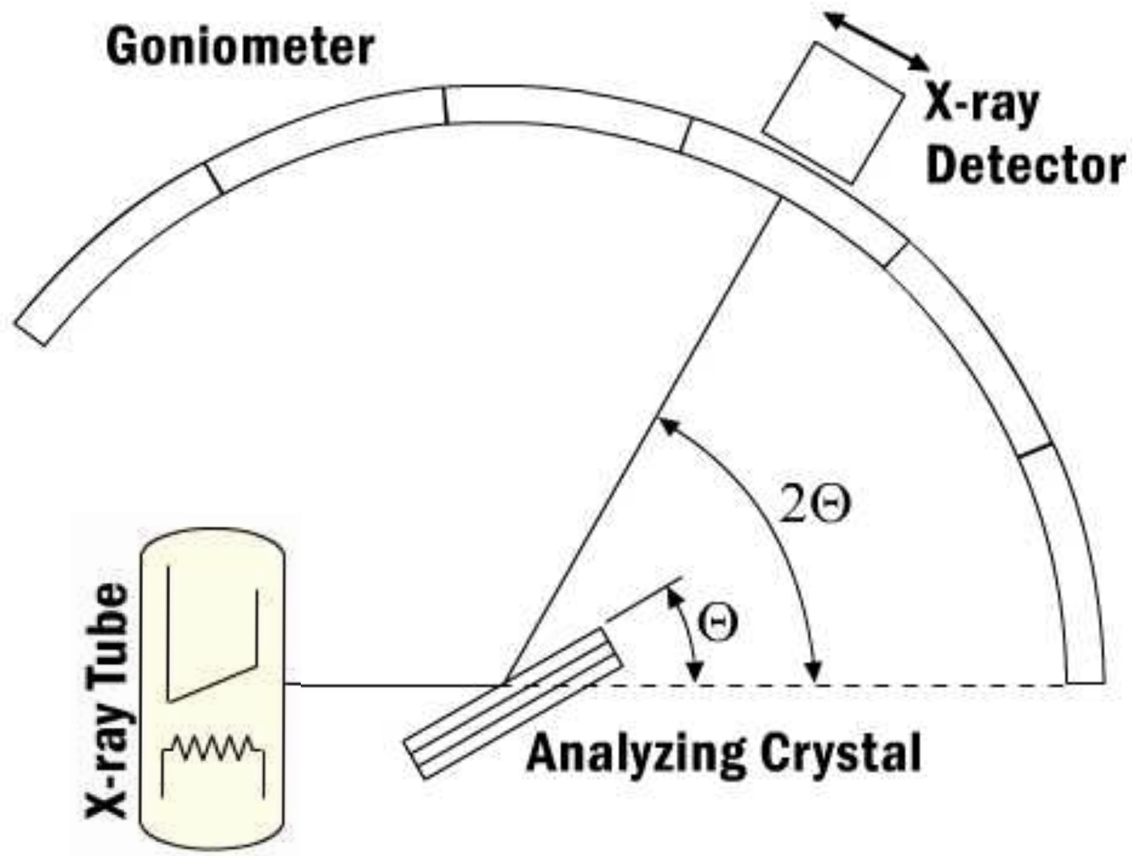


Warning: diffraction from slits follows a different formula, without the factor of 2, and θ is measured from the normal not surface.

Bragg X-Ray Spectrometer

Rotate the planes (analyzing crystal) by θ, and move the X-ray detector by 2θ.

The detector will see X-rays only with $\lambda = 2d\sin\theta$ (assuming $n = 1$).



If you know the analyzing crystal plane spacing d, you can find the wavelength.

Even if you don't know d, you at least get a <u>relative</u> wavelength.

