PHYS 250

Lecture 3.1

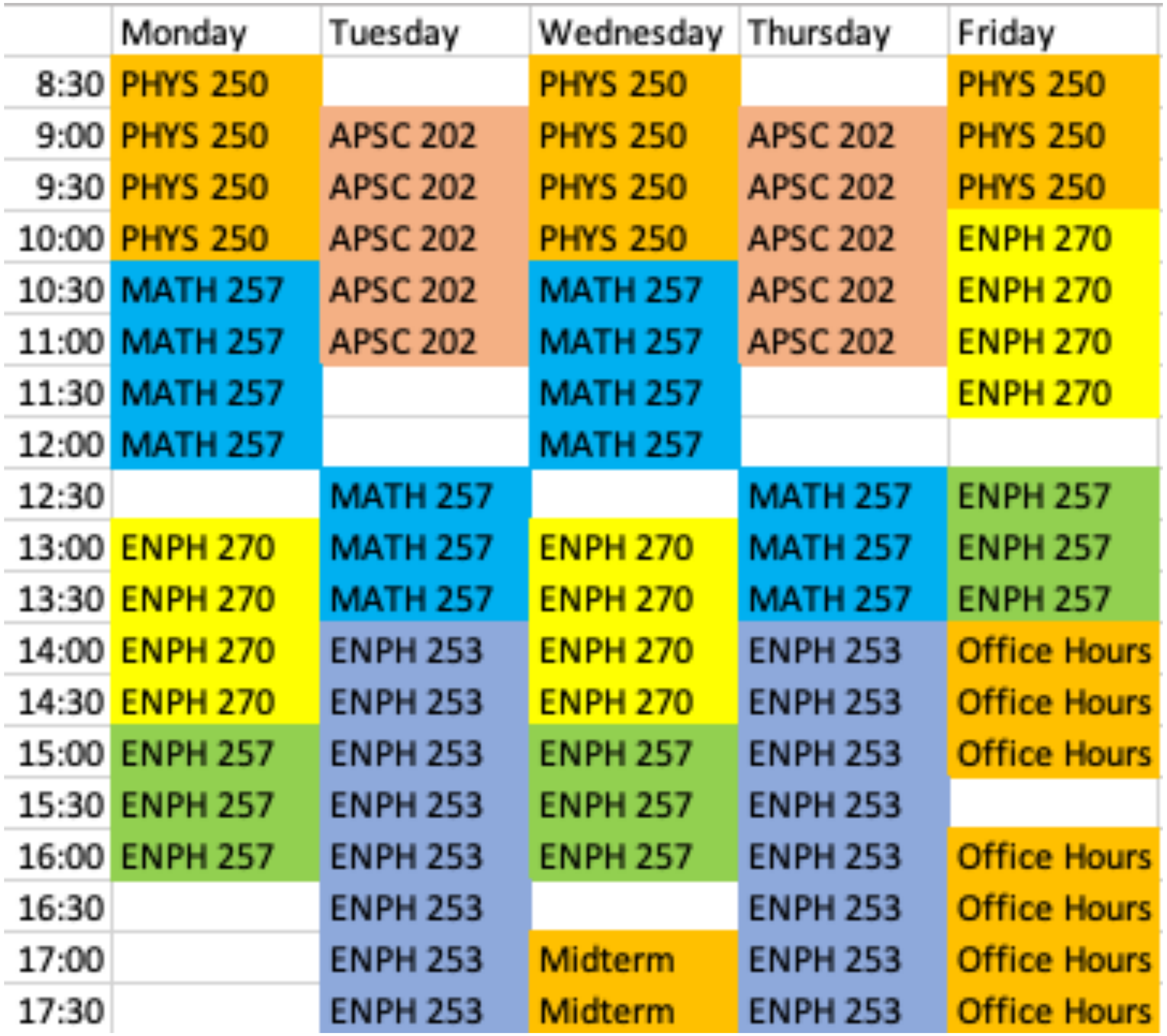
Atoms 1

Today

Administrivia
Measuring Light Wavelengths
Spectral Lines: Emission and Absorption
History of Knowledge of Atomic Structure
Bohr Model
X-Ray Emission Lines
Franck-Hertz Experiment
de Broglie Matter Waves

Midterm Exam

Midterm exam has been moved to Wednesday June 4 at 5-6 PM.
It covers Relativity, Photons, and Atoms (this week).



	Monday	Tuesday	Wednesday	Thursday	Friday
8:30	PHYS 250		PHYS 250		PHYS 250
9:00	PHYS 250	APSC 202	PHYS 250	APSC 202	PHYS 250
9:30	PHYS 250	APSC 202	PHYS 250	APSC 202	PHYS 250
10:00	PHYS 250	APSC 202	PHYS 250	APSC 202	ENPH 270
10:30	MATH 257	APSC 202	MATH 257	APSC 202	ENPH 270
11:00	MATH 257	APSC 202	MATH 257	APSC 202	ENPH 270
11:30	MATH 257		MATH 257		ENPH 270
12:00	MATH 257		MATH 257		
12:30		MATH 257		MATH 257	ENPH 257
13:00	ENPH 270	MATH 257	ENPH 270	MATH 257	ENPH 257
13:30	ENPH 270	MATH 257	ENPH 270	MATH 257	ENPH 257
14:00	ENPH 270	ENPH 253	ENPH 270	ENPH 253	Office Hours
14:30	ENPH 270	ENPH 253	ENPH 270	ENPH 253	Office Hours
15:00	ENPH 257	ENPH 253	ENPH 257	ENPH 253	Office Hours
15:30	ENPH 257	ENPH 253	ENPH 257	ENPH 253	
16:00	ENPH 257	ENPH 253	ENPH 257	ENPH 253	Office Hours
16:30		ENPH 253		ENPH 253	Office Hours
17:00		ENPH 253	Midterm	ENPH 253	Office Hours
17:30		ENPH 253	Midterm	ENPH 253	Office Hours

Final exam schedule is not yet available.

Other Issues?

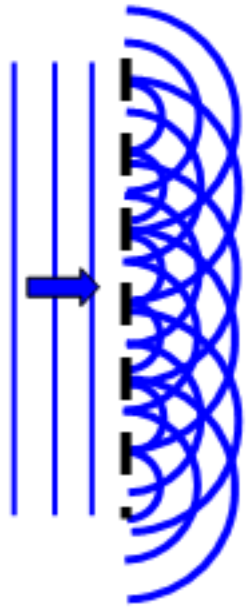
Worksheets? (Scores and mark-ups should be on Canvas)

Webwork? (Correct answers and solutions should be visible.
I'm still working on getting scores transferred to Canvas).

Clickers? (Scores should be visible. Can they be transferred to Canvas?)

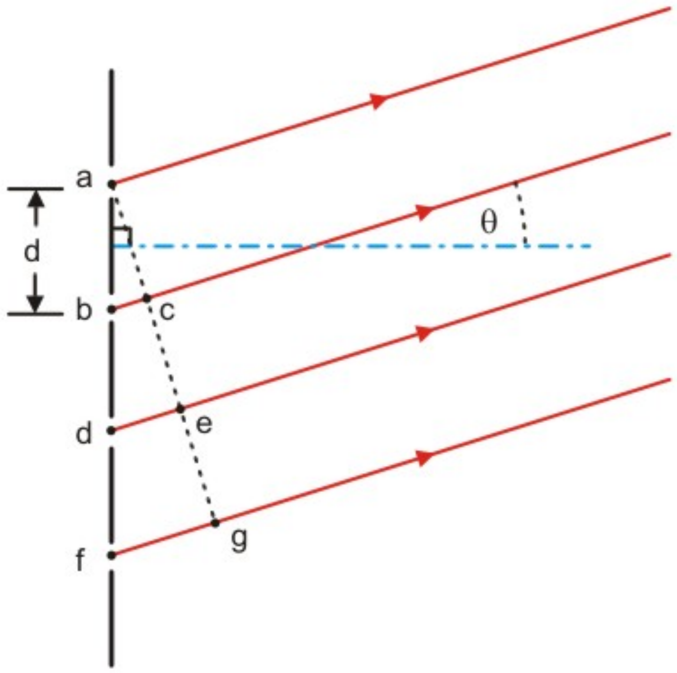
Measuring Light Wavelengths

If light goes through small slits, it emerges in circular waves.



In the forward direction, they are all in phase.
But they are also in phase for some other directions.

If the slit spacing is d, the path-length difference between slits is $\Delta x = d\sin\theta$.



If the wavelength is λ, the waves will be in phase if

$$d\sin\theta = n\lambda.$$

This is different from Bragg diffraction:
  θ is measured from the normal, not the surface.
  The factor of 2 is missing.

Diffraction Facts

$$d\sin\theta = n\lambda \Rightarrow \sin\theta = n\frac{\lambda}{d}$$

If $d >> \lambda$, then θ will be very small, and diffraction is invisible.

If $\lambda > d$, $\sin\theta > 1$, so diffraction is impossible.

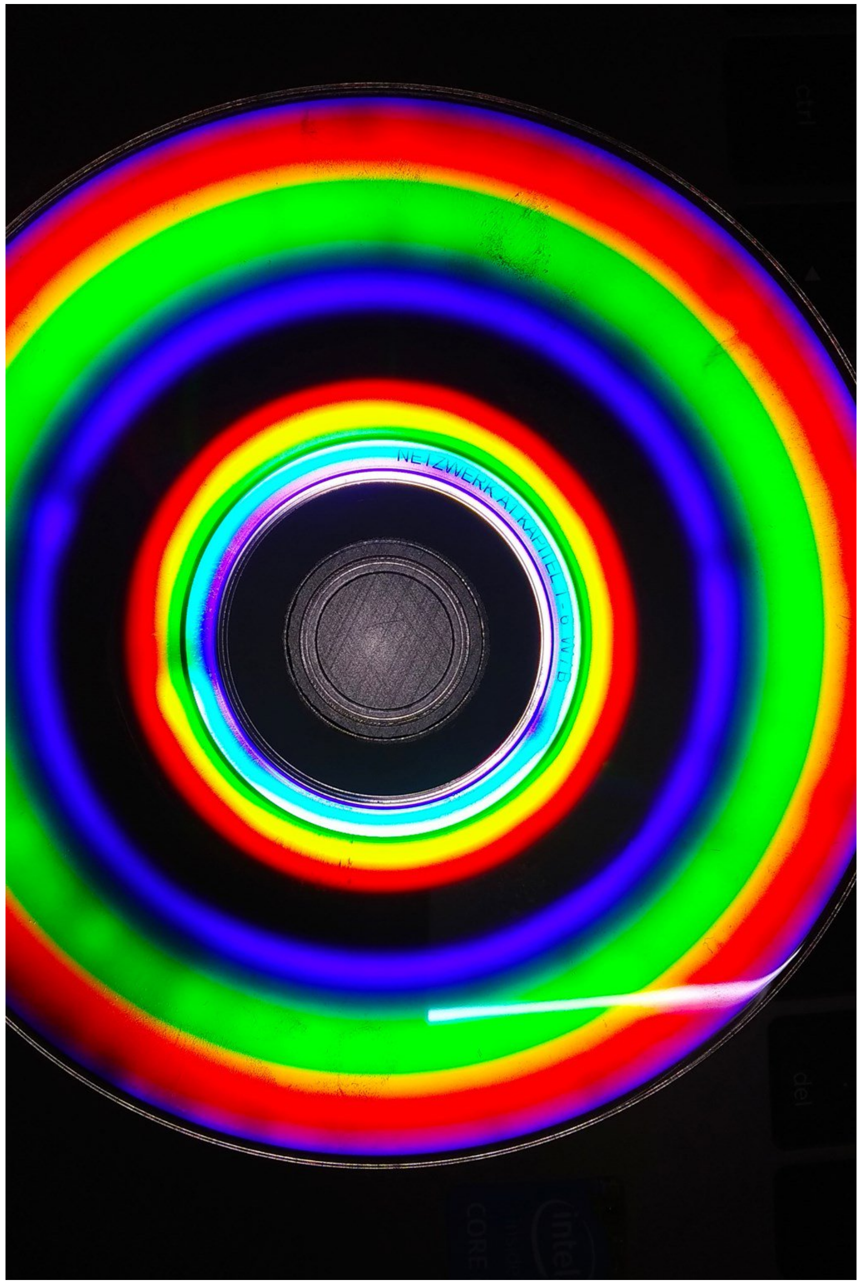
So the features we are diffracting from must be comparable to the wavelength for diffraction to be visible.

The $n = 0$ light goes straight through.

There is diffraction for $n = 1$, also for $n = 2$ at larger angle, etc.

CD Diffraction

Shine the flashlight from your phone
on the center of a CD from about 10 cm.



If you center it properly,
you get rings of color.

