PHYS 250 Worksheet 1 Solutions

Student ID	Family Name	First Name	Email

1. Barn and Pole.

A farmer has a barn with a front (entry) door and back (exit) door that are 4 meters apart. An athlete carries a 5 meter pole.

A. If the athlete then runs at 0.7 c, what length is the pole to the farmer?

$\beta = u/c = 0.7$ and $\gamma = \left(1-\beta^2\right)^{-1/2} = 1.40$.
The apparent length of the pole is $L = L_0/\gamma = 5/1.40 = 3.571$ m

B. Can the farmer close the front (entry) door without opening the back (exit) door, with the running athlete completely inside the barn?

Yes. There will be 4.0 – 3.571 = 0.429 m space to spare.

C. If the athlete runs at 0.7 c, what length is the barn to him?

The apparent length of the barn is $L = L_0/\gamma = 4/1.40 = 2.857$ m

D. In the frame of the athlete, is it possible for both ends of the pole to be inside the barn at the same time?

No. The athlete sees his pole as 5 meters long, and the barn as 2.857 meters long. So both ends of the pole cannot be inside the barn at the same time.

E. In the farmer's frame, the front (entry) door is at $x = 0$ and the back (exit) door is at $x = 4$ m. At $t = 0$, the back end of the pole is at $x = 0$, and the farmer presses a button that simultaneously (in the farmer's frame) closes the front (entry) door and opens the back (exit) door.

What are x' values in the athlete's frame of the the ends of his pole at $t' = 0$?
What are the x' values of the front (entry) door opening and back (exit) door closing, at $t = 0$?

In the athlete's frame, the ends of the pole are <u>always</u> at $x' = 0$ and $x' = 5$ m.
The Lorentz transform formulas are $ct' = \gamma(ct - \beta x)$ and $x' = \gamma(x - \beta ct)$.
The exit door closes at $t_{exit} = 0$ but $x_{exit} = 4$, so $x'_{exit} = \gamma(x_{exit} - \beta ct_{exit}) = 1.40(4 - 0) = 5.6\text{ m}$.
The entry door closes at $x_{entry} = t_{entry} = 0$ so $x'_{entry} = t'_{entry} = 0$.

F. Explain the events from the athlete's perspective.

When the exit door opens, the athlete says the tip of his pole is at $x' = 5$,
and the exit door is at $x' = 5.6$, so his pole hasn't hit the exit door before it opens.

The athlete says the exit door opens at $ct'_{exit} = \gamma(ct_{exit} - \beta x_{exit}) = 1.40(0 - 0.7 \cdot 4) = -3.92$.
Converting that into time gives $t'_{exit} = -3.92/2.998 \times 10^8 = -13.08\text{ ns}$.

The entry door closes at $t'_{entry} = 0$, so to the athlete, there are 13 nanoseconds when both doors are open. During that time, both ends of the 5 meter pole are outside the 2.857 meter barn.

The athlete considers the clearance when the exit door opens to be 5.6 – 5.0 = 0.6 meters.

Note that $0.6/\gamma = 0.6/1.4 = 0.429$, which is exactly the clearance between the pole and the exit door that the farmer saw when he closed the entry door.

2. A starship approaches your starship with $\gamma = 2.0$. (No, γ isn't the Star Trek warp-factor). The approaching starship fires at you a beam of photons that have energy of 10×10^6 electron-Volts per photon.

A. What is the velocity in your frame of the approaching starship in meters/second ? (8 points) (Don't worry about the sign).

$$\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{1 - 1/2^2} = 0.8660 \quad v = \beta c = 0.8660 \cdot 2.998 \times 10^8 = 2.596 \times 10^8 \text{ m/s}$$

B. What is the velocity in your frame of the approaching photons in meters/second? (6 points) (Don't worry about the sign).

Photons always move at the speed of light, so $v = 2.998 \times 10^8$ m/s

C. What is the momentum of each photon, in the frame of the approaching starship ? (8 points) (Easier in units based on electron-Volts and speed-of-light c. Don't worry about the sign.)

$E^2 = (pc)^2 + (mc^2)^2$ and photons have $m = 0$, so $E = pc \rightarrow p = \dfrac{E}{c} = 10$ MeV/c

If you insist on SI units,

$$p = \frac{10 \times 10^6 \text{ eV}}{2.998 \times 10^8 \text{ m/s}} \cdot \frac{1.602 \times 10^{-19} \text{ Joule}}{1 \text{ eV}} \cdot \frac{1 \text{ kg-m/s}^2\text{-m}}{1 \text{ Joule}} = 5.34 \times 10^{-21} \text{ kg-m/s}$$

D. Is the photon energy in <u>your</u> frame higher, lower, or equal to 10 MeV ? (6 points)

The starship is approaching, so the photons are blue-shifted to shorter wavelength, so their energy is higher. Or, you could do the calculation below.

E. What is the photon energy in <u>your</u> frame? (8 points) (Easier in units based on electron-Volts and speed-of-light c.)

The Lorentz transform is $\dfrac{E'}{c} = \gamma \cdot \left(\dfrac{E}{c} + \beta p \right)$. The sign is + because the starship is approaching.

Use $E = pc, \gamma = 2, \beta = 0.8660$ to get $E' = 2 \cdot (E + 0.8660 \cdot E) = 2 \cdot 1.8660 \cdot 10 \text{ MeV} = 37.32 \text{ MeV}$