X-Ray Tube Spectrum

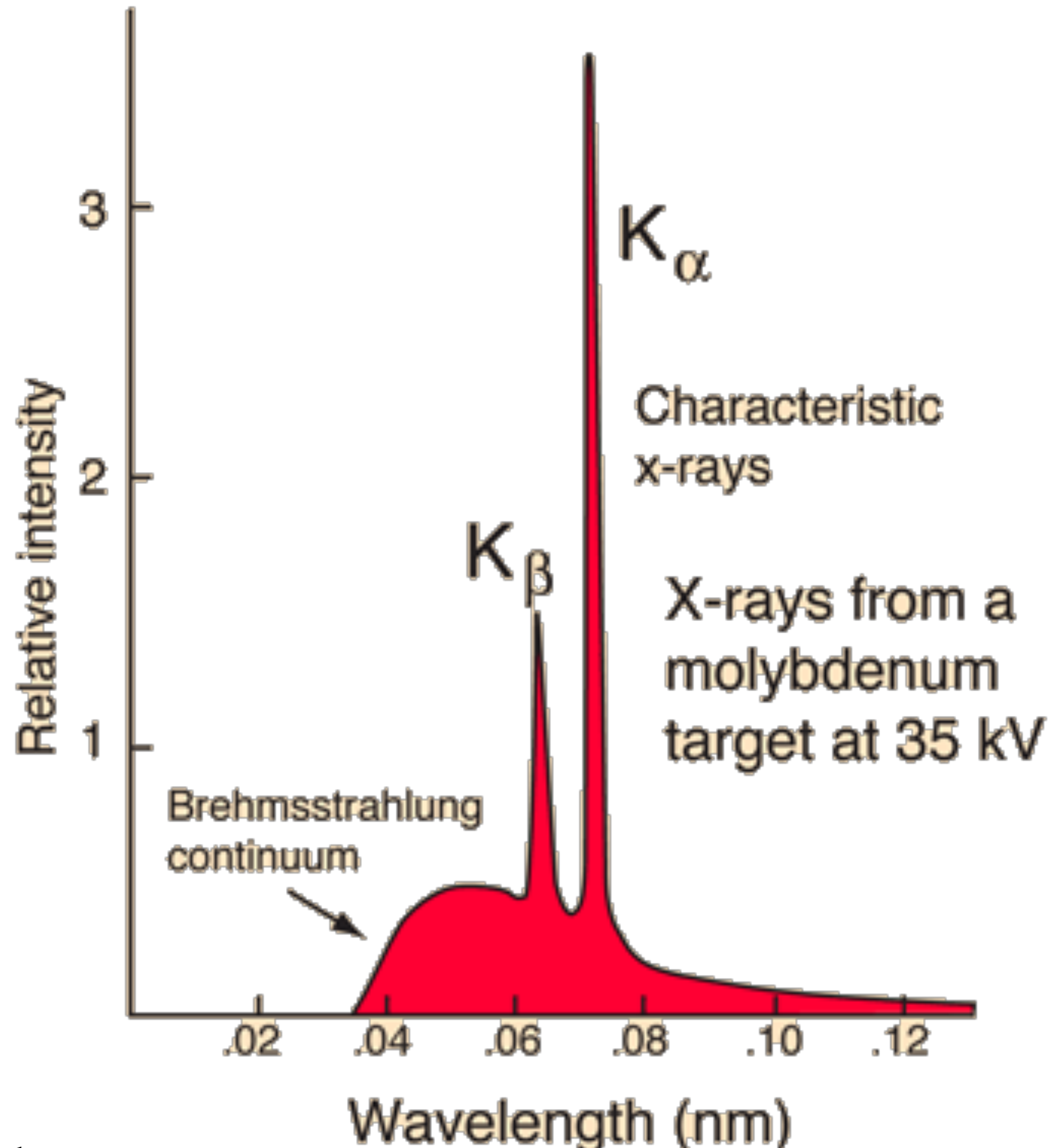
There is a smooth continuous spectrum from electrons hitting the metal anode, where they decelerate rapidly and radiate high-frequency photons.

The deceleration is sudden, so the spectrum is broad.

This is called the Bremsstrahlung continuum

There are usually some sharp peaks at longer wavelengths that depend on the anode material.

We'll talk more about them next week.



X-Ray Spectrum vs Voltage

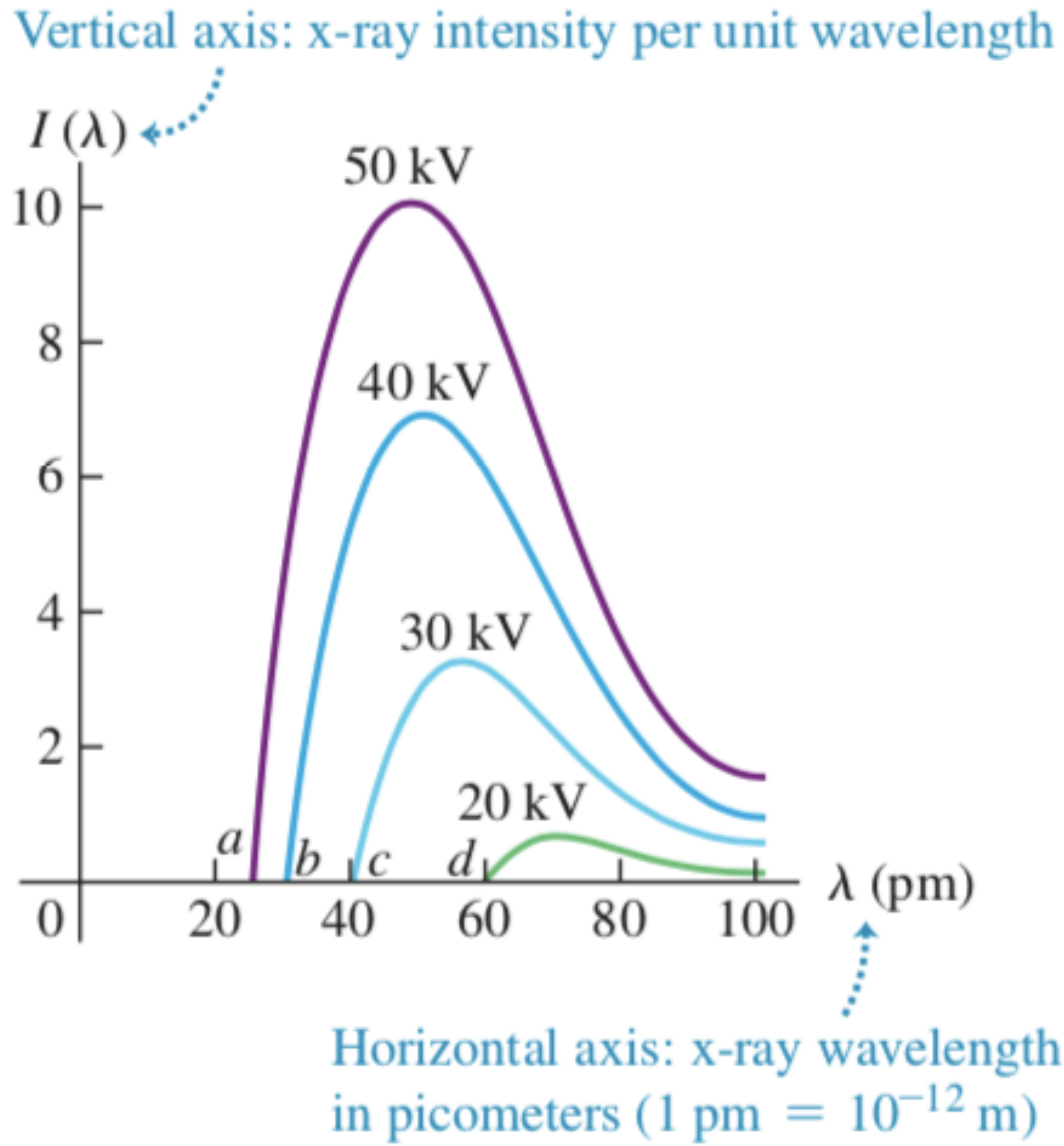
There is a minimum X-ray wavelength, which corresponds to a maximum X-ray frequency.