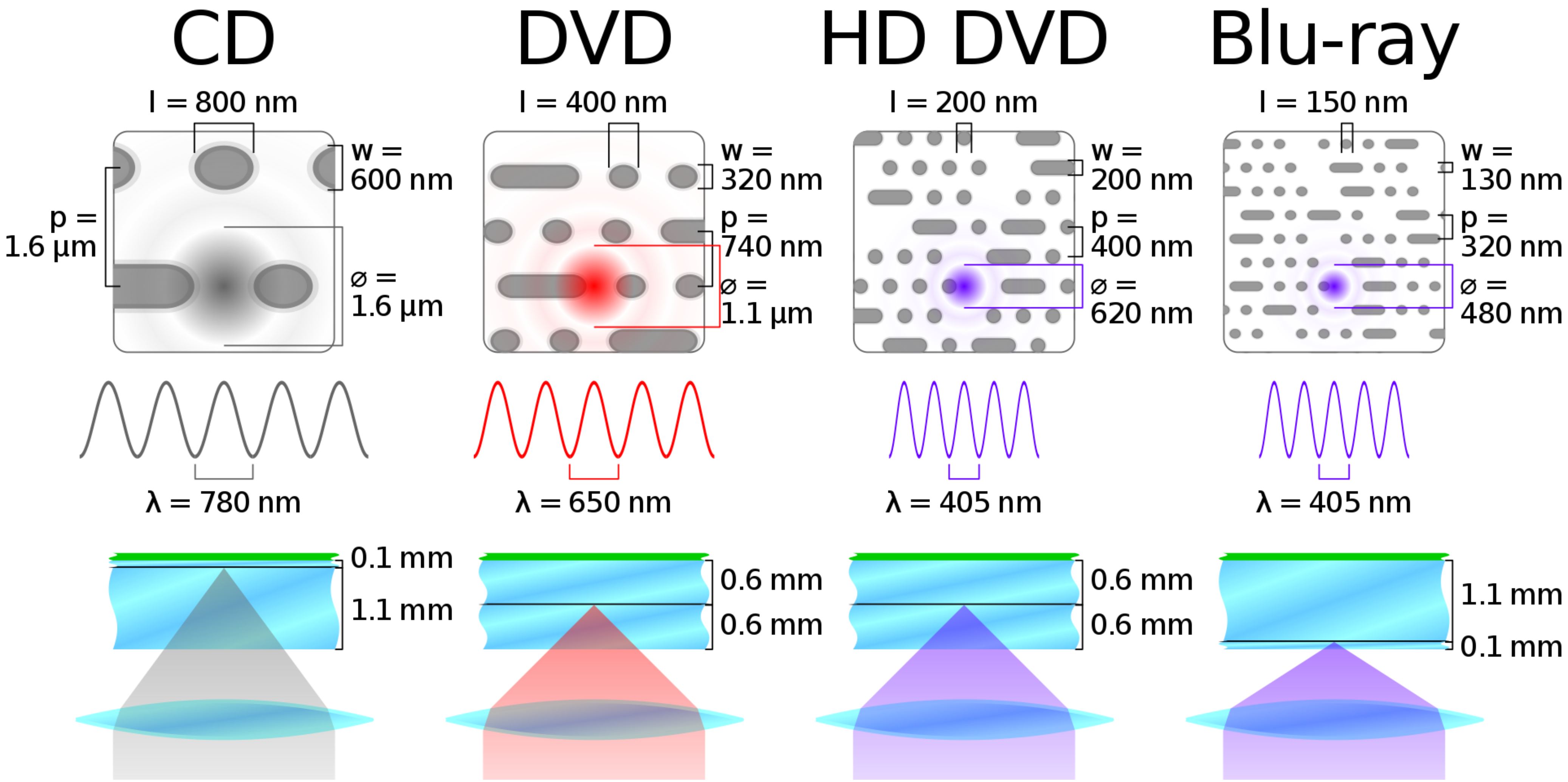
The tracks on a CD are 1600 nm
apart, which is comparable to the
wavelength of light.

The diffraction angle θ
depends on wavelength.

Purple or blue is small angle, red is large angle.

$n = 1$ is the small-angle ring, $n = 2$ is the larger angle.

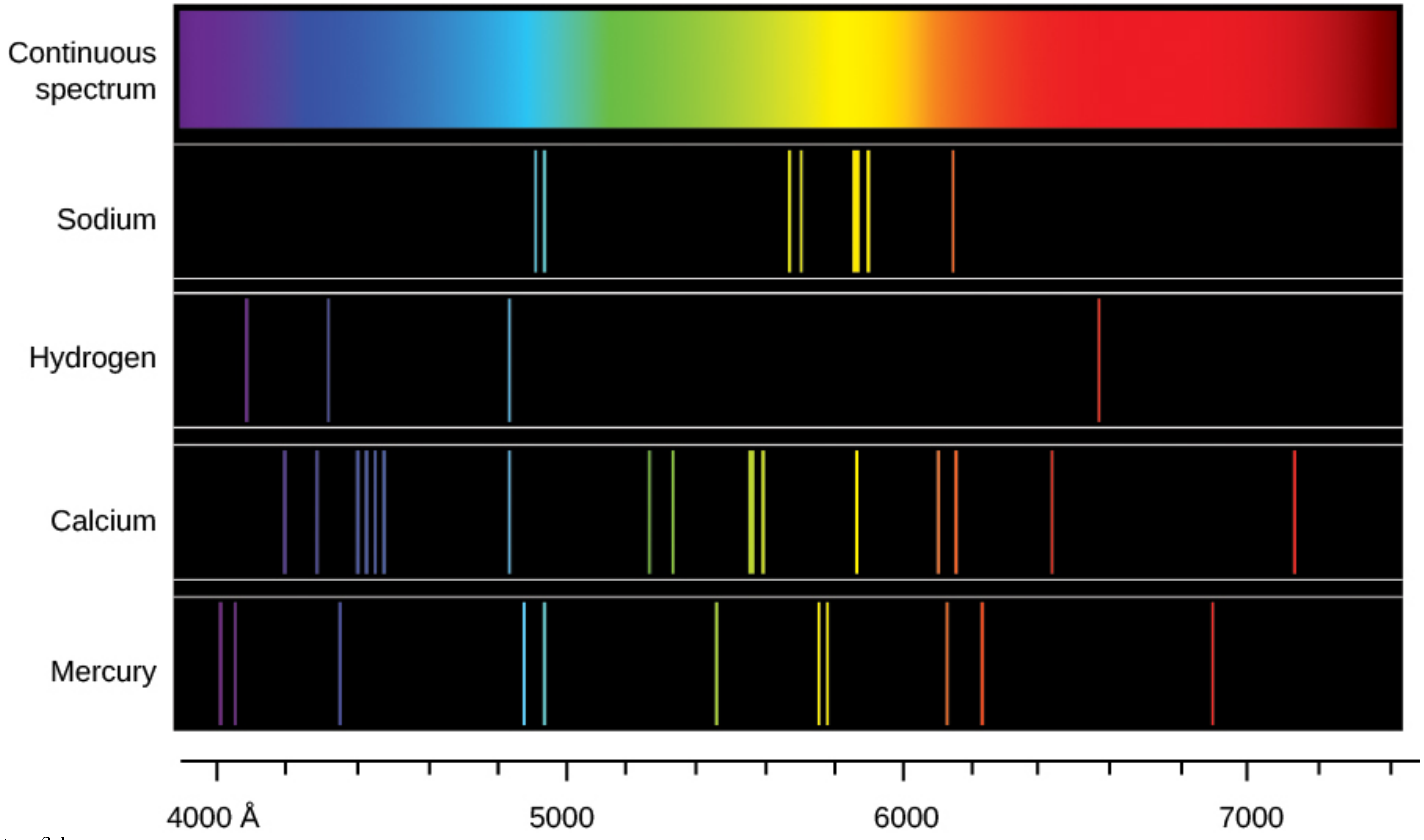
Other Discs


CD
l = 800 nm
w = 600 nm
p = 1.6 µm
⌀ = 1.6 µm
λ = 780 nm
0.1 mm
1.1 mm
DVD
l = 400 nm
w = 320 nm
p = 740 nm
⌀ = 1.1 µm
λ = 650 nm
0.6 mm
0.6 mm
HD DVD
l = 200 nm
w = 200 nm
p = 400 nm
⌀ = 620 nm
λ = 405 nm
0.6 mm
0.6 mm
Blu-ray
l = 150 nm
w = 130 nm
p = 320 nm
⌀ = 480 nm
λ = 405 nm
1.1 mm
0.1 mm

Spectral Emission Lines

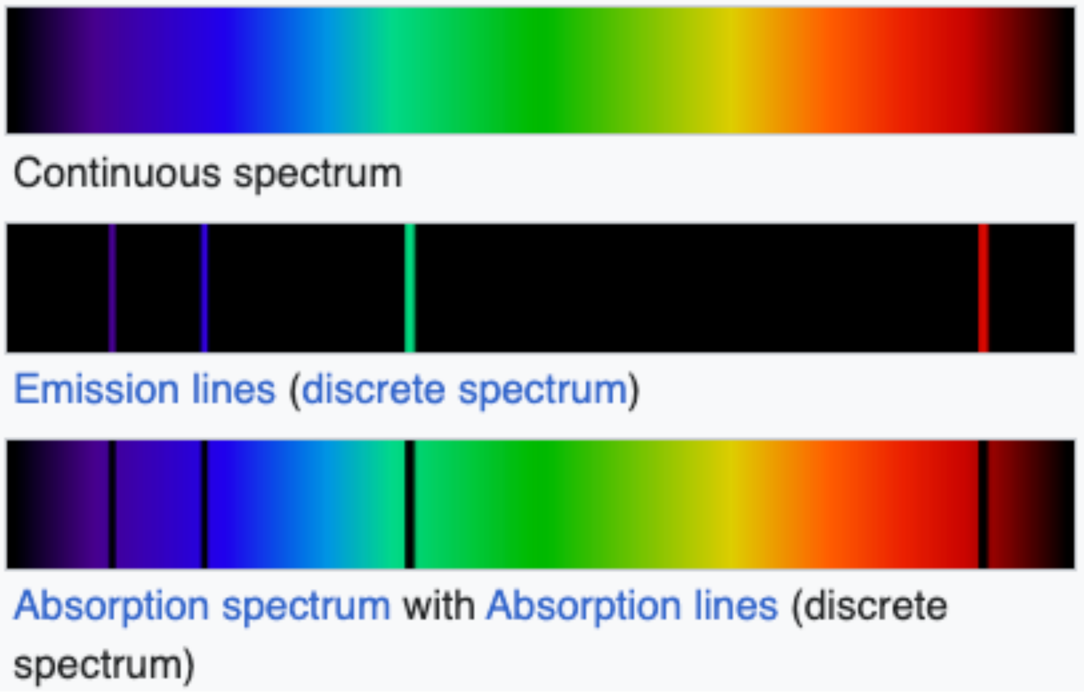
Hot dense objects emit black-body radiation, which has a continuous spectrum that depends on the temperature, but not on the substance.

Low density things, like gas in a electrical discharge tube, emit at only a few discrete wavelengths. For most atoms, there is no simple prediction.



Absorption vs Emission

A gas can also <u>absorb</u> light at exactly the same wavelengths that it can emit.

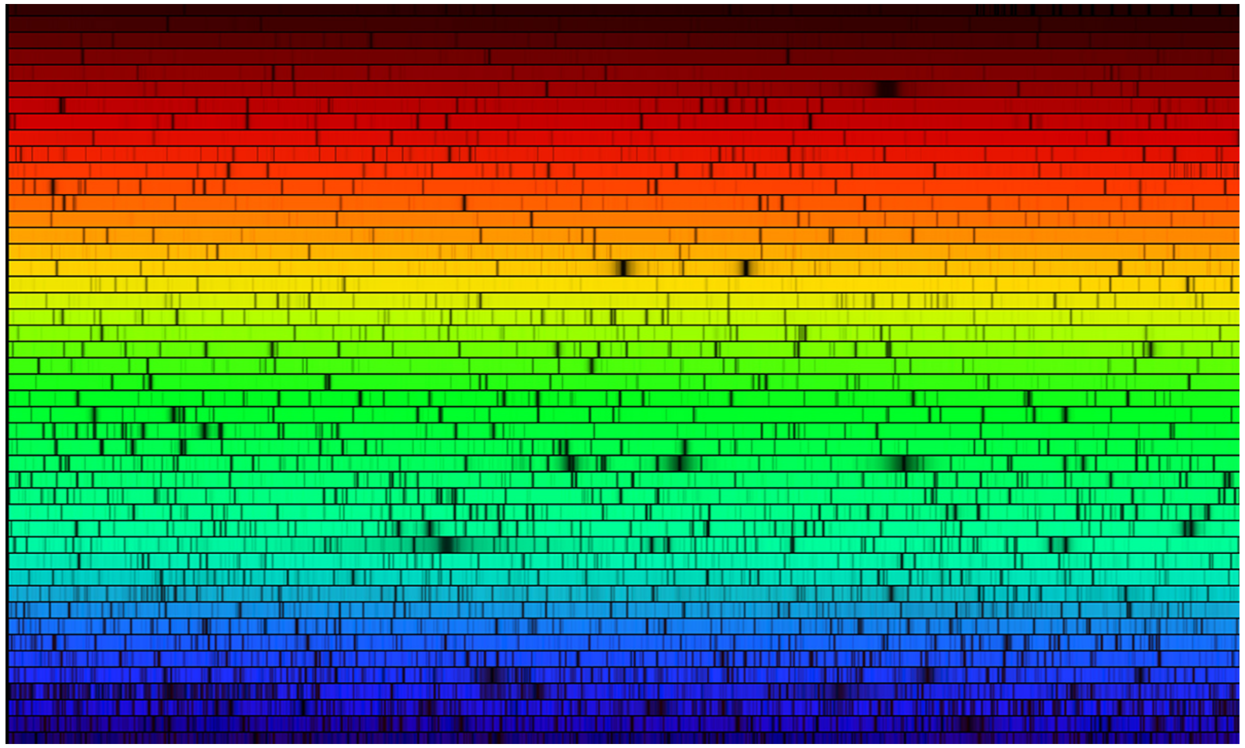


Continuous spectrum

Emission lines (discrete spectrum)

Absorption spectrum with Absorption lines (discrete spectrum)

This is a very high resolution spectrum from space of the Sun
(wrapped around many times)



The hot dense ionized gas gives a continuous black-body spectrum, but cooler gas above it absorbs specific wavelengths.

