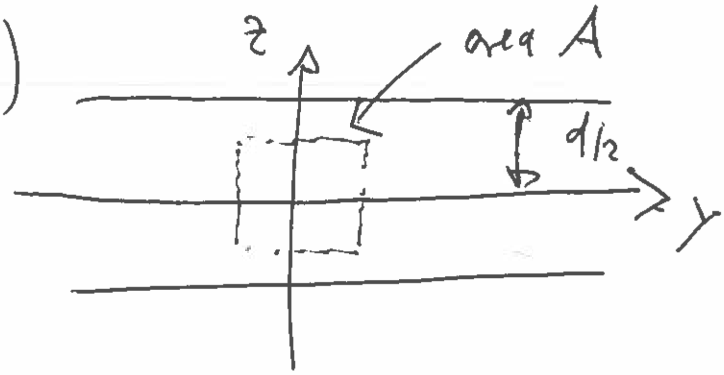
Sheet of charge:

a)



Gaussian pillbox

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow 2EA = \frac{2|z|A\rho_0}{\varepsilon_0}$$

$$E = |\vec{E}| = \frac{\rho_0}{\varepsilon_0}|z| \quad \left(|z| < \frac{d}{2}\right)$$

$$\boxed{\vec{E} = \frac{\rho_0}{\varepsilon_0} z\,\hat{z}} \quad \left(|z| < \frac{d}{2}\right)$$

for $|z| > \frac{d}{2}$ $\quad \oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow 2EA = \frac{1}{\varepsilon_0}\rho_0 A d$

$$E = |\vec{E}| = \frac{\rho_0}{\varepsilon_0}\frac{d}{2}$$

$$\vec{E} = \begin{cases} +\frac{\rho_0}{2}\frac{d}{\varepsilon_0}\,\hat{z} & z > \frac{d}{2} \\ -\frac{\rho_0}{2}\frac{d}{\varepsilon_0}\,\hat{z} & z < \frac{d}{2} \end{cases}$$

b) $V(z) = -\int_0^z \vec{E}\cdot d\vec{l} = -\int_0^z E_z\,dz$

$z>0$: $V\left(z < \frac{d}{2}\right) = -\int_0^z \frac{\rho_0}{\varepsilon_0} z\,dz = -\frac{\rho_0}{\varepsilon_0}\frac{z^2}{2}$

$$V\left(z>\frac{d}{2}\right) = V\left(\frac{d}{2}\right) - \int_{d/2}^{z} E_z\,dz = -\frac{\rho_0}{\varepsilon_0}\frac{1}{2}\left(\frac{d}{2}\right)^2 - \left(\frac{\rho_0 d}{2\varepsilon_0}\right)\left(z-\frac{d}{2}\right)$$

$$= -\frac{\rho_0 d z}{2\varepsilon_0} + \frac{\rho_0}{\varepsilon_0}\left(\frac{d}{2}\right)^2\left(1-\frac{1}{2}\right)$$

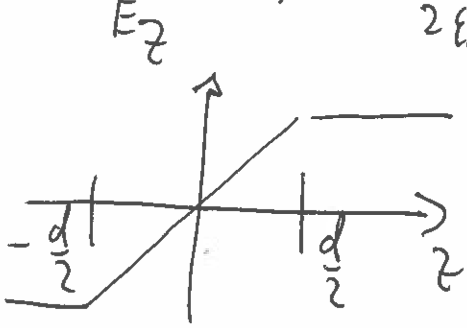
$$= -\frac{\rho_0 d z}{2\varepsilon_0} + \frac{\rho_0}{2\varepsilon_0}\left(\frac{d}{2}\right)^2$$

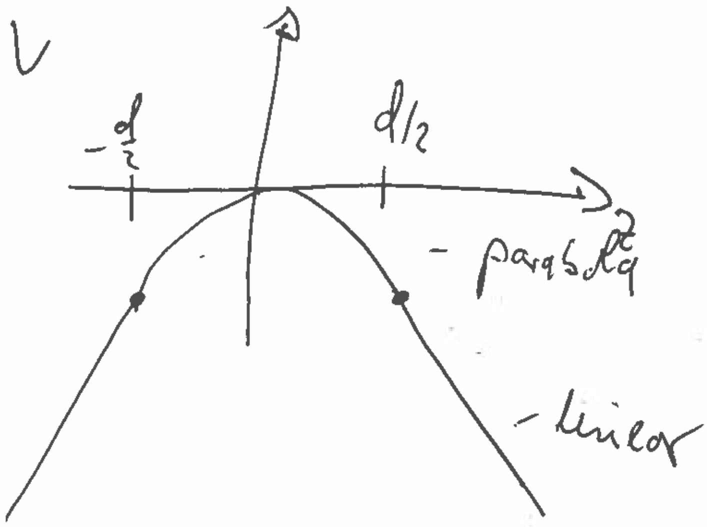
potential is symmetric about $z=0$, so for $z<0$:

$$V\left(z>-\frac{d}{2}\right) = -\frac{\rho_0}{\varepsilon_0}\frac{z^2}{2}$$

$$V\left(z<-\frac{d}{2}\right) = \frac{\rho_0 d z}{2\varepsilon_0} + \frac{\rho_0}{2\varepsilon_0}\left(\frac{d}{2}\right)^2$$

c)





d) Total charge $Q = \rho_0 A \cdot d = \sigma \cdot A$

$$\Rightarrow \boxed{\sigma = \rho_0 \cdot d}$$

Energy:

$$W = \frac{\varepsilon_0}{2}\int |\vec{E}|^2 d\tau = \frac{\varepsilon_0}{2}\int_{-\frac{d}{2}}^{\frac{d}{2}} A\left(\frac{\rho_0}{\varepsilon_0}\right)^2 z^2 dz$$

$$\frac{W}{A} = \frac{\varepsilon_0}{2}\left(\frac{\rho_0}{\varepsilon_0}\right)^2\left(\frac{z^3}{3}\right)\Bigg|_{-\frac{d}{2}}^{\frac{d}{2}} = \frac{\rho_0^2}{2\varepsilon_0}\frac{1}{3}\left(\frac{d^3}{8}+\frac{d^3}{8}\right) = \frac{\rho_0^2}{24\varepsilon_0}d^3$$

or

$$W = \frac{1}{2}\int \rho V d\tau = \underbrace{\frac{A}{2}\int_{-\frac{d}{2}}^{\frac{d}{2}} \rho_0\left(-\frac{\rho_0}{\varepsilon_0}\frac{z^2}{2}\right)dz}_{=\text{bulk}} + \underbrace{\frac{1}{2}\sigma_s A\cdot\left(-\frac{\rho_0}{2\varepsilon_0}\frac{d^2}{4}\right)\cdot 2}_{=\text{surface}}$$

$\sigma_s = -\frac{\rho_0 d}{2}$ to make system charge neutral

$$\frac{W}{A} = \frac{-\rho_0^2}{4\varepsilon_0}\left(\frac{z^3}{3}\right)\Bigg|_{-\frac{d}{2}}^{+\frac{d}{2}} + \frac{1}{2}\frac{\rho_0^2}{2\varepsilon_0}\frac{d^3}{4} = -\frac{1}{48}\frac{\rho_0^2 d^3}{\varepsilon_0} + \frac{1}{16}\frac{\rho_0^2 d^3}{\varepsilon_0}$$

$$= \frac{1}{24}\frac{\rho_0^2 d^3}{\varepsilon_0}$$

Cylinder:

a) $Q = \lambda_0 L + \rho_0 \pi a^2 L = 0 \quad \Rightarrow \rho_0 = -\dfrac{\lambda_0}{\pi a^2}$

b) $\oint \vec{E}\, d\vec{a} = E\, 2\pi s L = \dfrac{Q_{enc}}{\varepsilon_0} \qquad (s < a)$

$= (\rho_0 \pi s^2 L + \lambda_0 L)/\varepsilon_0$

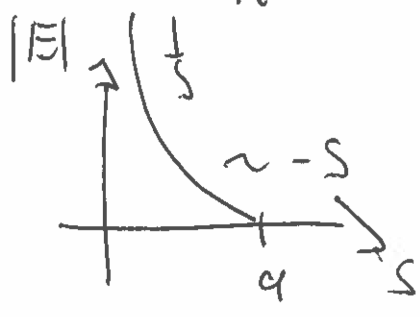
$= \left(-\dfrac{\lambda_0}{\pi a^2} \pi s^2 L + \lambda_0 L\right)/\varepsilon_0$

$= \left(\lambda_0 L \left(1 - \dfrac{s^2}{a^2}\right)\right)/\varepsilon_0$

So $\vec{E}_{in} = \dfrac{\lambda_0}{2\pi\varepsilon_0}\left(\dfrac{1}{s} - \dfrac{s}{a^2}\right)\hat{s}$

$\vec{E}_{out} = 0 \quad$ since $Q_{enc} = 0$

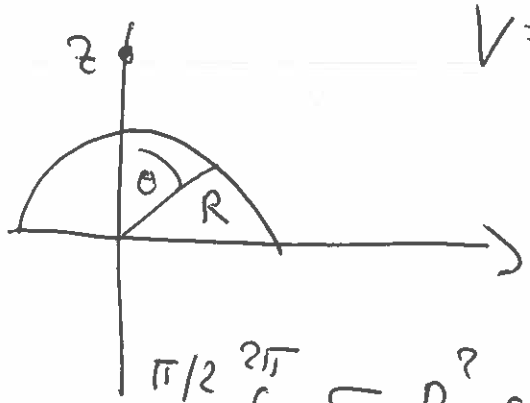
c)



d) System still charge neutral $\Rightarrow \vec{E}_{out}$ unchanged, $\vec{E}_{out} = 0$
metal screens field on inside, charges rearrange and go to surfaces, metal polarizes; $\vec{E}_{in} = 0$ now!

Hemisphere:

a)



$$V = K\int \frac{dq}{|\mathscr{R}|} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, da}{|\mathscr{R}|}$$

$$|\mathscr{R}| = \sqrt{z^2 + R^2 - 2Rz\cos\theta}$$

$$V = K\int_0^{\pi/2}\int_0^{2\pi} \frac{\sigma R^2 \sin\theta\, d\theta\, d\phi}{\sqrt{z^2+R^2-2zR\cos\theta}} = K\sigma 2\pi R^2 \cdot \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{z^2+R^2-2zR\cos\theta}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\sigma 2\pi R^2 \cdot \frac{1}{zR}\left[\sqrt{z^2+R^2-2zR\cos\theta}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\frac{R}{z}\left[\sqrt{z^2+R^2} - \sqrt{z^2+R^2-2zR}\right]$$

b) infinite sheet: $\vec{E} = \pm\frac{\sigma}{2}\hat{z}$

$$V_{sh} = -\frac{\sigma}{2}|z| + \text{const}$$

$$V_{total} = V_{hem} + V_{sh}$$