The minimum wavelength goes down as the voltage goes up.

$$qV = \frac{hc}{\lambda_{\text{min}}} = hf_{\text{max}}$$

Photoelectric effect in reverse !

Technically, the work function "sucks" the cathode ray electron into the anode, so the energy is $q\left(V + \phi_{\text{work}}\right) = hf_{\text{max}}$, but work functions are a few Volts, and X-ray tubes always work at many kV, so we neglect it.



Clicker Question

A typical dental X-ray machine operates at 70 kV.
What is the minimum photon wavelength?

A. 17.7 femtometers $\left(17.7\times10^{-15}\,\mathrm{m}\right)$

B. 17.7 picometers $\left(17.7\times10^{-12}\,\mathrm{m}\right)$

C. 17.7 nanometers $\left(17.7\times10^{-9}\,\mathrm{m}\right)$

D. 56.5 nanometers

E. 56.5 picometers

Clicker Answer

A typical dental X-ray machine operates at 70 kV.
What is the minimum photon wavelength?

B. 17.7 pm

We have $qV = \dfrac{hc}{\lambda_{\text{min}}} = hf_{\text{max}} \rightarrow \lambda_{\text{min}} = \dfrac{hc}{qV}$

qV is simply 70×10^3 electron-volts, so

$$\lambda_{\text{min}} = \frac{1240 \text{ eV-nm}}{70\times10^3 \text{ eV}} = 1.771\times10^{-2} \text{ nm} = 17.71 \text{ pm.}$$

Scattering of EM Waves

EM waves makes charged particles oscillate at the same frequency as the wave.

The particles radiate EM energy at the same frequency as the wave.

So reflected or scattered light has the same wavelength as the incident light.

Classically, there is a slight net force along the wave direction, so there can be a Doppler shift, but it's tiny small for frequencies in the visible range or lower.

Using Bragg spectrometers, people noticed that when X-rays scatter through a large angle, the wavelength changes significantly

Arthur Compton explained this, and got a Nobel Prize for it.

Photon Momentum-Wavelength Relation

Relativity: All particles obey $E^2 = (pc)^2 + (mc^2)^2$.

Photons are massless, so they obey $E = pc$.

Quantum: Photons obey $E = hf = \frac{hc}{\lambda}$.

So $E = pc = \frac{hc}{\lambda} \rightarrow \boxed{p = \frac{h}{\lambda}}$

4-Vector Momentum and Dot Product

It's useful to combine the momentum and energy of a particle into a <u>4-vector</u>

$$\underset{\rightarrow}{P} = \left(\frac{E}{c}, \vec{P} \right)$$

It's conventional to call relativistic energy *E*/*c* the zeroth component, and the relativistic momentum components 1, 2, and 3.

My convention is to put an arrow <u>below</u> a 4-vector, and <u>above</u> a 3-vector.

It's useful to define a <u>4-vector dot-product</u> as

$$\underset{\rightarrow}{P}_1 \cdot \underset{\rightarrow}{P}_2 = \left(\frac{E_1}{c}, \vec{P}_1 \right) \cdot \left(\frac{E_2}{c}, \vec{P}_2 \right) = \frac{E_1}{c}\frac{E_2}{c} - \vec{P}_1 \cdot \vec{P}_2 = \frac{E_1}{c}\frac{E_2}{c} - \left|\vec{P}_1\right| \cdot \left|\vec{P}_2\right| \cos\theta_{12}$$