Absorption Bands

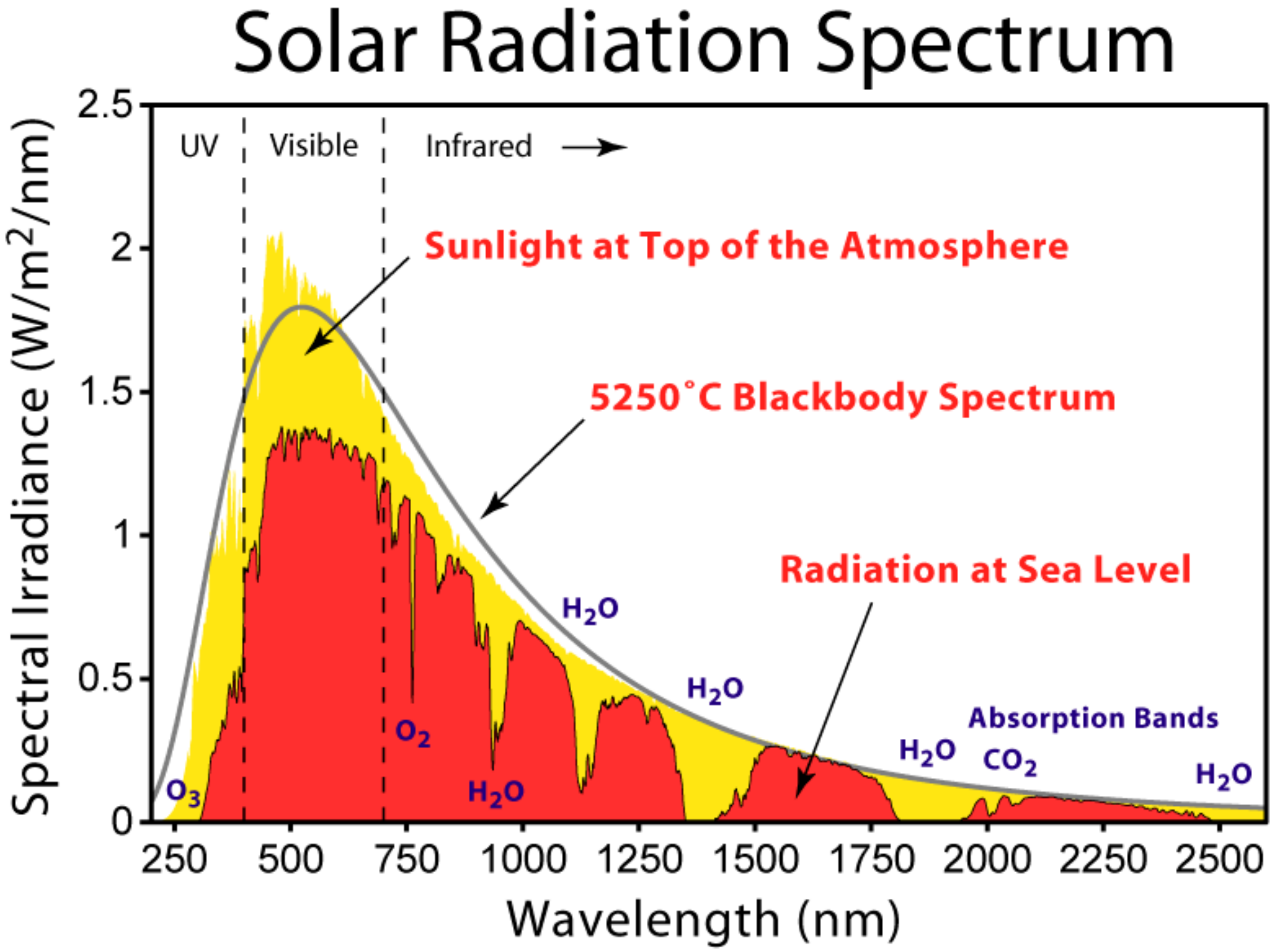
Atomic emission and absorption are due to "vibrations of electrons."
Electrons have low mass, so the $\omega = \sqrt{k/m}$ frequency is high.

Cooler atoms form molecules. The atoms in molecules can also "vibrate."
Atoms have much higher mass than electrons, so the frequencies are lower.

Molecules can also rotate.
That turns out to give absorption "lines" that are so close together that they look continuous, and they are called <u>absorption bands</u>.

The absorption bands are at longer than the human-visible spectrum.

Ozone (O_3) absorbs ultraviolet.
Water and CO_2 absorb infrared.



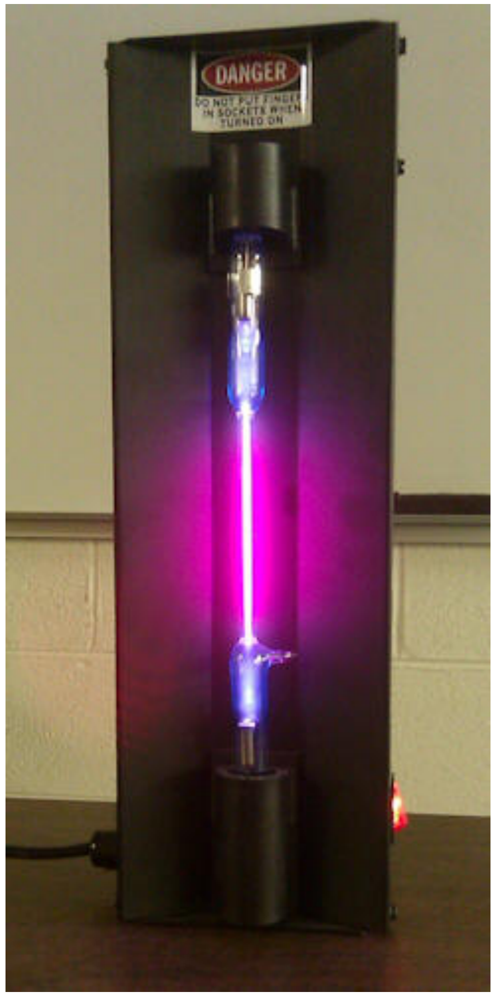
Hydrogen Emission Lines

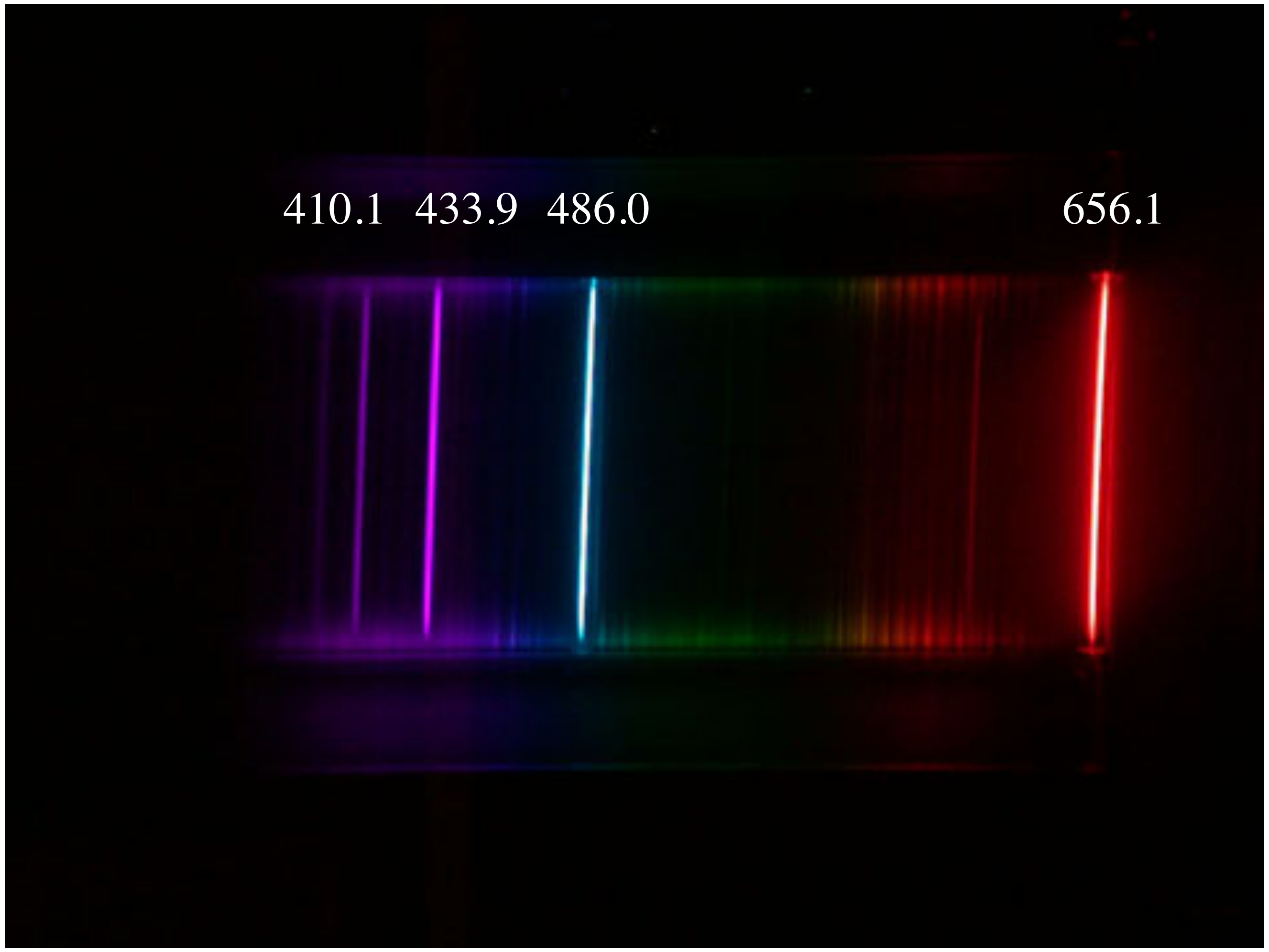
In 1865, Johann Balmer found a simple empirical formula for hydrogen lines.

$$\lambda = B\frac{n^2}{n^2-4} \quad \text{with } B = 364.5 \text{ nm and } n > 2$$

$$n=3: \quad 364.5\frac{3^2}{3^2-4} = 656.1 \qquad n=4: \quad 364.5\frac{4^2}{4^2-4} = 486.0$$

$$n=5: \quad 364.5\frac{5^2}{5^2-4} = 433.9 \qquad n=6: \quad 364.5\frac{6^2}{6^2-4} = 410.1$$





Clicker Question

The Balmer series of spectral lines of Hydrogen continues into the ultraviolet.

What is the shortest Balmer series wavelength?

A. 91.13 nm

B. 182.3 nm

C. 364.5 nm

D. 410.1 nm

E. 820.2 nm

Clicker Answer

The visible Balmer series lines of Hydrogen continues into the ultraviolet.

What is the shortest Balmer series wavelength?

C. 364.5 nm

Balmer says $\lambda = B\dfrac{n^2}{n^2-4}$ with $B = 364.5$ nm with $n > 2$

Increasing n decreases the wavelength.

Taking n to infinity gives $\lambda = 364.5$ nm

Improved Balmer Formula

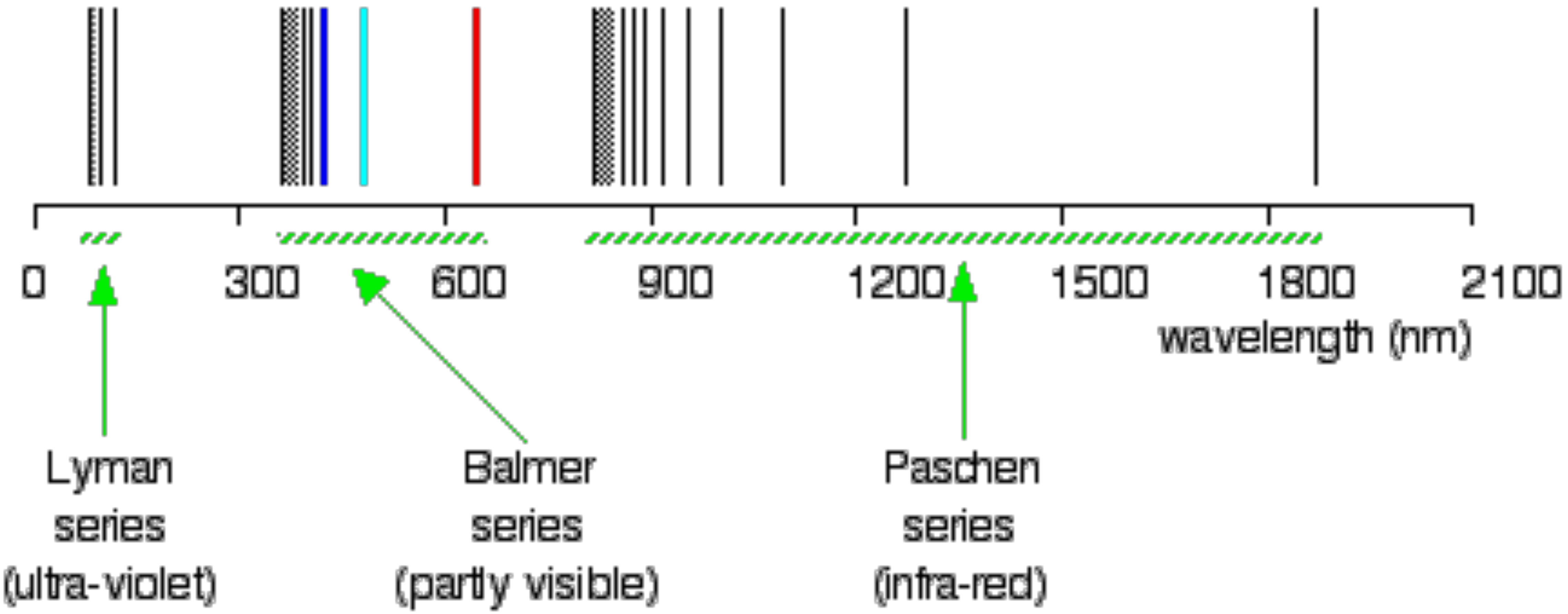
Other hydrogen lines in the ultraviolet and infrared were found later. They also fit a slightly modified formula.

$$\lambda = \frac{B}{4}\frac{n^2m^2}{n^2-m^2} = \quad \text{with } B = 364.5\text{ nm and } n > m$$

$m = 1:$ Lyman series (ultraviolet)

$m = 2:$ Balmer series (visible)

$m = 3:$ Paschen series (infrared)



Rydberg Formula

In 1888, Johannes Rydberg was studying the spectra of “alkali” atoms (sodium, potassium, cesium) because they seemed simple like hydrogen and proposed a variation on Balmer, using $1 / \lambda$ instead of λ :

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ with } n_2 > n_1$$

$$R = 1.097 \times 10^7 \text{ m}^{-1} = \frac{1}{91.13 \text{ nm}} = \frac{4}{B}$$

This predicts (some) alkali atom spectral lines pretty well.

The modern explanation is that the inner electrons “screen” the nuclear charge so even though the total nuclear charge is many protons, it looks like the same charge as a single proton, to an electron far from the atom.

