PHYS 250 Worksheet 5 Solutions

1. Square Well Bound States

A. Using the "Bound States" PhET simulation, what are the bound state energy levels for an electron in a single square well with height of 10 eV (the default) and width of 0.5 nm (not the default)?

From the simulation, E1 = 0.97 eV, E2 = 3.78 eV, E3 = 7.99 eV

B. What are the corresponding energy levels for an infinite square well of width 0.5 nm? (This is a paper calculation, not the simulation). How do you explain the difference from A?

$$E_n = n^2 \frac{1}{2m}\left(\frac{\pi\hbar}{w}\right)^2 = n^2 \frac{1}{2mc^2}\left(\frac{\pi\hbar c}{w}\right)^2 = n^2 \frac{1}{2\cdot 0.511\times10^6\text{ eV}}\left(\frac{\pi\cdot 197.3\text{ eV-nm}}{0.5\text{ nm}}\right)^2 = n^2\cdot 1.504\text{ eV}$$

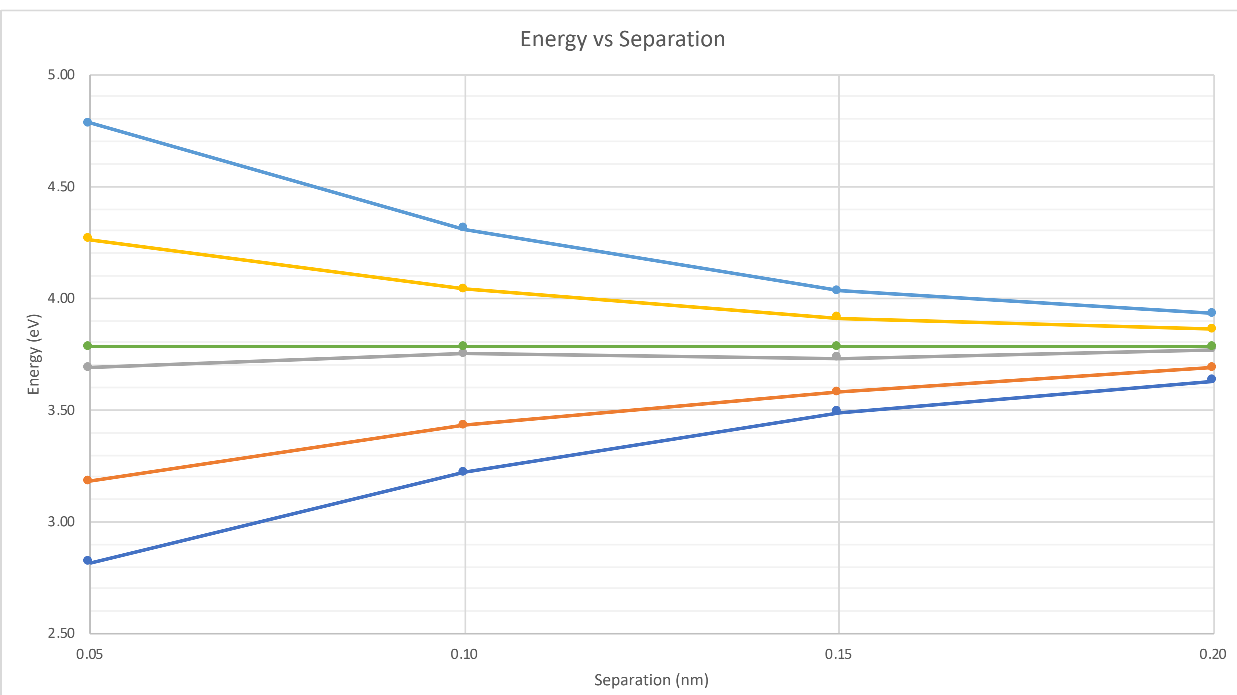
$E_1 = 1.504$ eV, $E_2 = 6.015$ eV, $E_3 = 13.53$ eV

The wavefunctions spread outside the potential well, so the wavelength is longer, so the energies are lower.

B. What are the energies of states 6-10 for the case of 5 square wells with height of 10 eV and width of 0.5 nm (all default values), for separations of 0.05, 0.10, 0.15, and 0.20 nm? Make a table of your results, including a column for E2 in a single well.

Separation	E6	E7	E8	E9	E10	E2 (Single)
0.05	2.82	3.18	3.69	4.26	4.78	3.78
0.10	3.22	3.43	3.75	4.04	4.31	3.78
0.15	3.49	3.58	3.73	3.91	4.03	3.78
0.20	3.63	3.69	3.77	3.86	3.93	3.78

C. Make a graph of your results, including E2 for a single well as a horizontal line.



2. Lasers

Background information:

For equilibrium of an excited state, the excitation rate and the decay rate must be the same.