The "square" of a 4-vector is the dot-product with itself

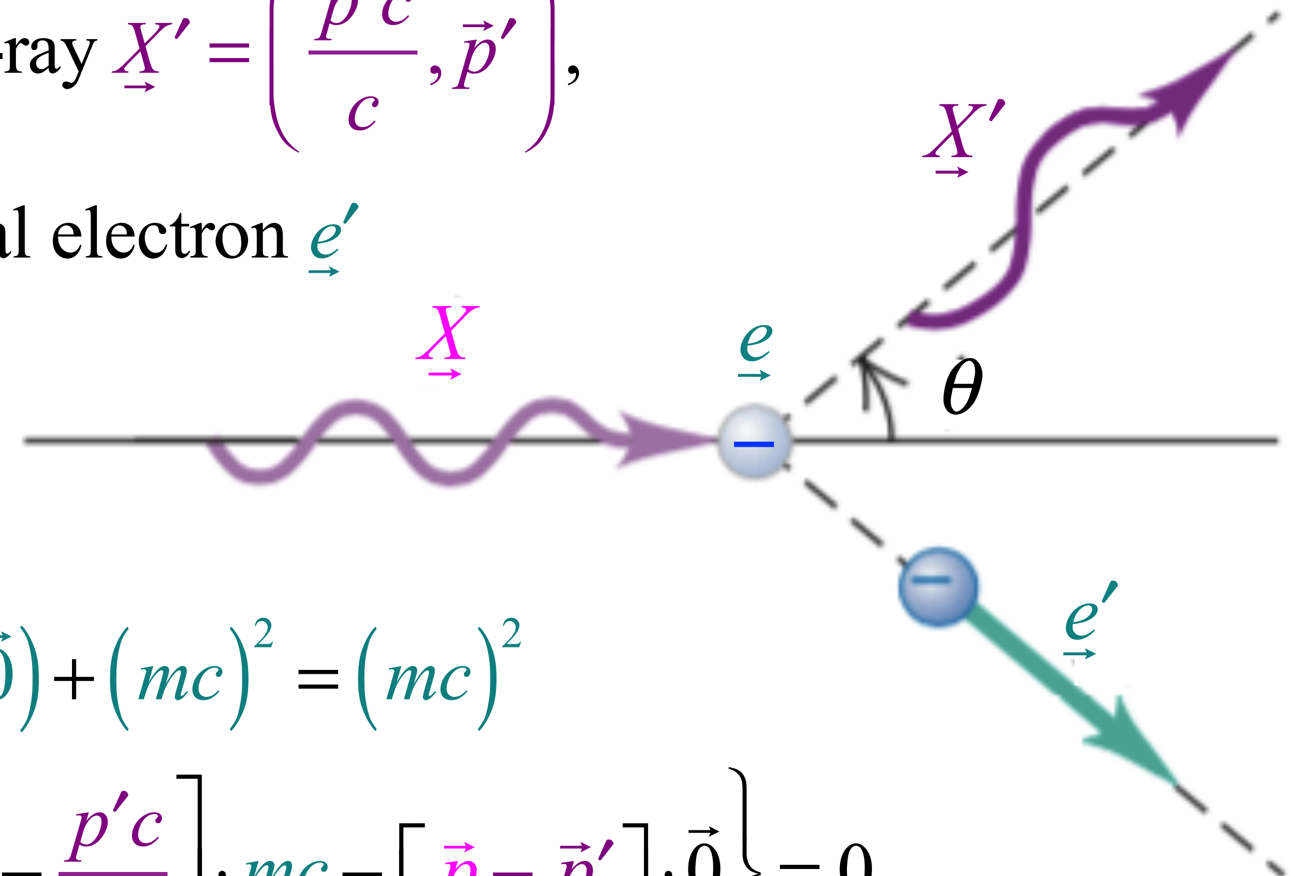
$$\underset{\rightarrow}{P}^2 = \frac{E^2}{c^2} - \vec{P} \cdot \vec{P} = \frac{E^2}{c^2} - \left|\vec{P}\right|^2$$

Since $E^2 = \left(Pc\right)^2 + \left(mc^2\right)^2$, this is equivalent to $\underset{\rightarrow}{P}^2 = \left(mc\right)^2$

Compton Kinematics

Incident X-ray $\underrightarrow{X} = \left(\frac{pc}{c}, \vec{p} \right)$, scattered X-ray $\underrightarrow{X}' = \left(\frac{p'c}{c}, \vec{p}' \right)$,

initial electron at rest $\underrightarrow{e} = \left(mc, \vec{0} \right)$, and final electron $\underrightarrow{e}'$



$$\underrightarrow{X} + \underrightarrow{e} = \underrightarrow{X}' + \underrightarrow{e}' \quad \rightarrow \quad \left[\underrightarrow{X} - \underrightarrow{X}' \right] + \underrightarrow{e} = \underrightarrow{e}'$$

$$\left[\underrightarrow{X} - \underrightarrow{X}' \right]^2 + 2\left[\underrightarrow{X} - \underrightarrow{X}' \right] \cdot \underrightarrow{e} + \underrightarrow{e}^2 = \underrightarrow{e}'^2$$

$$\left[\underrightarrow{X}^2 - 2\underrightarrow{X} \cdot \underrightarrow{X}' + \underrightarrow{X}'^2 \right] + 2\left[\underrightarrow{X} - \underrightarrow{X}' \right] \cdot \left(mc, \vec{0} \right) + \left(mc \right)^2 = \left(mc \right)^2$$

$$\left[0 - 2\left(\frac{pc}{c}\frac{p'c}{c} - pp'\cos\theta \right) + 0 \right] + 2\left\{ \left[\frac{pc}{c} - \frac{p'c}{c} \right] \cdot mc - \left[\vec{p} - \vec{p}' \right] \cdot \vec{0} \right\} = 0$$

$$pp'\left(1 - \cos\theta \right) = mc\left[p - p' \right]$$

$$1 - \cos\theta = mc\frac{p - p'}{pp'} = mc\left(\frac{1}{p'} - \frac{1}{p} \right) = mc\left(\frac{\lambda'}{h} - \frac{\lambda}{h} \right) \quad \text{using } \; p = \frac{h}{\lambda}$$

$$\lambda' - \lambda = \frac{h}{mc}\left(1 - \cos\theta \right) \qquad \text{Compton's Equation}$$

Compton Wavelength

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$\frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240 \text{ eV-nm}}{0.511 \times 10^6 \text{ eV}/c^2 \cdot c^2} = 2.426 \times 10^{-3}\,\text{nm}$$

$$= 2.426 \text{ pm} = 2.426 \times 10^{-12}\,\text{m}$$

The effect is noticable for X-rays in the picometer range (70 kV is 17 pm), very hard to see in the nanometer range (1 keV), and negligible for light.

