PHYS 301: Electricity and Magnetism
Term 1, 2023/24
FIRST MIDTERM EXAM, October 17, 2023
Time: 60 min

NAME: **Student Number:**

This is a closed book exam. Calculators are allowed but not needed. Show all your work as partial credit will be given only if your reasoning is clear. Possibly useful formulae follow:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\mathbf{r}')d\tau'}{|\mathbf{R}|^2}\hat{\mathbf{R}} \tag{1}$$

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r}) \tag{2}$$

$$V(b) - V(a) = -\int_a^b \mathbf{E}\cdot d\mathbf{l} \tag{3}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho(\mathbf{r}')d\tau'}{|\mathbf{R}|} \tag{4}$$

$$W = \frac{1}{2}\int \rho V d\tau \tag{5}$$

$$W = \frac{\epsilon_0}{2}\int E^2 d\tau \tag{6}$$

$$\nabla^2 V(\mathbf{r}) = -\rho/\epsilon_0 \tag{7}$$

$$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n \tag{8}$$

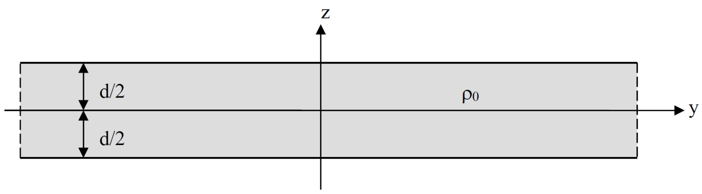
$$E^{\perp}_{above} - E^{\perp}_{below} = \sigma/\epsilon_0 \qquad E^{||}_{above} = E^{||}_{below} \tag{9}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} + \frac{\text{quadrupole}}{r^3} + ...\right) \tag{10}$$

$$\mathbf{p} = \sum_i q_i \mathbf{r_i} \quad \text{(discrete charges)} \tag{11}$$

$$\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau' \quad \text{(continuous charges)} \tag{12}$$

Problem 1:
An infinite slab of thickness d is in the xy-plane and has constant uniform charge density ρ_0.



(a)[2 pts] Compute the electric field everywhere in space. Please show your work and reasoning.

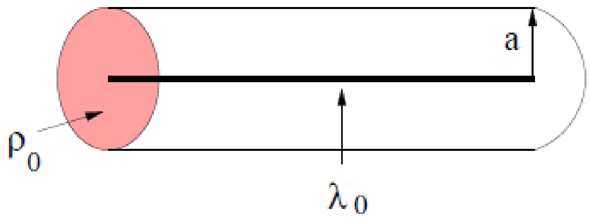
(b) [2 pts] Compute the voltage everywhere in space, with the zero of voltage at $z = 0$.

(c) [2 pts] Sketch $E_z(z)$ and $V(z)$.

(d) [2 pts] If d is made smaller and smaller, the slab becomes more and more like a plane of surface charge density σ. What is the relation between σ, d and ρ_0?

(e) [2 pts] Still with the slab of finite thickness d, we add uniformly charged plates on both sides with opposite charge density so that the entire system becomes charge neutral. What is the electrostatic energy per unit area of this system?

Problem 2:
An infinite linear charge distribution with density λ_0 is located on the axis of an infinite solid cylinder (an insulator) of uniform charge density ρ_0 and radius a. The net charge per length of the combined charge distributions is zero.



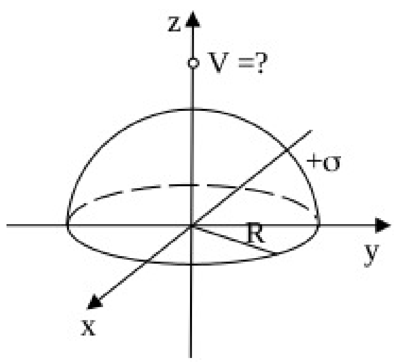
(a) [1 pts] Find the relation between ρ_0 and λ_0.

(a) [3 pts] Find the electric field $\vec{E}$ both inside and outside of the solid cylinder. Use your result from (a) to eliminate ρ_0 from the answer.

(b) [2 pts] Sketch the magnitude of E as a function of the radial distance s from the axis of the cylinder.

(c) [2 pts] The solid cylinder is now replaced with a conductor, but still carrying the same charge density ρ_0 (The line charge λ_0 remains on the axis). Does your answer to part (a) change? If so, what is the new electric field inside and outside? Please explain what happens, don't just state the answer.

Problem 3:
A hemisphere of radius R, centered on the origin and above the xy-plane, as shown below, has uniform surface charge density σ.



(a) [7 pts] Find the voltage on the z-axis at positions $z > R$.

(b) [3 pts] The hemisphere is now placed on top of an infinite sheet with the same charge density σ in the xy-plane (i.e. at $z = 0$). How does the voltage from part (a) change? Please answer even if you did not get a final answer for part (a).