PHYS 301: Electricity and Magnetism

Midterm 1

October 14th 2025

Duration: 1 hr

NAME: _____

Student Number: | | **Signature_____**

Please print your Student Number legibly in this box – we need it for proper scanning and uploading your exam!

- This exam consists of 3 questions, which add up to 30 pts.
- Part marks will be awarded for partially correct solutions. Make sure your work is clear and easy to read; don't skip steps. Include diagrams or brief explanations, if useful.

Please turn off and remove from the desk all cell phones, tablets and other communications devices!

Please note: you are not required to write this exam in series. Consider reading the entire exam first and beginning with what you feel most comfortable

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including cell phones), or other memory aid devices, other than those authorized by the examiners;
- speaking or communicating with other candidates; and
- purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

5. Please **don't detach any pages**, including "extra space" pages, from the exam booklet.
6. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
7. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

You can use extra pages at the end of the exam booklet. If you want them to be marked, write "see extra page" in the exam booklet, next to the question that you want us to mark on these extra pages.

Formulas that might (or might not) be useful

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \frac{1}{2}\log\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}\right)$$

$$\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{1}{3}\,(x^2-2a^2)\sqrt{x^2+a^2}$$

Problem 1 [4 pts]. A thin long wire is placed along the z-axis. It has a linear charge density

$$\lambda(z) = \lambda_0 \, e^{-|z|/a},$$

where λ_0 and a are some constants. We will use a cylindrical coordinate system to discuss the $\mathbf{E}$ field produced by this wire. **Note**: It's a qualitative problem. You **don't** need to compute $\mathbf{E}$, or even to set up an integral for it, you only need to think about the symmetry of the problem. A detailed explanation of your answer is **not** mandatory, though very welcome (for your own benefit, not for marks) if time permits.

a) **[2 pts]** Given the geometry of the charge distribution, which component or components of $\mathbf{E}$ will be non-zero in cylindrical coordinates?

b) **[2 pts]** On which coordinates (s, ϕ, z) will this component, or each of these components, depend?

Problem 2 [11 pts]. A sphere of radius R is charged with a positive uniform volume charge density ρ_0. After a negative uniform surface charge density, $-\sigma_0$, is added to the surface of the sphere, the electric field outside the sphere does not change the direction, but drops by a factor of 3. In the questions below, we assume that both ρ_0 and $-\sigma_0$ are present.

a) [2 pts] Find σ_0 in terms of ρ_0 and R.

b) [4 pts] Calculate and sketch the radial component of the electric field as a function of r. Label the characteristic values on both axes. Show your work.

c) [4 pts] What is the electric potential energy stored in this sphere?

d) [1 pt] Can this sphere be made of a conducting material? Explain your answer.

Problem 3 [15 pts]. A disk of radius R carries a non-uniform surface charge density $\sigma(s) = \sigma_0 s^2/R^2$. It lays in the (x, y) plane, at $z = 0$, with its center at $(x = 0, y = 0)$.

a) [2 pts] Find the total charge on the disk.

b) [3 pts] Set up an expression for the electric potential at the axis of the disk, at a distance z from its center. You don't need to compute the integral at this stage.

c) [4 pts] Consider the limit $z \gg R$. Approximate the integrand of the expression from part b) and find the potential up to the leading term. Is your result consistent with part a)? **Explain why yes or why no**.

d) [5 pts] Now I want you to find the electric field at the axis of the disk in the limit $z \ll R$. **First**, comment on the structure of the field (which component or components are non-zero, and on which variables they depend). **After that**, proceed with computing the $\mathbf{E}$ field. There are many approaches to solving this part, please explore!

e) [1 pt] What can you say about how E_z at the axis of the disk changes across the disk? In other words, does E_z have a jump at $z = 0$, or is it continuous? Explain. **Note**: You can answer this question even if you are not sure in your solution to part d).

Extra page. If you want your work on it to be marked, indicate this clearly next to the question you are solving.

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