If the mass in the formula is for a whole atom, rather than for a single electron, the maximum wavelength shift is thousands of times smaller.

Compton Scattering Experiment

Use an X-ray tube which has a very narrow and strong peak from atomic physics.

The X-rays will scatter from a target (carbon here) at all angles.

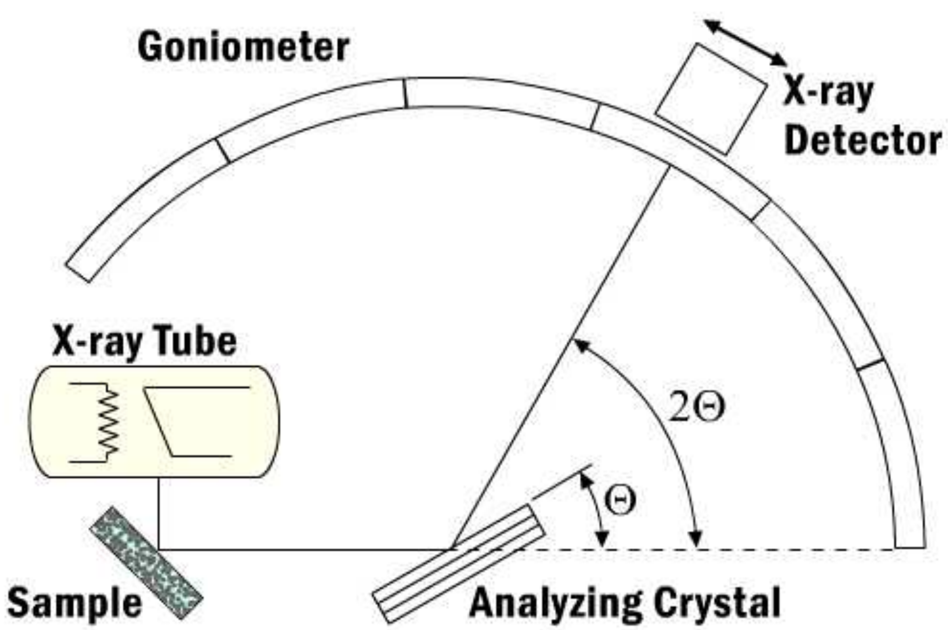
Pass the X-rays through a collimator to select a particular scattering angle, and into a Bragg spectrometer to measure the wavelength by scanning.

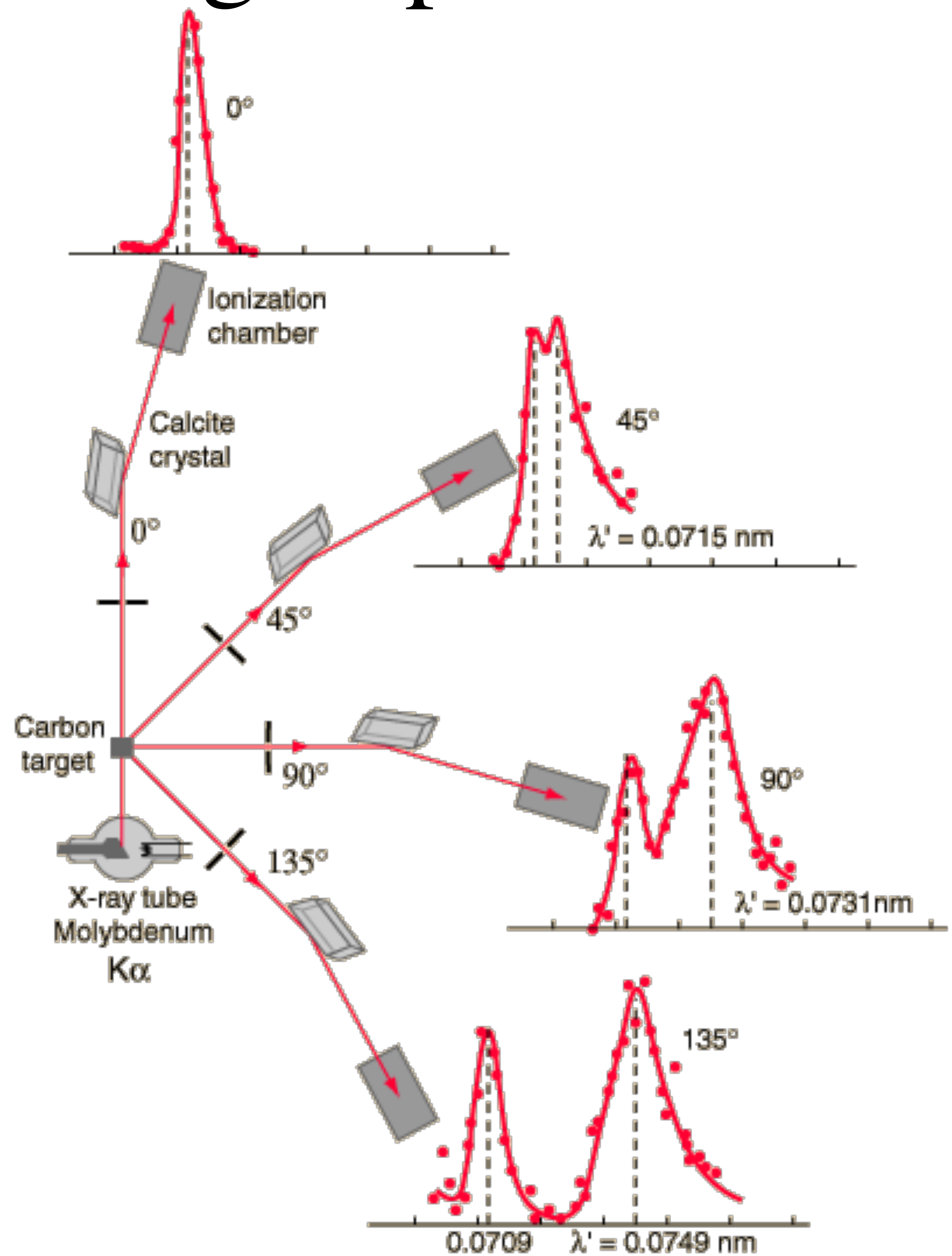
Repeat the experiment at a range of collimator positions.

