Approximations: Tutorial

Intro to approximations.

In physics, **approximations** and **limiting cases** are often used to give us more insight into a system. We will use them often in this course (you have already seen a few in lecture). We will practice them here.

Making an approximation does **not** mean just equating to zero the parameters which are small. In fact, making an approximation means neglecting something **in comparison** with something.

Consider the expression $X = \frac{Aa}{A^2+a^2}$ and assume that $A \gg a$. The sign "≫" is **different** from ">", it indicates that you can neglect a in comparison with A.

a) Can you set $a = 0$ in the denominator of this expression? Why or why not?

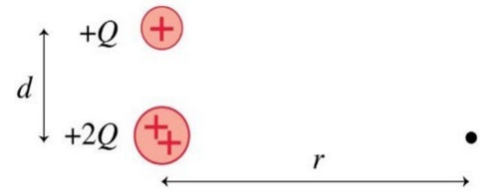
b) Can you set $a = 0$ in the numerator of this expression? Why or why not?

c) Write down an approximated expression for X assuming that $A \gg a$.

d) Calculate X with the parameters $A = 1000$, $a = 0.001$ before and after applying the approximation. Based on this, was your approximation valid?

Note: Yes, 10^{-6} is a small number, but this is not enough to say that it is zero. The charge on an electron or proton is much smaller, 1.60×10^{-19} C, but we still consider these as non-zero charges. You could neglect 10^{-6}, for example, **<u>in comparison</u>** with 10^{-3}, but if 10^{-6} is the only number you have, you cannot neglect it – it is your answer.

More practice with approximations.



Let's look at this system of charges. You can compute the electric field of these two charges at the black dot to have the following Cartesian components:

$$\vec{E} = \hat{\boldsymbol{x}}\, E_x + \hat{\boldsymbol{y}}\, E_y, \text{ with } E_x = \frac{KQr}{(r^2 + d^2)^{3/2}} + \frac{2KQ}{r^2} \text{ and } E_y = -\frac{KQd}{(r^2 + d^2)^{3/2}}.$$

a) Consider the limiting case $r >> d$. What would this physically represent? What are your approximated expressions for the electric field components? What does the result remind you of?

b) Consider the limiting case $d >> r$. What would this physically represent? What are your approximated expressions for the electric field components? What does the result remind you of?

c) You should now have found that, in the two cases presented here, you recover the field from a point charge of either +2Q or +3Q. Does it make sense or not?

Solutions

Intro to approximations.

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Consider the expression $X = \frac{Aa}{A^2+a^2}$ and assume that $A \gg a$. The sign "≫" is **different** from ">", it indicates that you can neglect a in comparison with A.

a) Can you set $a = 0$ in the denominator of this expression? Why or why not?

Yes: $a \ll A \Rightarrow a^2 \ll A^2 \Rightarrow$

$A^2 + a^2 \approx A^2$

b) Can you set $a = 0$ in the numerator of this expression? Why or why not?

No. Here a is not being compared with anything $\Rightarrow$ we have to keep it, even if we know that it is small.

c) Write down an approximated expression for X assuming that $A \gg a$.

$$X = \frac{A \cdot a}{A^2 + \cancel{a^2}} = \frac{A \cdot a}{A^2} = \frac{a}{A}.$$

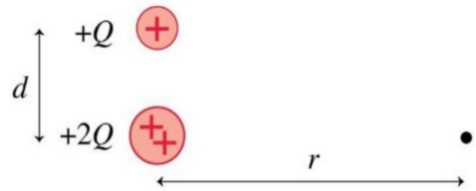
d) Calculate X with the parameters $A = 1000$, $a = 0.001$ before and after applying the approximation. Based on this, was your approximation valid?

$$X_{exact} = \frac{1000 \times 0.001}{1000^2 + 0.001^2} = \frac{1}{10^6 + 10^{-6}} = 0.9999\ldots \times 10^{-6}$$

$$X_{approx} = \frac{0.001}{1000} = 1 \times 10^{-6}$$ – practically the same

Note: Yes, 10^{-6} is a small number, but this is not enough to say that it is zero. The charge on an electron or proton is much smaller, $1.60\times10^{-19}\ C$, but we still consider these as non-zero charges. You could neglect 10^{-6}, for example, **<u>in comparison</u>** with 10^{-3}, but if 10^{-6} is the only number you have, you cannot neglect it – it is your answer.

More practice with approximations.



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a) Consider the limiting case $r >> d$. What would this physically represent? What are your approximated expressions for the electric field components? What does the result remind you of?

$$(r^2 + \cancel{d^2})^{3/2} \approx (r^2)^{3/2} = r^3$$

$$E_x \approx \frac{KQr}{r^3} + \frac{2KQ}{r^2} = \frac{3KQ}{r^2}$$ ⇒ like a "big" point charge +3Q

$$E_y \approx -\frac{KQd}{r^3} = -\frac{KQ}{r^2}\left(\frac{d}{r}\right) << E_x$$ ⇒ field is almost radial (again, as for a "big point charge).

since $d/r << 1$

b) Consider the limiting case $d >> r$. What would this physically represent? What are your approximated expressions for the electric field components? What does the result remind you of?

$$(\cancel{r^2} + d^2)^{3/2} \approx d^3$$

$$E_x \approx \frac{KQr}{d^3} + \frac{2KQ}{r^2} = \frac{2KQ}{r^2}\left(1 + \cancel{\frac{1}{2}\frac{r^3}{d^3}}\right) \approx \frac{2KQ}{r}$$ ⇒ Again, a "point charge", but now it is +2Q

$$E_y \approx \frac{KQ}{d^2} << E_x$$ ⇒ again, the field is almost radial

c) You should now have found that, in the two cases presented here, you recover the field from a point charge of either +2Q or +3Q. Does it make sense or not? YES:

- In case a), when $r >> d$, we are far away from the system of charges, and we "do not see" the details of its geometry. All what we "see" is that there is a net charge +3Q.

- In case b), when $d >> r$, increasing d essentially means removing the top charge further and further away from the system ⇒ all what we "see" is that the remaining field is created by the charge +2Q.