Improved Rydberg Formula

If you can manage to remove all but one electron from an atom, which you can do by getting it very hot, then a modified Rydberg formula predicts the spectrum as exactly as it does for hydrogen:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \text{ with } n_2 > n_1$$

$$R = 1.097\times 10^7\ \text{m}^{-1} = \frac{1}{91.13\ \text{nm}}$$

$$Z = \text{ atomic number (nuclear charge)}$$

This was verified for Li^{+2}, Be^{+3}, etc.

It works for He^{+1} too, but no one had any at the time.

Rydberg and Photon Energy

Using Planck's formula $E = hf = \frac{hc}{\lambda}$ we can turn Rydberg's formula into a statement about photon energies:

$$E = \frac{hc}{\lambda} = hcRZ^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$= 1240 \text{ eV-nm} \cdot \frac{1}{91.13 \text{ nm}} \cdot Z^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

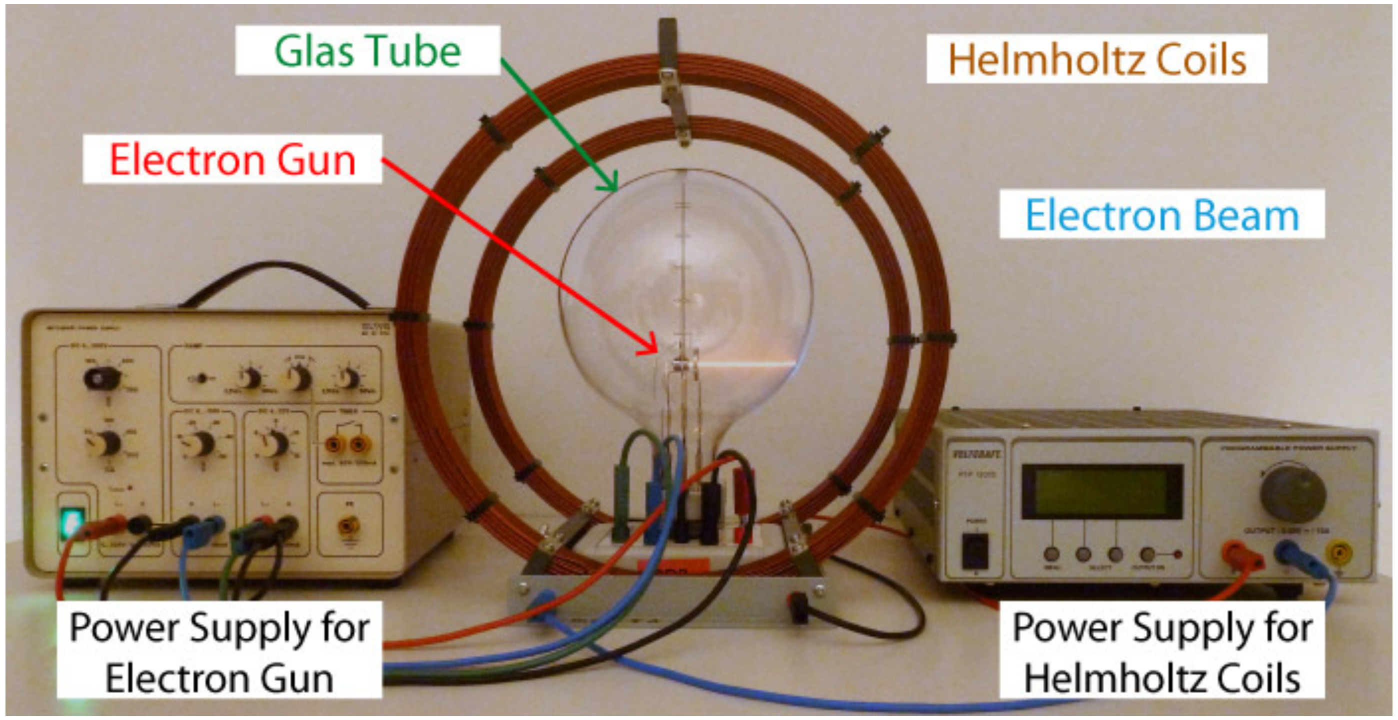
$$= 13.598 \text{ eV} \cdot Z^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

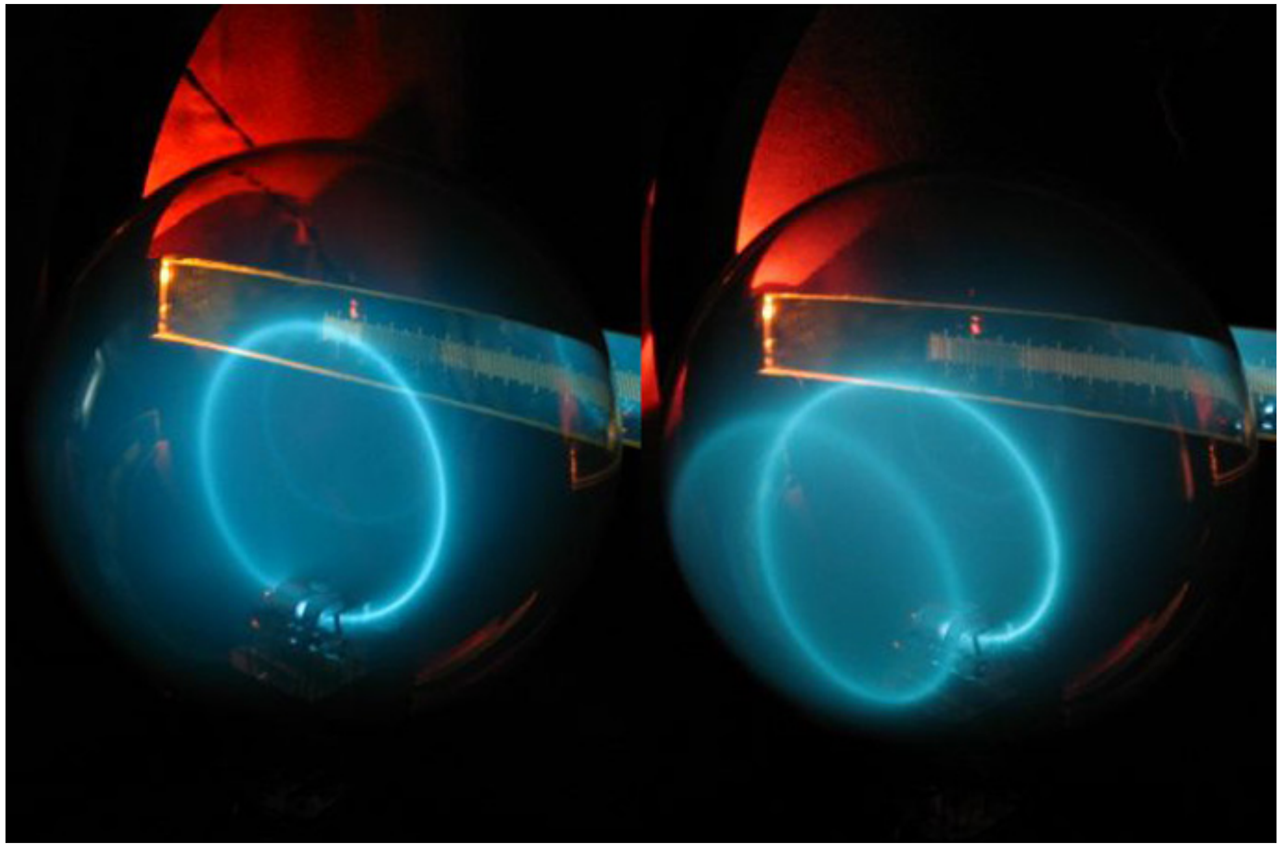
Cathode Rays

Spectral lines come from gas-discharge tubes. With better vacuum and higher voltage, we get cathode rays, and X-rays. At first, neither was understood.

X-rays were not deflected by magnetic or electric fields, so they must be neutral.

Cathode rays are deflected by magnetic fields, so they must be charged.



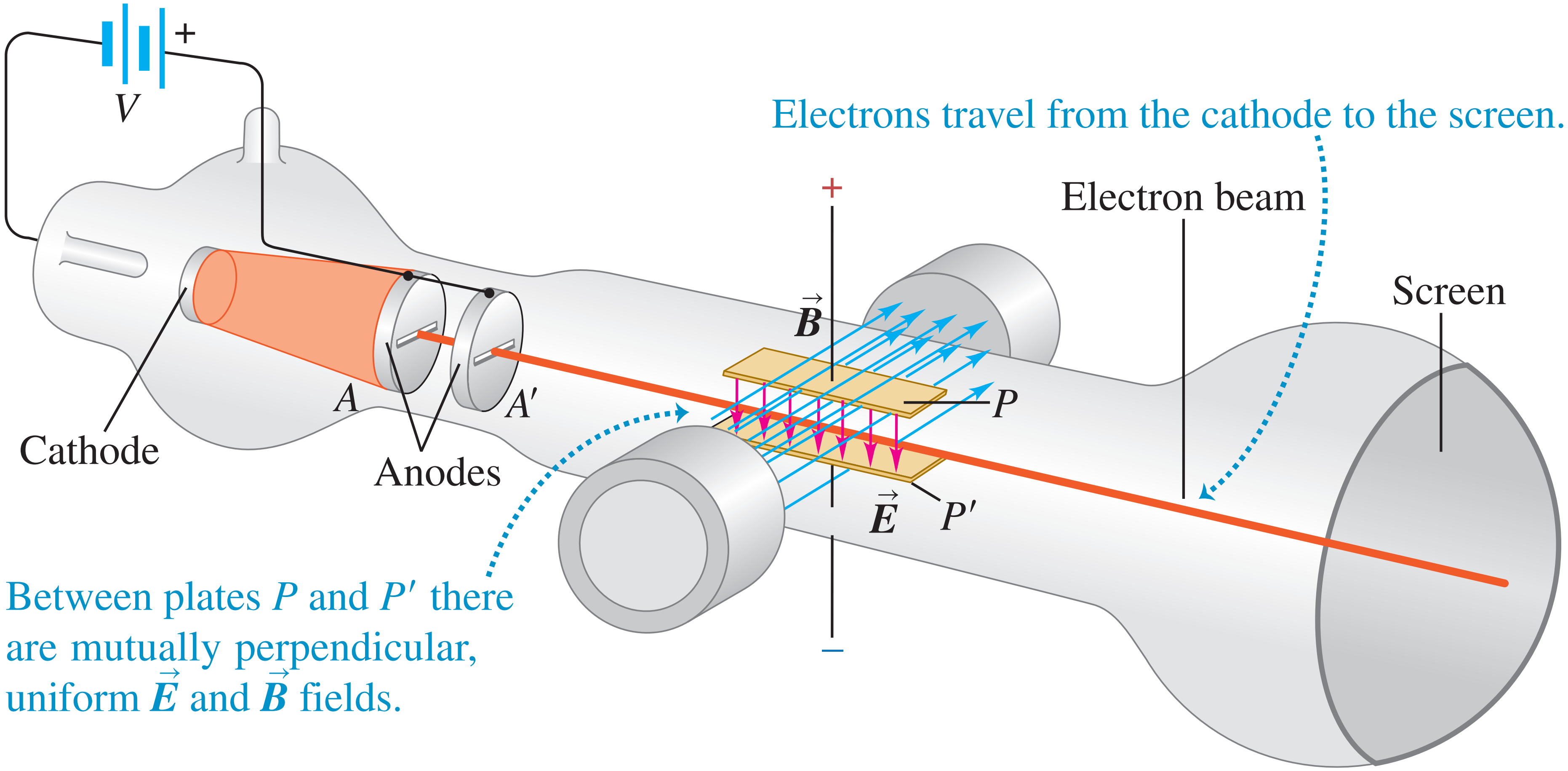


JJ Thomson Experiment

Cathode rays did not seem to be deflected by electric fields, which was strange.

The explanation is that the vacuum in cathode ray tubes was not very good,
so the residual gas got ionized, and shorted out any electric field.

JJ Thomson had much better vacuum, and set up an electric field
at right angles to the magnetic field, so the deflections were in the same plane,
and opposite to each other.



Electron Charge to Mass Ratio

Thomson adjusted the fields so the deflection cancelled.
This determines the velocity without knowing the mass.

$$F_{\text{magnetic}} = qvB = F_{\text{electric}} = qE \quad \rightarrow \quad v = \frac{E}{B}$$

Then he could determine the charge to mass ratio of the electron

$$\text{Energy } U = qV = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{E}{B}\right)^2 \quad \rightarrow \quad \frac{q}{m} = \frac{1}{2V}\left(\frac{E}{B}\right)^2$$

Electron Charge to Mass Ratio 2

Chemists could measure how much mass of a metal could be electroplated
by a Coulomb (Amperes times seconds) of charge.

So chemists knew the charge to mass ratio for the elements,
but they still didn't know the absolute mass of any atom.

The highest charge to mass ratio was for hydrogen (charge 1, and lowest mass).

The ratio for cathode rays was ~ 2000 times higher than for hydrogen !

JJ Thomson also showed that cathode rays had the same charge to mass ratio,
no matter what the cathode material was.

Thomson’s Plum-Pudding Atom

Chemists knew atoms had a size (but not what it was).

Cathode rays travelled farther through gases than they would
if cathode rays were the same size as atoms, so they must be smaller.

Thomson guessed that atoms were balls of positive charge,
with his negative electrons inside.