Many of the X-rays scatter off whole atoms with negligible wavelength shift.

But electrons that scatter from a single electron will have a wavelength shift that depends on the scattering angle, giving a second peak at longer wavelength.

Compton Scattering Experiment





Photons are Particles, and Waves

X-ray photons scatter as if they are particles with momentum as well as energy (albeit with zero rest mass!)

But in order for the Bragg spectrometer to work, they must also be waves !

In the classical wave model, the amplitude of the wave can be anything.

In the quantum wave model, the amplitude is quantized such that $E = nhf$.

But the value of h is so tiny, usually n is huge, so E is essentially continuous.

Photon Flux and Density

An EM wave has power flux in Joules/m^2/second (or Watts/m^2).

Divide by 1.602×10^{-19} Joules/eV to get eV/m^2/second.

Divide by photon energy in eV = $hf = hc/\lambda$
to get the flux of photons/m^2/second.

Flux is density times velocity, and $v = c$,
so photon <u>density</u> is photon <u>flux</u> divided by c.

Solar Photon Flux and Density

The power flux from the Sun is about 1.4 kW/m^2 on Earth.

The energy of a typical solar photon is about 1.4 eV.

The solar photon <u>flux</u> is then about

$$\frac{1.4\times10^{3}\ \text{J/m}^2/\text{s}}{1.602\times10^{-19}\ \text{J/eV}\cdot 1.4\ \text{eV/photon}} = 6.24\times10^{21}\ \text{photons/m}^2/\text{s}$$

That corresponds to a photon <u>density</u> of

$$\frac{6.24\times10^{21}}{2.998\times10^{8}} = 2.03\times10^{13}\ \text{photons/m}^3$$

Clicker Question

A green laser pointer produces photons with wavelength 532 nm.

How many photons per second does a 5 mW green laser pointer produce?

A. 1.34×10^{16} photons/second

B. 1.34×10^{19} photons/second

C. 7.27×10^{16} photons/second

D. 7.27×10^{19} photons/second

E. None, because light is waves

Clicker Answer

A green laser pointer produces photons with wavelength 532 nm.

How many photons per second does a 5 mW green laser pointer produce?

A. 1.34×10^{16} photons/second

$$E = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{532 \text{ nm}} = 2.331 \text{ eV}$$

$$\frac{5\times10^{-3} \text{ J/s}}{1.602\times10^{-19} \text{ J/eV}} = 3.121\times10^{16} \text{eV/s}$$

$$\frac{3.121\times10^{16} \text{eV/s}}{2.331 \text{ eV}} = 1.339\times10^{16} \text{ photons/second}$$

Detecting Photons

The rod cells in your eyes can give a nerve impulse from a single photon.
But signal processing requires several adjacent rod cells to fire in a short time.
And the efficiency for a photon to get to the retina and hit the right molecule
in the rod cell is fairly low. So dozens of photons are required to see a flash.

It takes about 4 photons for a silver halide grain in photo film to be developable.
Space resolution can be as small as microns.
(A good image requires weaker development which requires more light.)