If there are N atoms in a state with lifetime τ, there will be N/τ spontaneous emission photons produced per second.

The Law of Collisions is $\rho\sigma\lambda = 1$ where ρ is the density in atoms per volume, σ is the collision cross-section per atom with dimensions of area, and λ is the mean free path with dimensions of length. If there are initially N_0 photons, the number that avoid colliding with atoms after travelling distance x is $N(x) = N_0 \exp(-x/\lambda)$.

The cross-section σ for stimulated emission equals the cross-section for absorption.

Laser information:

A laser tube is 30 cm long with a transverse area of 1 cm^2. It contains a gas of atoms with energy levels E_3, E_2, E_1, and ground state E_0. The spontaneous emission transitions are 3 to 2 with a lifetime of 10^{-5} s, 2 to 1 with a lifetime of 10^{-3} s, and 1 to 0 with a lifetime of 10^{-6} s.

A. If an electrical discharge excites $X = 10^{18}$ gas atoms per second from state 0 to state 3, what will be the equilibrium populations of levels 3, 2, and 1 ? Assume no stimulated emission.

For level 3, $\frac{N_3}{\tau_3} = X \to N_3 = X \cdot \tau_3 = \left(10^{18}\ \text{s}^{-1}\right)\cdot\left(10^{-5}\ \text{s}\right) = 10^{13}$

Those transitions put X atoms per second into level 2, so $N_2 = X \cdot \tau_2 = \left(10^{18}\ \text{s}^{-1}\right)\cdot\left(10^{-3}\ \text{s}\right) = 10^{15}$

That puts X atoms per second into level 1, so $N_1 = X \cdot \tau_1 = \left(10^{18}\ \text{s}^{-1}\right)\cdot\left(10^{-6}\ \text{s}\right) = 10^{12}$

B. Using your above result for the populations, if the stimulated-emission cross-section for state 2 to state 1 transitions is $(0.1\text{ nm})^2$ per atom, what is the mean free path a photon to cause stimulated emission, and the mean free path to be absorbed?

$$1 = \rho\sigma\lambda \quad \rho = \frac{N}{\text{Volume}} \to \lambda = \frac{1}{\rho\sigma} = \frac{\text{Volume}}{N}\cdot\frac{1}{\sigma}$$

$$\sigma = \left(0.1\text{ nm}\right)^2 = \left(10^{-10}\text{ m}\right)^2 = \left(10^{-8}\text{ cm}\right)^2 = 10^{-16}\text{ cm}^2$$

$$\lambda = \frac{\left(30\text{ cm}\right)\cdot\left(1\text{ cm}^2\right)}{N}\cdot\frac{1}{10^{-16}\text{ cm}^2} = \frac{3\times10^{17}\text{ cm}}{N}$$

$$\text{Stimulated emission: } \lambda = \frac{3\times10^{17}\text{ cm}}{N_2} = \frac{3\times10^{17}\text{ cm}}{10^{15}} = 300\text{ cm}$$

$$\text{Absorption: } \lambda = \frac{3\times10^{17}\text{ cm}}{N_1} = \frac{3\times10^{17}\text{ cm}}{10^{12}} = 3\times10^{5}\text{ cm}$$

C. Could this device work as a laser if there were no mirrors? Why?

With these populations, the mean free path for stimulated emission is much less than for absorption, so absorption is fairly unimportant.

But photons from 2 to 1 spontaneous transitions are in random directions, so hardly any are aimed along the tube. And even if they are aimed along the tube, the average distance they travel inside the tube is half its length. And that is much less than the mean free path for stimulated emission. So it would not work as a laser without mirrors.

D. If one end had a 100% reflective mirror and the other end had a 90% reflective mirror, a photon would take about 10 round-trip passes through the 30 cm tube before it escaped. Would that configuration work as a laser?

That would make the mean free path of a photon before escaping the mirrors about 600 cm, which is twice as long as the mean free path for stimulated emission. So there would be laser gain.

E. How long would it take a single photon to become 2 photons by stimulated emission?

The doubling time would be the time it takes to travel one mean free path of 300 cm = 3 m, or $t = \dfrac{3\text{ m}}{3\times10^{8}\text{ m/s}} = 10^{-8}\text{ s}$. So once the chain reaction gets going, it builds up very fast!

F. About how many photons per second would the laser produce?

The electrical discharge puts 10^{18} atoms per second into level 3, which puts 10^{18} atoms per second into level 2. Essentially all of them undergo stimulated emission due to the intense light inside the tube, so about 10^{18} photons per second would be produced.