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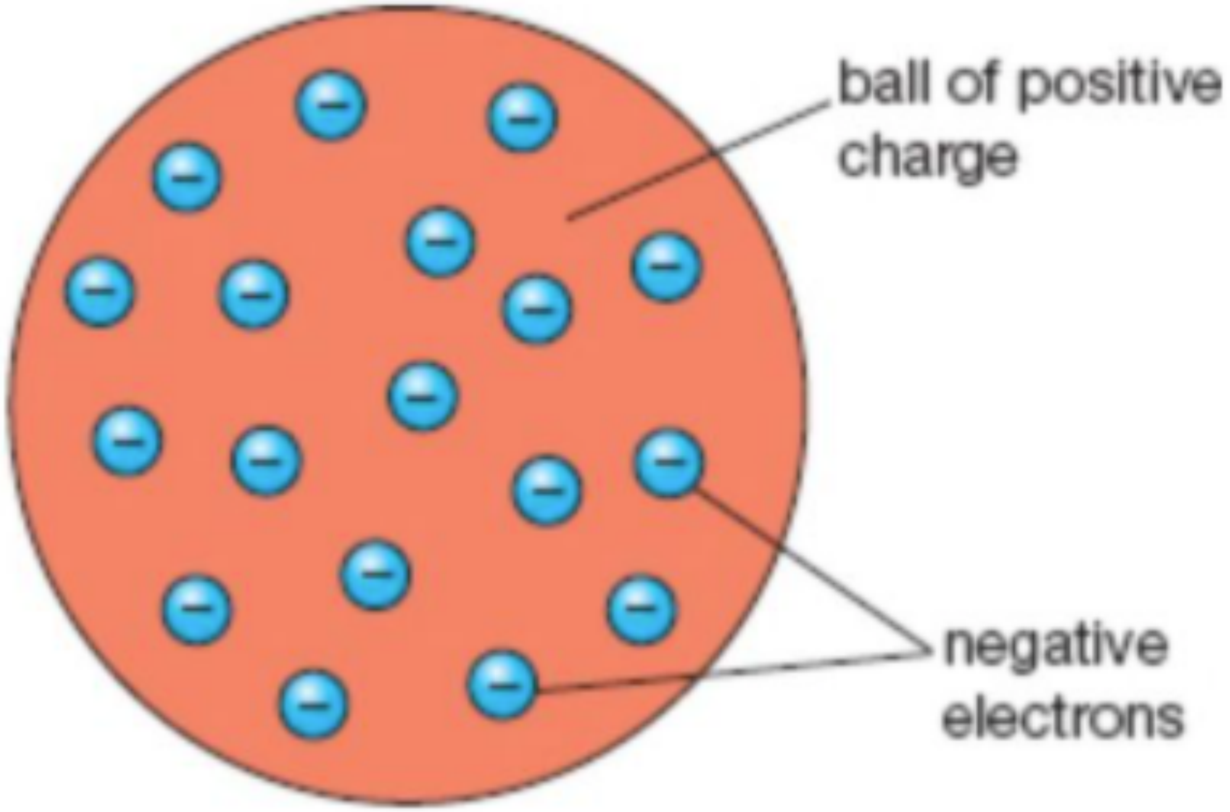
with his negative electrons inside.

Maybe oscillations of the electrons
inside the positive ball could
explain spectral lines?



Scattering Models

The electric field outside an atom should be zero
because the positive and negative charges would cancel.

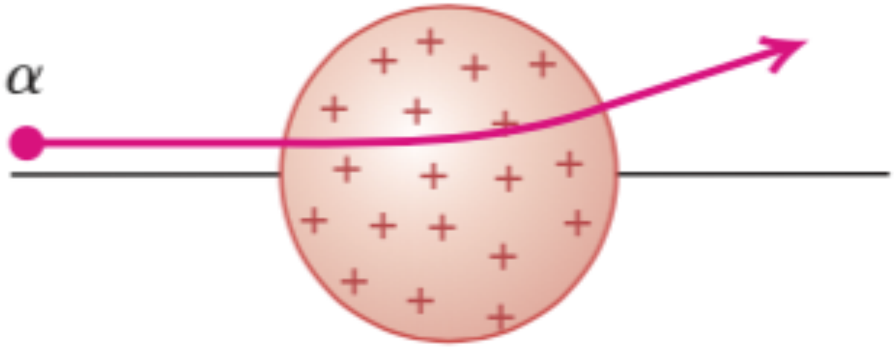
Inside the atom, the electric field should mostly cancel, except near an electron.

But electrons have low mass, so they shouldn't deflect the much heavier alpha particles.

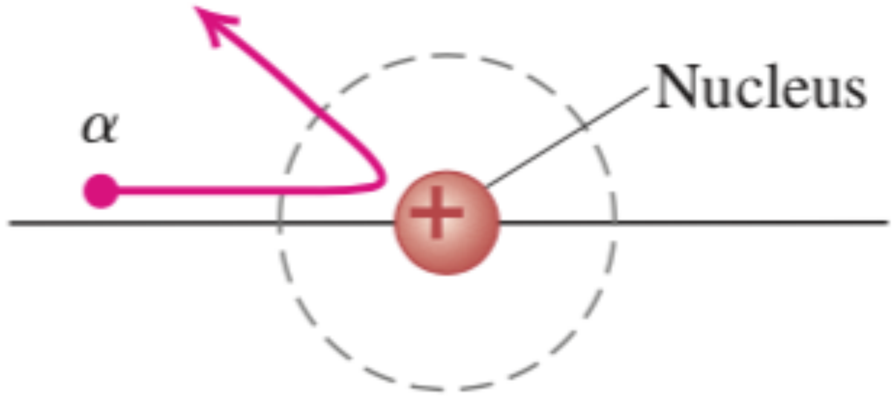
Rutherford found that quite a few alpha particles scattered through large angles.

That would only be possible
if the positive charge, and mass,
were concentrated in a small volume.

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.



(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).

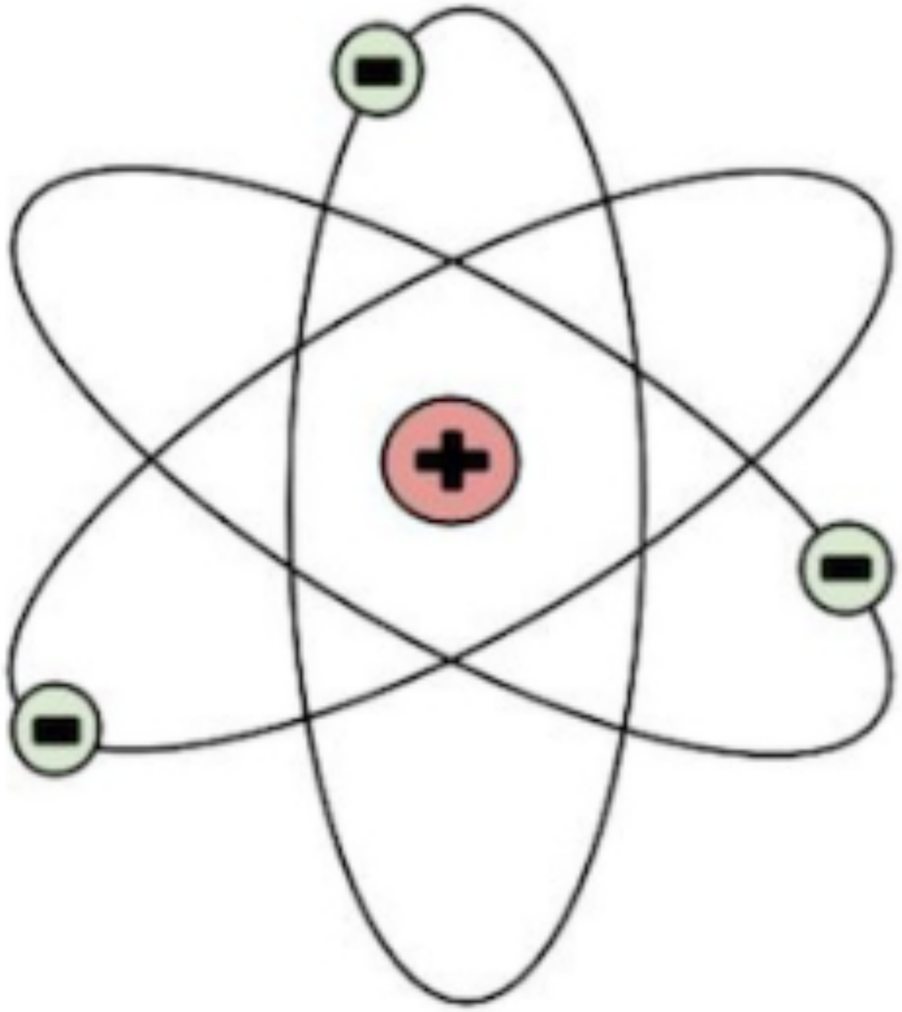


Rutherford Atom

All positive charge, and almost all of the mass, is concentrated in a small nucleus.

Electrons orbit the positive charge like planets.

All atoms of an element have the same charge, and same number of electrons



Rutherford Orbit Energy

With the usual convention that potential energy is zero at infinity,

and for an attractive force, the potential energy at r is $E_{\text{potential}} = -\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r}$.

The kinetic energy is $E_{\text{kinetic}} = \frac{1}{2}mv^2$, so $E_{\text{total}} = E_{\text{kinetic}} + E_{\text{potential}} = \frac{1}{2}\left(mv^2\right) - \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r}$

Combine Coulomb's Law $F = \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r^2}$ and Newton's Law $F = ma$

with acceleration in a circular orbit $a = \frac{v^2}{r}$ to get $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r^2} = m\frac{v^2}{r}$,

and multiply both sides by r: $\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r} = mv^2$.

Plug into the total energy: $E_{\text{total}} = \frac{1}{2}\left(\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r}\right) - \frac{qQ}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{1}{2}\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r}$

Orbit Energy from Angular Momentum

Angular momentum is $\vec{L} = \vec{r} \times \vec{p} \rightarrow L = rmv \rightarrow v = \dfrac{L}{mr}$.

Plug v into the relation for a circular orbit $\dfrac{qQ}{4\pi\varepsilon_0}\dfrac{1}{r} = m\left(v^2\right)$ and solve for 1/r:

$$\frac{qQ}{4\pi\varepsilon_0}\frac{1}{r} = m\left(\frac{L^2}{m^2 r^2}\right) \rightarrow \frac{qQ}{4\pi\varepsilon_0} = \frac{L^2}{mr} \rightarrow \frac{1}{r} = \frac{qQ}{4\pi\varepsilon_0}\frac{m}{L^2}$$

Plug into the energy:

$$E_{\text{total}} = -\frac{1}{2}\frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{r}\right) = -\frac{1}{2}\frac{qQ}{4\pi\varepsilon_0}\left(\frac{qQ}{4\pi\varepsilon_0}\frac{m}{L^2}\right)$$

$$= -\frac{m}{2}\left[\frac{qQ}{4\pi\varepsilon_0}\right]^2\frac{1}{L^2}$$

Rutherford Atom Problems

Orbits can be any size, so atoms of a given element should have a range of sizes.

Elements form nice crystals, so atoms of an element are all the same size.

The period of the orbit depends on the size of the orbit.
So if light frequency is the same as the orbit frequency,
there should be a continuous spectrum, not discrete lines.

Circular orbital motion is accelerated motion.
And Maxwell says that accelerated charges radiate energy.
It turns out that the orbits should decay very rapidly.

$$t = \left|E\right|^{-3} \cdot \frac{2m^2c^3}{256\pi\varepsilon_0 q} = \left|E_{\text{eV}}\right|^{-3} \cdot 39.2\text{ ns-eV}^3$$

If E_0 is –1 eV, the decay time is 39.2 nanoseconds.
If E_0 is –13.6 eV, the decay time is 15.6 picoseconds.

Bohr Model

In 1913, Niels Bohr and Rutherford incorporated Planck's constant into the orbiting-electron model in a way that fixed a lot of the defects, and also predicted the Rydberg spectrum, and the Rydberg (or Balmer) constant.

They postulated that the angular momentum of the orbits could only be integer multiples of Planck's constant h, but divided by 2π (so energies are right).

This $\frac{h}{2\pi}$ is used so much that it is abbreviated as $\frac{h}{2\pi} = \hbar$.

Plug $L = n\hbar$ and $Q = Zq$ into $E = -\frac{m}{2}\left[\frac{qQ}{4\pi\varepsilon_0}\right]^2 \frac{1}{L^2}$ to get

$$E = -\frac{m}{2}\left[\frac{qZq}{4\pi\varepsilon_0}\right]^2 \frac{1}{n^2\hbar^2} = -\frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0\hbar}\right]^2 \frac{Z^2}{n^2}$$

Bohr Model Energies

The value of $\hbar = \frac{h}{2\pi} = \frac{6.626\times 10^{-34}}{2\pi} = 1.055\times 10^{-34}\ \text{J-s}$.

Evaluating the constants in $E = -\frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0\hbar}\right]^2 \frac{Z^2}{n^2}$ gives

$$\frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0\hbar}\right]^2 = \frac{9.109\times 10^{-31}}{2}\cdot\left[\frac{\left(1.602\times 10^{-19}\right)^2}{4\pi\cdot 8.854\times 10^{-12}\cdot 1.055\times 10^{-34}}\right]^2 = 2.177\times 10^{-19}\ \text{J}$$

Converting into electron Volts gives $\frac{2.177\times 10^{-19}\ \text{J}}{1.602\times 10^{-19}\ \text{J/eV}} = 13.598\ \text{eV}$.

Compare $E = -13.6\ \text{eV}\cdot\frac{Z^2}{n^2}$ to Rydberg's $E = 13.6\ \text{eV}\cdot Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

The spectral line energies are differences of Bohr orbit energies!

Bohr Model Radius

Take the energy-radius relation $E = \frac{1}{2}\frac{qZq}{4\pi\varepsilon_0}\frac{1}{r}$ and solve for $r = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0}\frac{Z}{E}$.

Plug in the Bohr energy $E = \frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0}\right]^2\frac{1}{\hbar^2}\frac{Z^2}{n^2}$ to get

$$r = \frac{1}{2}\frac{q^2}{4\pi\varepsilon_0}\frac{Z}{\frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0}\right]^2\frac{1}{\hbar^2}\frac{Z^2}{n^2}} = \frac{n^2}{Z}\frac{\hbar^2}{m}\frac{4\pi\varepsilon_0}{q^2}$$

The constant is

$$\frac{\hbar^2}{m}\frac{4\pi\varepsilon_0}{qq} = \frac{\left(1.055\times10^{-34}\right)^2}{9.109\times10^{-31}}\frac{4\pi\cdot8.854\times10^{-12}}{\left(1.602\times10^{-19}\right)^2} = 5.297\times10^{-11}\text{ m} = 52.97\text{ pm}$$

Overall $r = 52.97\text{ pm}\cdot\frac{n^2}{Z}$. For $n = Z = 1$, the diameter is 106 pm.