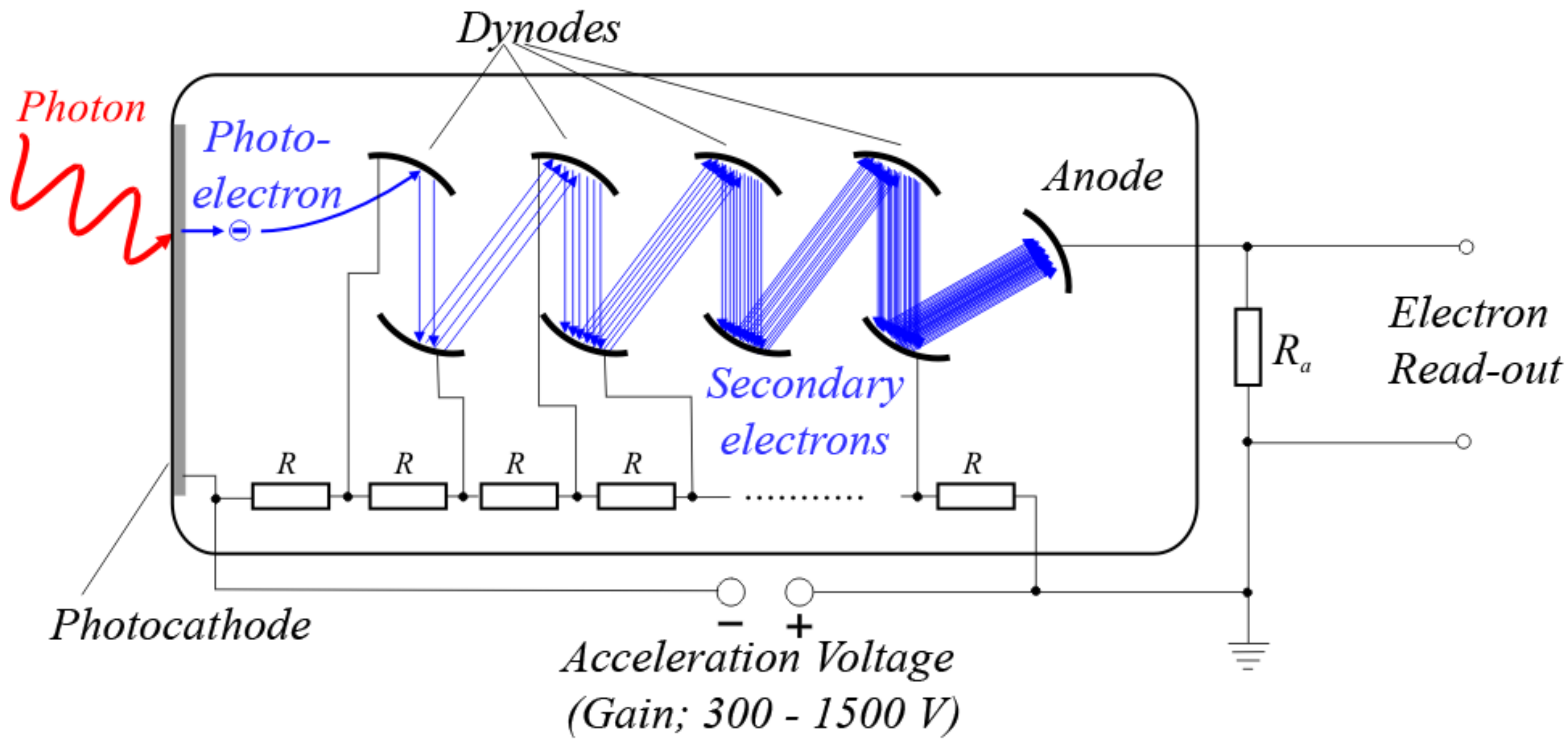
A photomultiplier tube can detect single photons (with limited efficiency).

A microchannel plate can detect single photons with sub-millimeter resolution.

Single X-ray photons produce an electron with enough energy to knock electrons
out of many more atoms, and that signal can be amplified by "gas gain."

Single gamma ray photons are fairly easy to detect.

