PHYS 250 Worksheet 3 Solutions

1. A spring that can pivot around the origin at $\vec{r} = 0$ can exert force $-k\vec{r}$ on mass m. If the angular momentum L of circular orbits of the mass is quantized to $L = n\hbar$ with integer n,

A. What are the allowed energies?

For a circular orbit with radius r, $F = ma \rightarrow -kr = -m\frac{v^2}{r} \rightarrow v^2 = \frac{k}{m}r^2 \rightarrow v = \sqrt{\frac{k}{m}} \cdot r$.

The angular momentum is $\vec{L} = \vec{r} \times \vec{p} = rmv$.

Plug in the above: $L = rm \cdot \left(\sqrt{\frac{k}{m}} \cdot r \right) = \sqrt{km} \cdot r^2 \rightarrow r^2 = \frac{L}{\sqrt{km}}$

The kinetic energy is $T = \frac{1}{2}m \cdot \left(v^2\right) = \frac{1}{2}m \cdot \left(\frac{k}{m}r^2\right) = \frac{1}{2}kr^2$.

The potential energy is $V\left(r\right) = -\int_{r=0}^{r} \vec{F} \cdot d\vec{r} = -\int_{r=0}^{r} \left(-kr\right) \cdot dr = \frac{1}{2}kr^2$.

The total energy is $E = T + V = \frac{1}{2}kr^2 + \frac{1}{2}kr^2 = kr^2$.

Plug in the r vs L relation from above: $E = k \cdot \frac{L}{\sqrt{km}} = L \cdot \sqrt{\frac{k}{m}}$.

Plug in $L = n\hbar$ to get $E = n \cdot \hbar \cdot \sqrt{\frac{k}{m}}$.

The harmonic oscillator frequency in radians/second is $\omega = \sqrt{\frac{k}{m}}$, so $E = n \cdot \hbar\omega$.

Unlike Hydrogen, the energy is positive, and proportional to n instead of $-1/n^2$. There is no upper limit. And there is no reason to exclude $n = 0$.

B. What are the allowed radii?

$r^2 = \frac{L}{\sqrt{km}} = \frac{n\hbar}{\sqrt{km}} \rightarrow r = \sqrt{\frac{\hbar}{\sqrt{km}}} \cdot \sqrt{n}$. The radius grows as $\sqrt{n}$ instead of n^2 .

We can also write this as $r = \sqrt{\frac{\hbar \cdot \omega}{\sqrt{km} \cdot \sqrt{\frac{k}{m}}}} \cdot \sqrt{n} = \sqrt{\frac{\hbar\omega}{k}} \cdot \sqrt{n}$

2. What is the de Broglie wavelength of a proton with kinetic energy of 3 MeV ?

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m} \rightarrow p = \sqrt{2m \cdot KE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m \cdot KE}} = \frac{hc}{\sqrt{2mc^2 \cdot KE}} \quad \text{Proton mass } = 938.3\ \text{MeV/c}^2$$

$$\lambda = \frac{1240\ \text{eV-nm}}{\sqrt{2 \cdot 938.3\times10^6\ \text{eV} \cdot 3\times10^6\ \text{eV}}} = 1.653\times10^{-5}\ \text{nm} = 16.53\ \text{fm}$$

3. The energy levels for Unobtanium (Z = imaginary) are shown here.
Unobtanium has a single electron.

$E_4 = -2$ eV

$E_3 = -5$ eV

$E_2 = -10$ eV

$E_1 = -20$ eV

A. Photons from the Unobtanium $n = 3$ to $n = 1$ and $n = 3$ to $n = 2$ transitions eject photoelectrons from an unknown metal, but photons from the $n = 4$ to $n = 3$ transition do not.

What can you say about the work function of the unknown metal?

3 to 1 is 15 eV and 3 to 2 is is 5 eV. So the work function is < 5 eV.
4 to 3 is 3 eV, so the work function is > 3 eV.

B. If photons with energy 18 eV shine on Unobtanium gas, what are the possible radiated photon energies?

An 18 eV photon would excite electrons from the $n = 1$ level to the $n = 4$ level.

The n = 4 electron could fall back to the $n = 1$ level, emitting an 18 eV photon.

But it could also fall back to the $n = 2$ level, emitting an 8 eV photon.
From the $n = 2$ level, it could fall back to $n = 1$, emitting a 10 eV photon.

The $n = 4$ electron could also fall back to the $n = 3$ level, emitting a 3 eV photon.
The $n = 3$ electron could fall back down to $n = 1$, emitting a 15 eV photon.
But the $n = 3$ electron could also fall back to the $n = 2$, emitting a 5 eV photon.

So overall, we expect to see photons with energy 18, 15, 10, 8, 5, and 3 eV.