Bohr Orbit Velocity

Start with the angular momentum definition $L = r \cdot mv \rightarrow v = \frac{L}{m} \cdot \frac{1}{r}$

and the Coulomb orbit relation $\frac{1}{r} = \frac{Zq^2}{4\pi\varepsilon_0} \frac{m}{L^2}$ to get $v = \frac{L}{m} \cdot \frac{Zq^2}{4\pi\varepsilon_0} \frac{m}{L^2} = \frac{Z}{L} \frac{q^2}{4\pi\varepsilon_0}$

Plug in angular momentum $L = n\hbar$ to get $v = \frac{Z}{n} \frac{q^2}{4\pi\varepsilon_0 \hbar}$ or $\beta = \frac{v}{c} = \frac{Z}{n} \frac{q^2}{4\pi\varepsilon_0 \hbar c}$.

Evaluate the constants:

$$\frac{q^2}{4\pi\varepsilon_0 \hbar c} = \frac{\left(1.602 \times 10^{-19}\right)^2}{4\pi \cdot 8.854 \times 10^{-12} \cdot 1.055 \times 10^{-34} \cdot 2.998 \times 10^{8}} = 7.293 \times 10^{-3}$$

So $\beta = 7.293 \times 10^{-3} \cdot \frac{Z}{n}$. For Hydrogen with $Z = n = 1$, relativity is negligible.

But for Uranium, $Z = 92$, and $n = 1$, we get $\beta = 0.6709$, which gives $\gamma = 1.349$. So relativity is not negligible for heavy elements.

Reduced Mass

We actually cheated in the classical orbit calculation, because we treated the electron as being in orbit around a fixed centre. In reality, the electron and the proton both orbit around the centre of mass of the system.

If the calculation is done right, for Hydrogen the electron mass gets replaced by the reduced mass $\mu = \frac{m_e \cdot m_p}{m_e + m_p}$. Since $m_p \approx 2000 \cdot m_e$, this is a very small change from the electron mass in $E_n = -\frac{m}{2}\left[\frac{q^2}{4\pi\varepsilon_0\hbar}\right]^2 \frac{Z^2}{n^2}$

For ionized Helium, $\mu = \frac{m_e \cdot m_{He}}{m_e + m_{He}}$, which is even closer to the electron mass.

But spectroscopy can be done precisely enough that it was noticable that the Helium energies were not exactly $Z^2 = 4$ times the Hydrogen energies, but including the reduced masses explained the differences.

Bohr Model Pros

The quantization of angular momentum defines specific orbits
with specific predicted energies and radii.

The predicted energies come out exactly right for hydrogen,
and the other cases where the Rydberg formula works.

But note that Bohr needed the fudge factor of 2π to get the energies right,
without explaining why.

The lowest energy orbit has a definite size, so all atoms are the same size,
and the size is sensible.

Bohr Model Cons

Bohr atoms are planar, but atoms are spherical.

A charge moving in a circular orbit generates a magnetic moment.

Ground state hydrogen atoms do have a magnetic moment,
but it’s not the value predicted by the Bohr model.

In Schrodinger quantum mechanics, the ground state of hydrogen is spherical.

It has $L = 0$, so there is no orbital motion, and no magnetic moment from it.

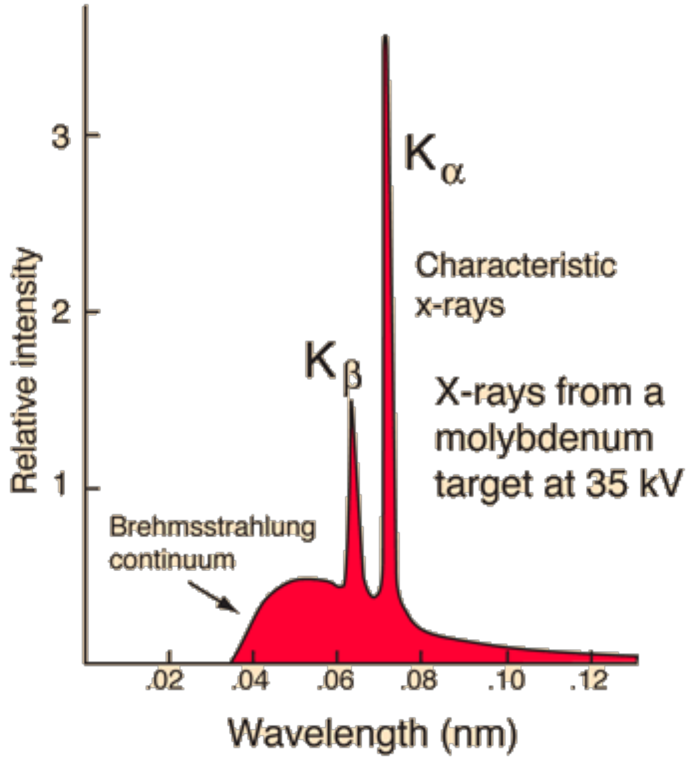
The magnetic moment of a hydrogen atom is due to the intrinsic magnetic moment of the electron, due to its “spin” angular momentum, plus a small contribution from the magnetic moment from the “spin” of the proton.

X-Ray Emission Lines

As the atomic number Z goes up, the Rydberg wavelengths go into the UV and eventually the X-ray range.

It also gets increasingly hard to get rid of all but one electron.

In 1913-14, Henry Moseley used Bragg diffraction to measure the X-ray emission lines from most elements.



He found the wavelength of the most prominent X-ray line (called Kα) varied smoothly like $1/Z^2$, for almost all the elements.

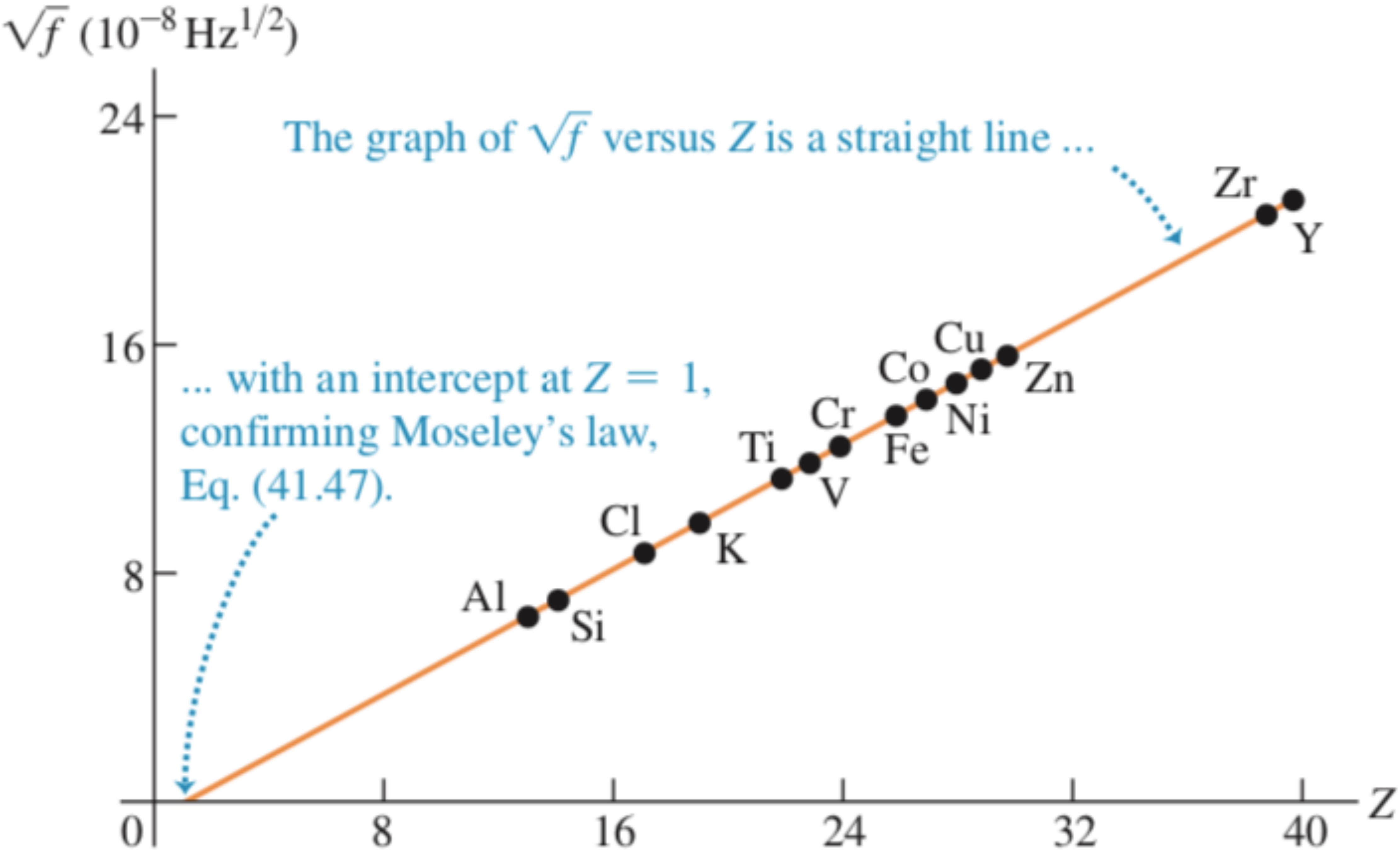
This was very different from the fairly random visible spectrum for different elements.

He did the classic “transform the data until it becomes a straight line” trick.

Moseley Plot

Measure Kα wavelength λ as a function of atomic number Z.

Plot $\sqrt{f} = \sqrt{c/\lambda}$ as a function of Z, and fit a line: $\sqrt{f} = a\cdot\left(Z-b\right)$ with $b = 1$



Moseley's Law

Squaring both sides of $\sqrt{f} = a \cdot (Z - b)$ gives $f = a^2 \cdot (Z - b)^2$.

Multiplying both sides by h gives $E = hf = ha^2 \cdot (Z - b)^2$.

If we use eV as the energy unit, and $b = 1$, Moseley's fit becomes

$$E = 10.2\ \text{eV} \cdot (Z - 1)^2$$

This is Moseley's Law (for Kα X-ray emission).

Moseley’s Law 2

The atomic weights aren’t always in the same order as their atomic numbers.

An example is cobalt ($Z = 27$, $W = 58.9$) and nickel ($Z = 28$, $W = 58.7$)

Moseley was able to directly measure Z, rather than inferring it from masses.

There were some elements not known in 1914.

Moseley was able to definitely show that some Z values were missing.

Moseley would have surely won a Nobel Prize for his work,
but he was killed in WWI in the Battle of Gallipoli.

Interpreting Moseley’s Law

The improved Rydberg formula in terms of energy is

$$E_{\text{Rydberg}} = 13.6 \text{ eV } \cdot Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

If $n_1 = 1$ and $n_2 = 2$, the bracket term is 1 – 1/4 = 3/4, and 3/4 x 13.6 = 10.2, which is the slope in Moseley’s Law.

This implies that the Kα line comes from an electron in the second Bohr orbit dropping into the first Bohr orbit.

Moseley’s Law has $\left(Z-1\right)^2$ because there is one electron in the first Bohr orbit, which partially “screens” the nuclear charge, reducing it by one unit.

Using Moseley's Law

You can excite atoms to emit their characteristic Kα line by shining them with X-rays with a higher energy, or spraying them with high voltage electrons.

Modern X-ray detectors can measure X-ray photon energies accurately without complicated Bragg setups.

So you can rather easily determine the elemental composition of a sample.