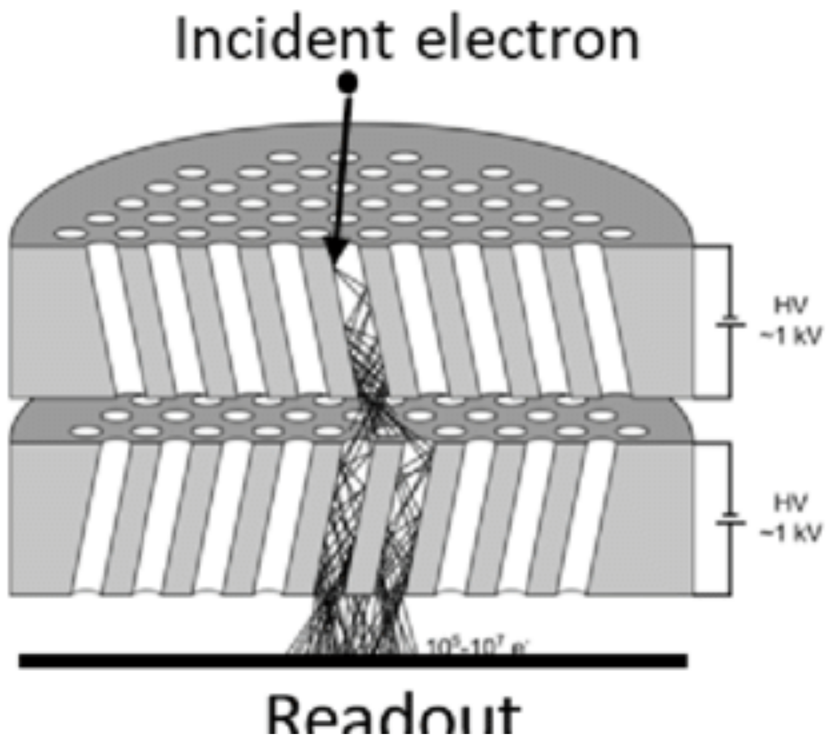
Photomultiplier Tube

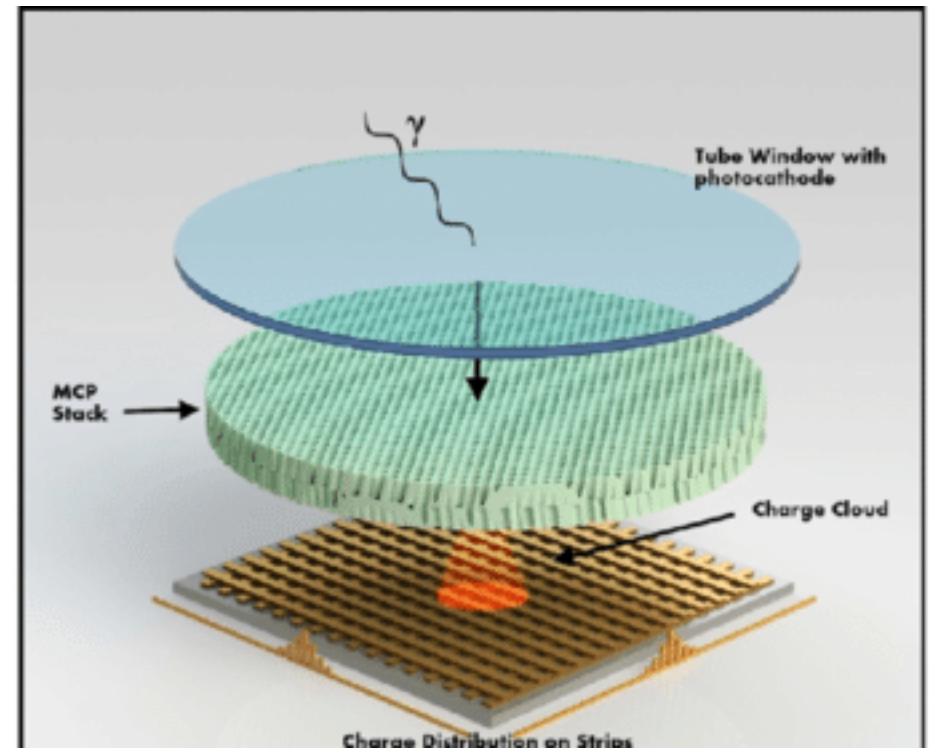


A photon hits a thin cesium layer inside a glass tube, making a single electron. That electron is pulled to a dynode where it makes many secondary electrons. Those are pulled to dynodes making more secondary electrons. 10^8 gain possible.

The "quantum efficiency" of a good tube is 20-30%.
Thermal noise is about 10 kHz.

Microchannel Plate Tube

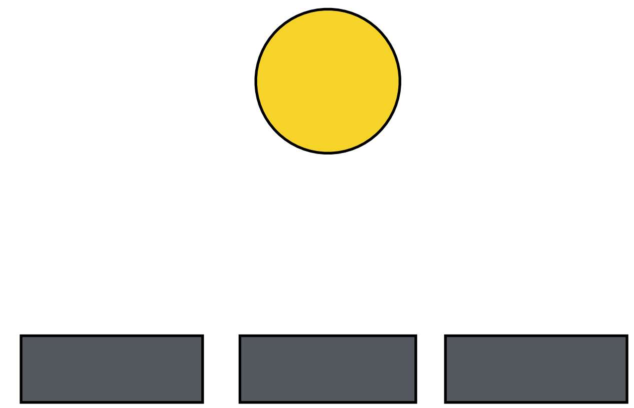




Photocathode like a photomultiplier tube, but the dynodes are replaced by "chevron plates" of resistive material with high secondary electron yield, and the final anode is replaced by crossed strips to localize the signal.

Photons and Interference

Set up a 2-slit interference experiment, and measure the light intensity using the photoelectric effect, so we see individual photons. Use dim light.

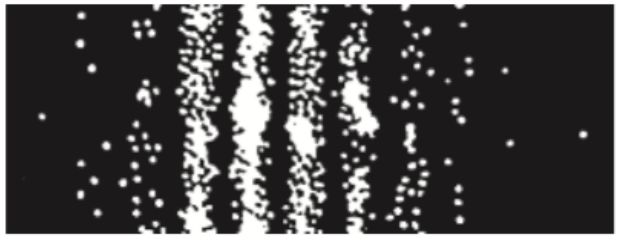


With only a few photons, it's confusing.

After 21 photons reach the screen

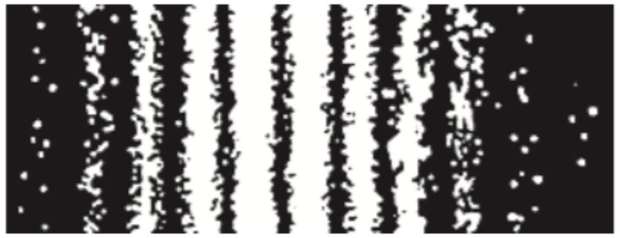
But with more and more photons, we see the standard interference pattern.

After 1000 photons reach the screen



Interference works even when the light is so dim that only one photon at a time is in the apparatus.

After 10,000 photons reach the screen



So each photon must go through both slits and interfere with itself !

Summary

$1\text{ eV} = 1.602\times10^{-19}\text{ Joule}\ \ (= \text{ electron charge in Coulombs})$

Planck: $E = nhf$ predicts the correct black-body radiation spectrum

Planck's Constant: $h = 6.626\times10^{-34}\text{ Joule-seconds} = 4.136\times10^{-15}\text{ eV-s}$

Frequently useful: $hc = 1240\text{ eV-nm}$

Planck-Einstein: $E = hf = \dfrac{hc}{\lambda}$

Einstein photoelectric: $E_{\text{max}} = hf - \phi_{\text{work}} = \dfrac{hc}{\lambda} - \phi_{\text{work}}$

X-ray Tube Spectrum: $hf_{\text{max}} = \dfrac{hc}{\lambda_{\text{min}}} = qV_{\text{tube}}$

Bragg's Law: $2d\sin\left(\theta_{\text{surface}}\right) = n\lambda$ $\qquad d =$ atomic plane spacing

Compton Effect: $\lambda' - \lambda = \dfrac{h}{mc}\cdot\left(1-\cos\theta\right) \qquad \dfrac{h}{mc} = 2.426\text{ pm}$

Photon Momentum: $E = pc \rightarrow \dfrac{hc}{\lambda} = pc \rightarrow p = \dfrac{h}{\lambda}$

For Next Time

WebWork 2 will be posted tonight, due next Monday midnight.

Worksheet on Friday, due at midnight.

Monday we start Atoms.