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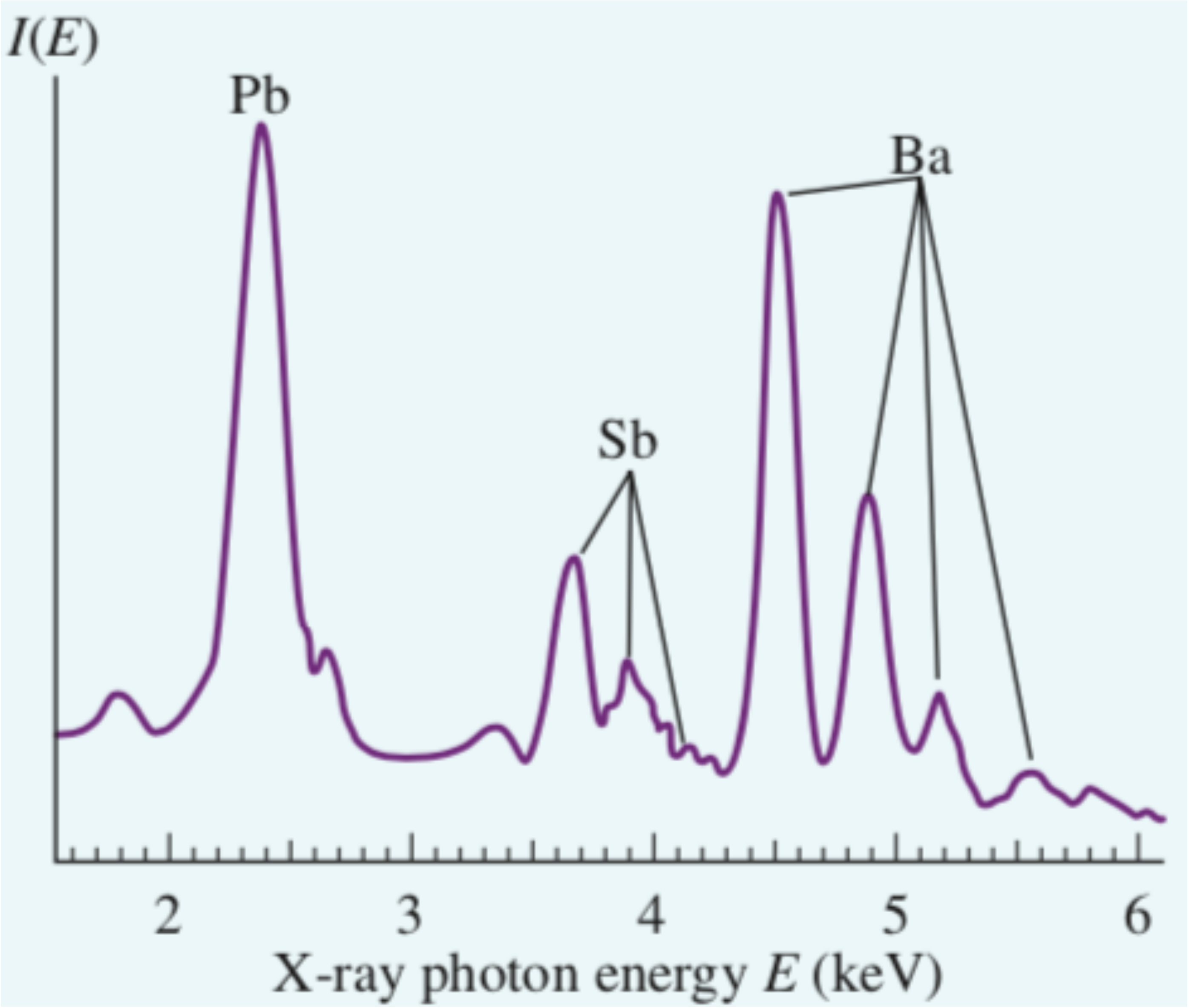
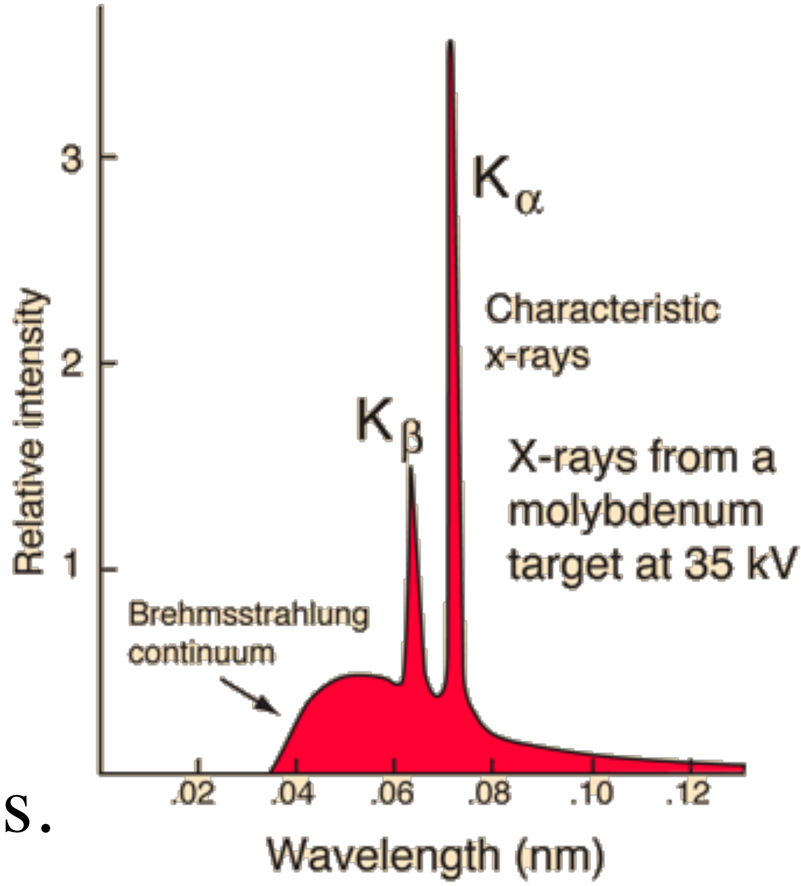


More Moseley's Law

There is also a $K\beta$ line at shorter wavelength (higher energy) from $n = 3$ to $n = 1$.

$$E = 13.6 \text{ eV} \left[\frac{1}{1} - \frac{1}{3^2} \right] (Z-1)^2 = 12.09 \text{ eV} \cdot (Z-1)^2$$

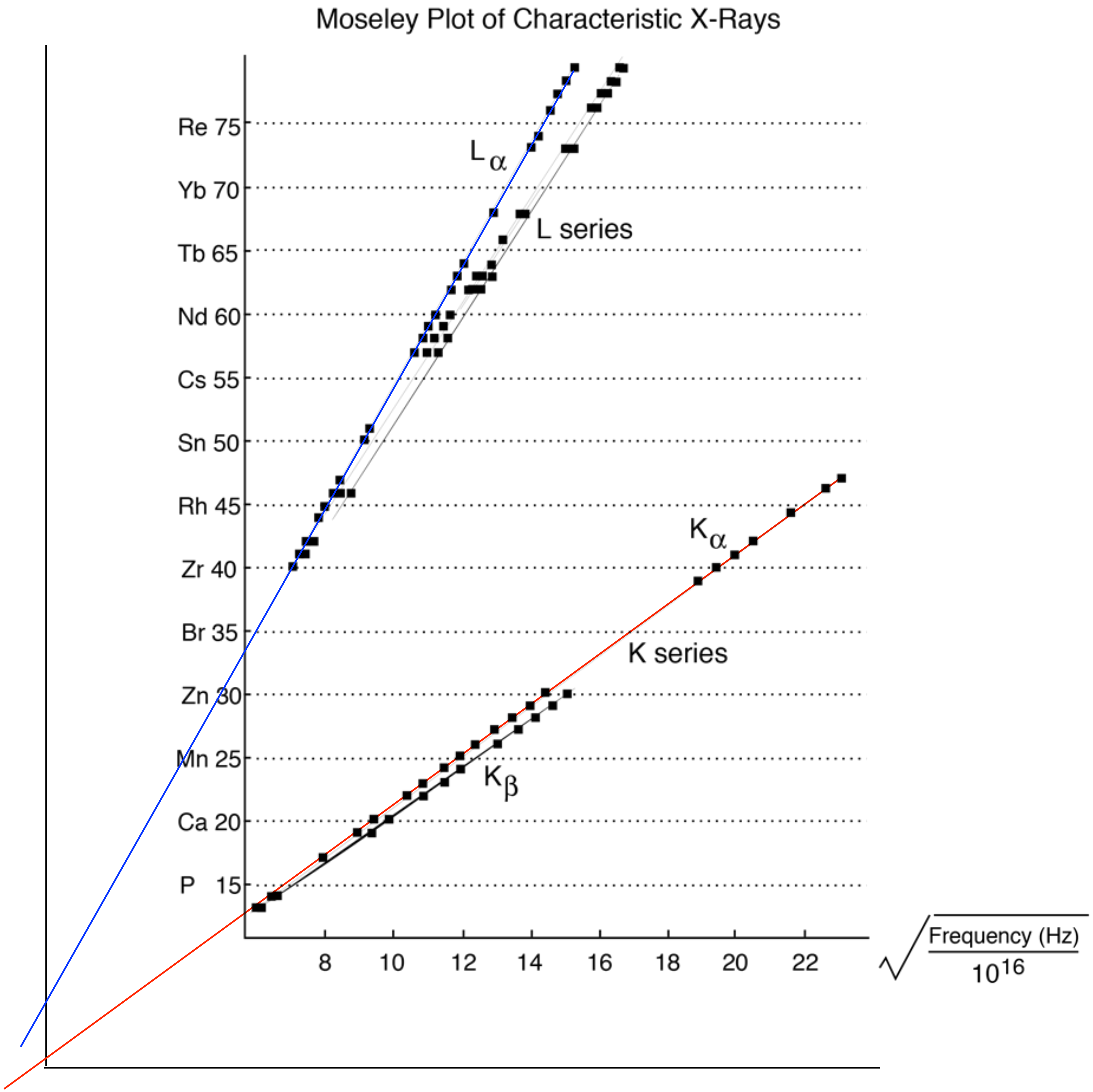


The transition from $n = 3$ to $n = 2$ is called the L-α line. There is a lot more "screening" in addition to the n changes.

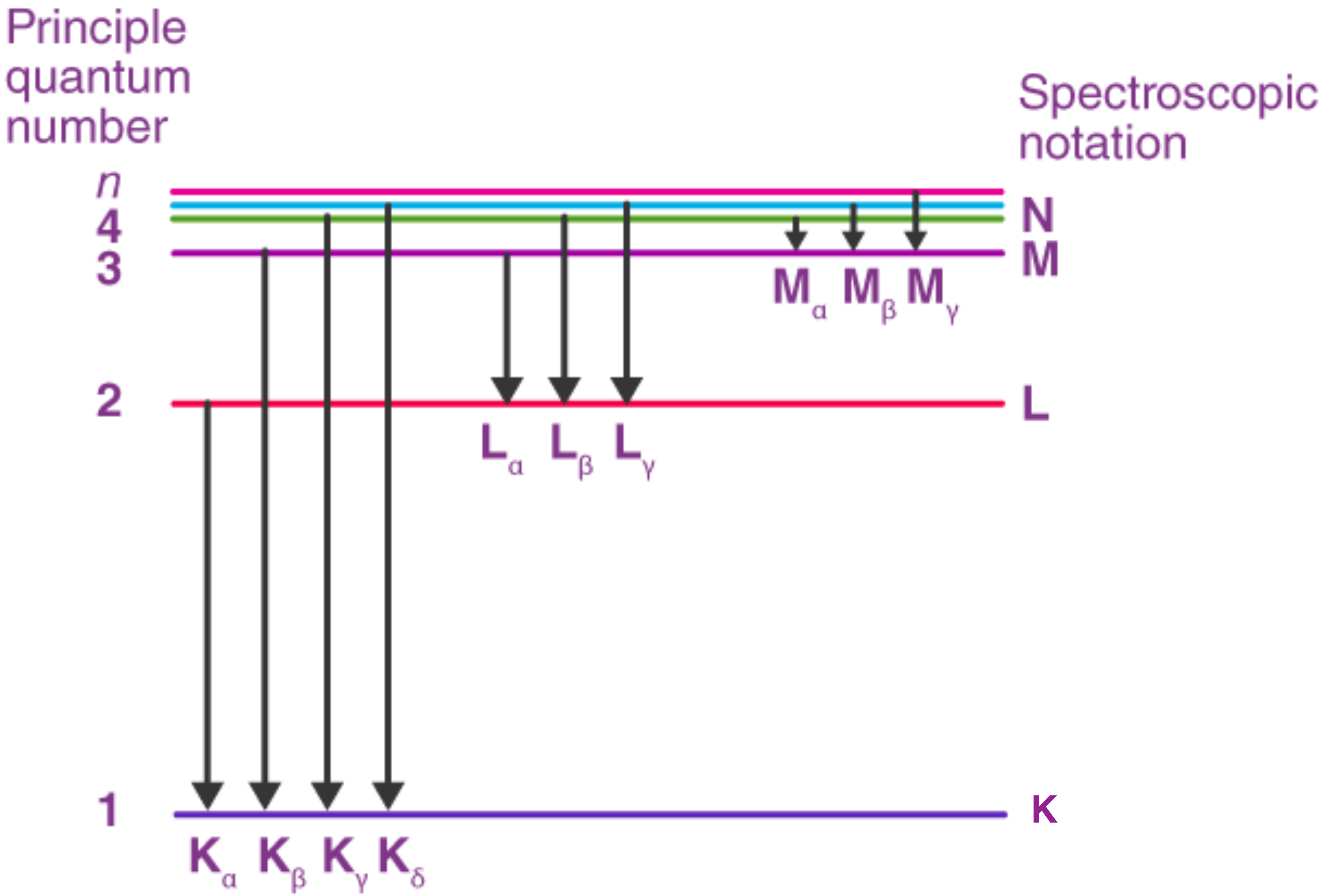
$$E = 13.6 \text{ eV} \left[\frac{1}{2^2} - \frac{1}{3^2} \right] (Z-7.4)^2 = 1.889 \text{ eV} \cdot (Z-7.4)^2$$

There are 2 electrons in a full n = 1 "shell," and 8 electrons in a full n = 2 "shell," so we would expect the screening with one electron missing to be 9. So 7.4 isn't too surprising.

Another Moseley Plot

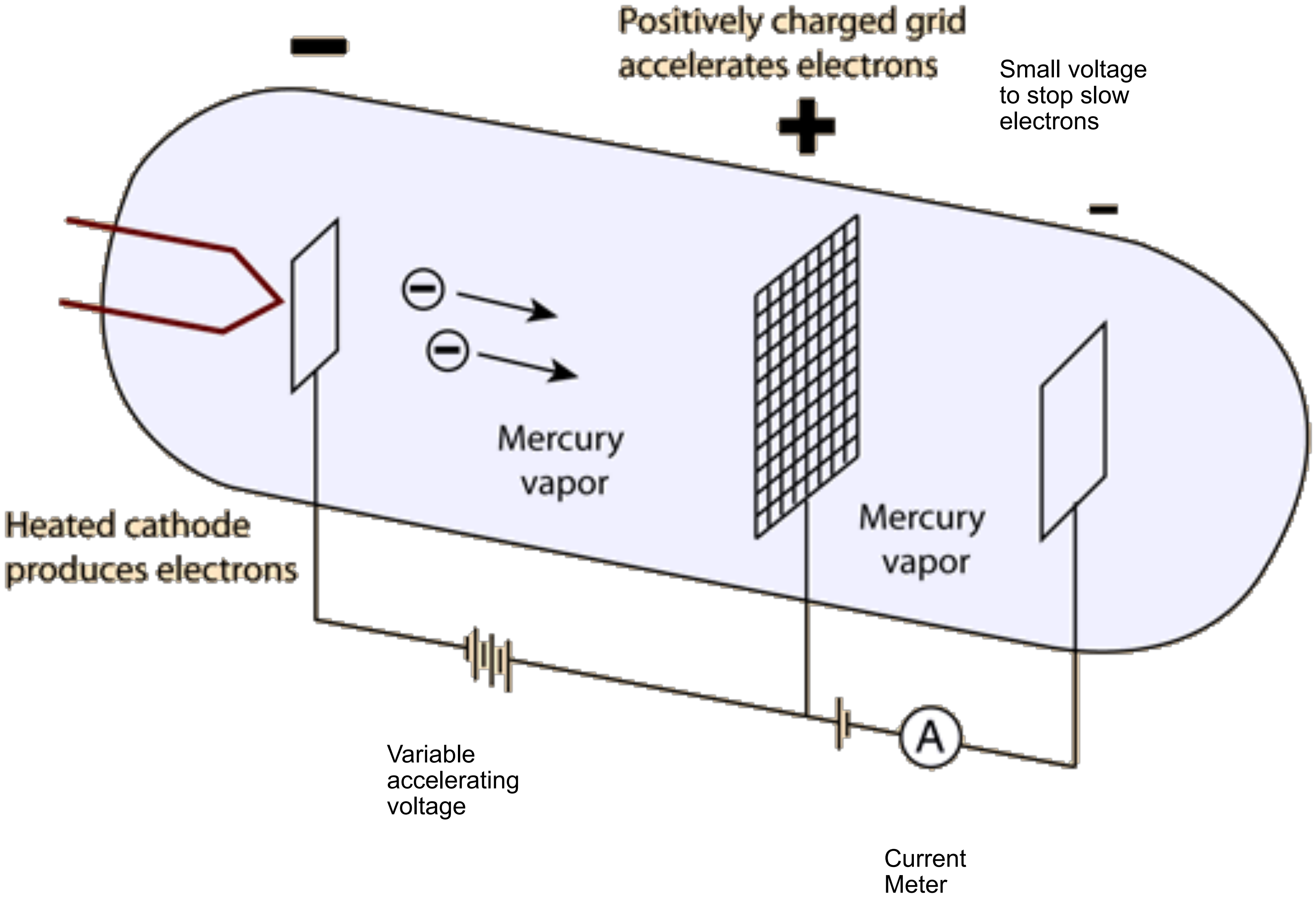


X-Ray Line Notation

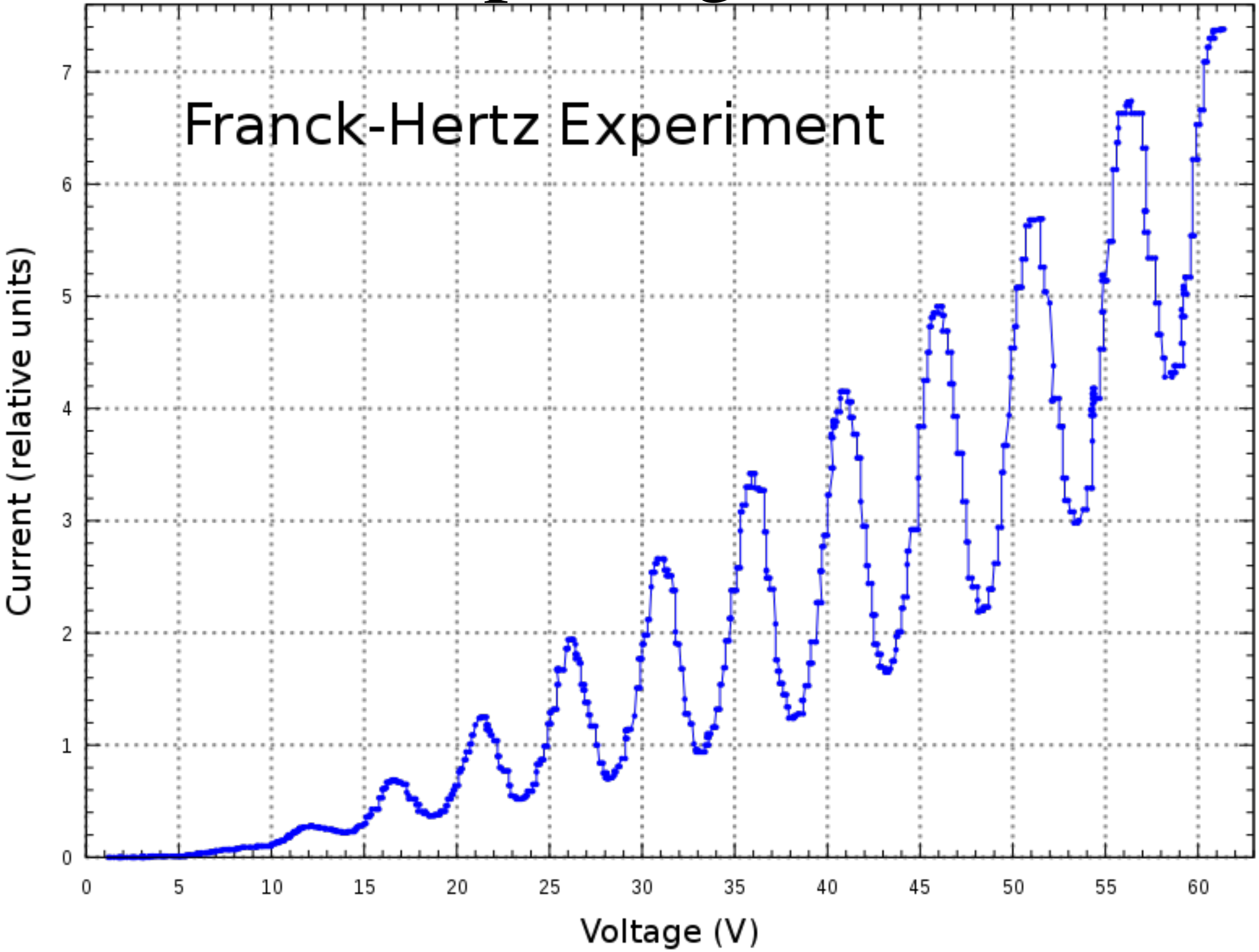


Franck-Hertz Experiment

James Franck & Gustav Hertz, 1914.
Tube with low pressure mercury vapour and electrodes.
Measure the current of electrons reaching the last anode as a function of the voltage between the cathode and grid.



Surprising Result



Peaks are 5 Volts apart.

Franck-Hertz Experiment

It was well known that mercury vapour has a spectral line at 250 nm (in the UV). This corresponds to

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV-nm}}{250\ \text{nm}} = 4.96\ \text{eV}$$

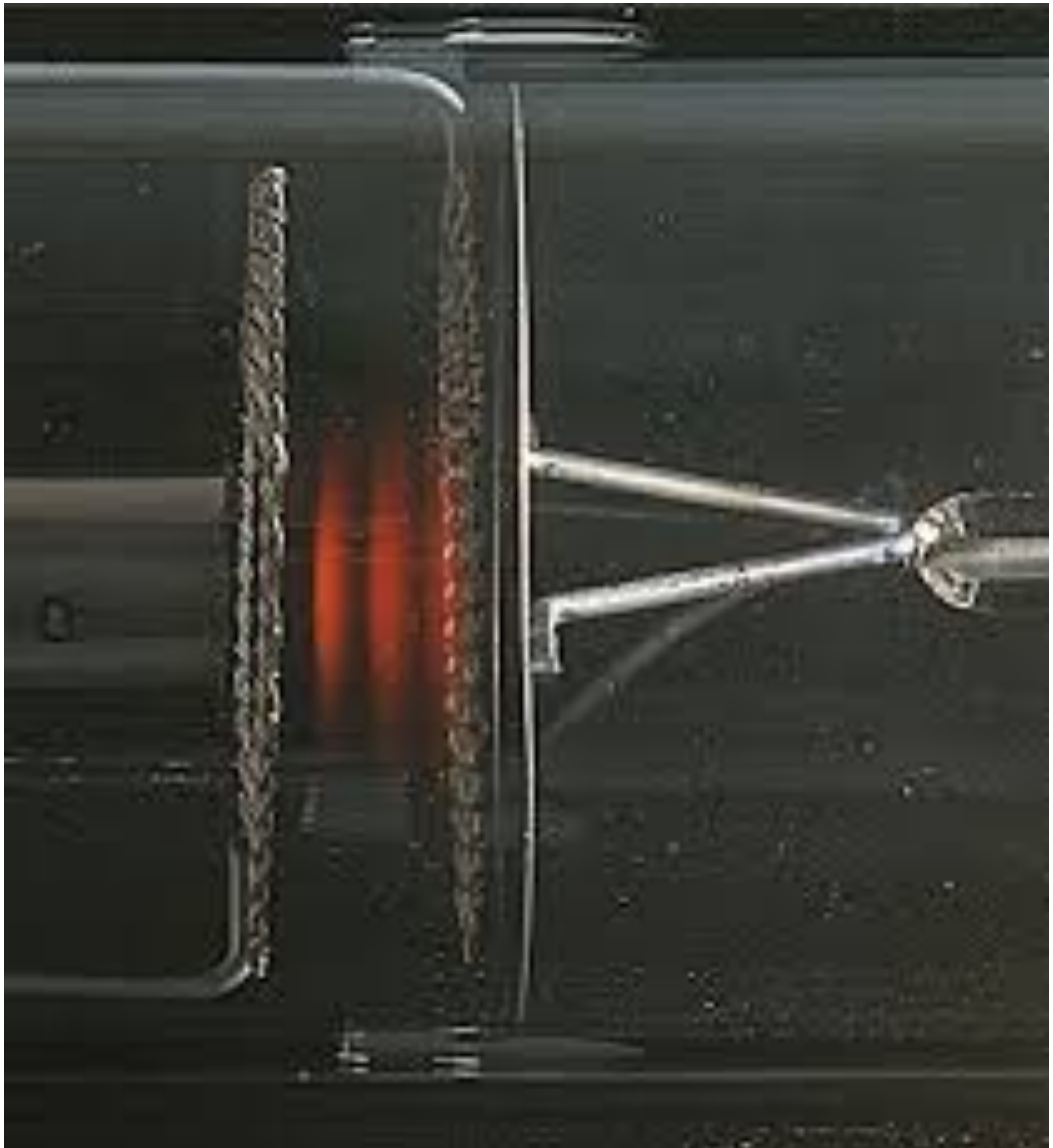
Increasing the voltage increases the current, until the electron energy reaches 5 eV. That's enough to excite the mercury atom to emit a photon.
The electron loses the 5 eV, meaning it comes to a stop.

The electron can gain another 5 eV, excite another mercury atom, and stop again.
Rinse and repeat.

Franck & Hertz verified that they saw the 250 nm spectral line in their tube.

Franck-Hertz in Neon

If you use neon gas instead of mercury vapor, the emitted photon is visible, and you can see alternate dark and bright regions.



Franck-Hertz Significance

It was the first clear evidence that the same discrete atomic energy levels, correlated with photon wavelengths via Planck's constant, also showed up in the interactions of electrons with atoms.

It got Franck and Hertz the 1925 Nobel Prize.

And it's a common undergraduate physics labs experiment.

de Broglie Waves

In Louis de Broglie's 1924 Ph.D. thesis, he proposed that particles, e.g. electrons, should have wave properties. He proposed that for both matter and photons,

$$\lambda = \frac{h}{p}$$

A particle in an orbit with radius r and angular momentum $L = rp$ would have a wavelength of $\lambda = hr / L$.

If we require an integer n wavelengths λ around the orbit circumference of $2\pi r$,

$$n\lambda = n\frac{hr}{L} = 2\pi r \quad \rightarrow \quad L = n\frac{h}{2\pi} = n\hbar$$

This is a much more natural way of getting the 2π fudge factor that we needed to get the Rydberg energy formula from Bohr.

de Broglie Waves 2

Making the electron into a wave also helps avoid the problem of the electron radiating away its energy.

A uniform and constant current following a circular path makes a magnetic field, but it doesn't radiate, even though the charges are accelerating. The radiation from different parts of the path cancel.

(If the de Broglie wave were a standard real sine wave, with positive, negative, and zero values, it wouldn't be uniform so it would still radiate.

In Schrodinger quantum mechanics, the wave is complex with the real and imaginary parts out of phase by 90°, so the probability of finding the electron at any place along the orbit is constant.)

de Broglie Wavelength

For photons, the momentum p to put into $\lambda = \dfrac{h}{p}$ is $p = \dfrac{E}{c}$, so $\lambda_{\text{photon}} = \dfrac{hc}{E_{\text{photon}}}$

For non-relativistic matter, the relation between kinetic energy and momentum is

$$E_{\text{kinetic}} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}, \text{ so } p = \sqrt{2mE_{\text{kinetic}}}$$

To use eV units, $\lambda_{\text{matter}} = \dfrac{c}{c}\dfrac{h}{\sqrt{2mE_{\text{kinetic}}}} = \dfrac{hc}{\sqrt{2mc^2 E_{\text{kinetic}}}}$

For an electron, $\lambda_{\text{matter}} = \dfrac{1240 \text{ eV-nm}}{\sqrt{2\cdot 0.511\times 10^6 \text{ eV/c}^2 \cdot c^2 \cdot E_{\text{eV}}}} = \dfrac{1.227\sqrt{\text{eV}}\cdot\text{nm}}{\sqrt{E_{\text{eV}}}}$

Clicker Question

What is the de Broglie wavelength of an electron with kinetic energy of 12.4 keV?

A. 0.100 nm

B. 100 nm

C. 98.95 pm

D. 11.02 pm

E. 1.716 pm

Clicker Answer

What is the de Broglie wavelength of an electron with kinetic energy of 12.4 keV?

D. 11.02 pm

$$\lambda = \frac{hc}{\sqrt{2mc^2 E_{\text{kinetic}}}} = \frac{1.227\ \sqrt{\text{eV}} \cdot \text{nm}}{\sqrt{E_{\text{eV}}}} \rightarrow \frac{1.227\ \sqrt{\text{eV}} \cdot \text{nm}}{\sqrt{12.4 \times 10^3\ \text{eV}}} = 1.102 \times 10^{-2}\ \text{nm} = 11.02\ \text{pm}$$

For Next Time

WebWork 2 is due at 11:59 PM tonight.

We will continue with Atoms (and nuclei) on Wednesday.

Webwork 3 will be posted Wednesday (I hope), due next Monday.

There will be another worksheet on Friday, due 11:59 PM.

Midterm will be Wednesday June 4 at 5-6 PM, covering through this week.

We will start Schrodinger Equation on Monday (not on the midterm).