PHYS 301: Electricity and Magnetism

Midterm 2

November 14th 2024

Duration: 1 hr

NAME: _____

Student Number: | | **Signature_____**

Please print your Student Number legibly in this box – we need it for proper scanning and uploading your exam!

- This exam consists of 3 questions, which add up to 30 pts.
- Part marks will be awarded for partially correct solutions. Make sure your work is clear and easy to read; don't skip steps. Include diagrams or brief explanations, if useful.

Please turn off and remove from the desk all cell phones, tablets and other communications devices!

Please note: you are not required to write this exam in series. Consider reading the entire exam first and beginning with what you feel most comfortable

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including cell phones), or other memory aid devices, other than those authorized by the examiners;
- speaking or communicating with other candidates; and
- purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

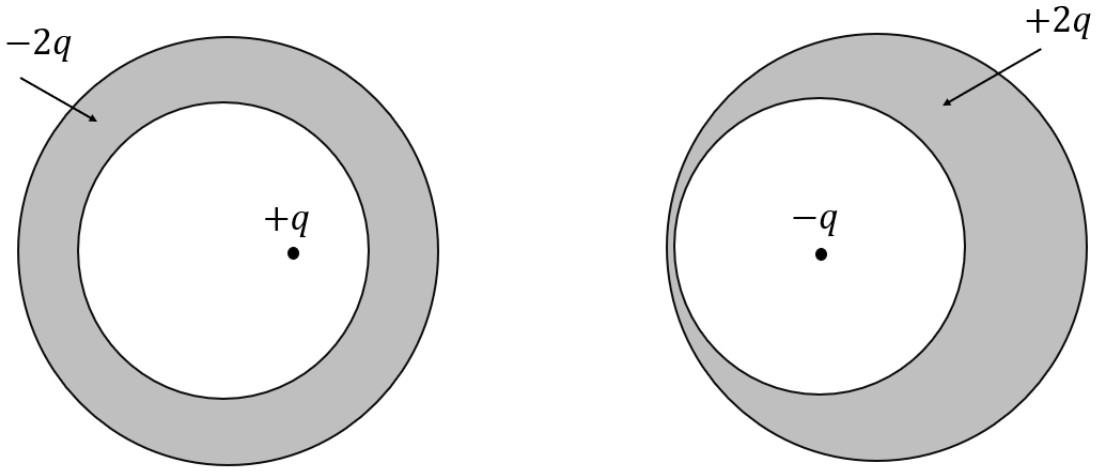
You can use extra pages at the end of the exam booklet. If you want them to be marked, write "see extra page" in the exam booklet, next to the question that you want us to mark on these extra pages.

Problem 1 [6 pts].

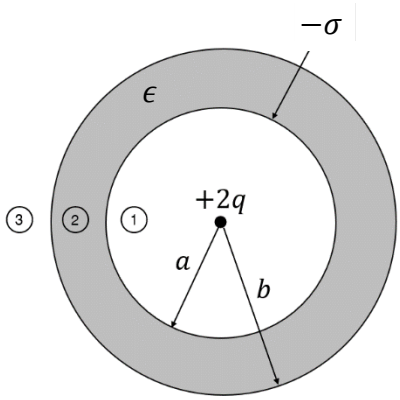
Two conducting spherical shells, one symmetric one asymmetric, have charges $+q$ and $-q$ fixed in their cavities, as shown. The charge $+q$ is shifted away from the center, and the charge $-q$ is at the center of the cavity. In addition to that, charge $-2q$ is added to the first shell, and charge $+2q$ is added to the second shell.

In this problem we assume that the distance between the two shells is much larger than their size, so that you can neglect direct interaction between charges sitting on different shells. **The figure is not to scale in this sense.**

a) [2 pts] Find the charge on all surfaces of both shells. Use clear notations.
b) [2 pts] Sketch charge distribution on each surface. We should see whether it is uniform or non-uniform.
c) [2 pts] Sketch electric field <u>everywhere in space</u>. If it is zero somewhere, state it.



Problem 2 [12 pts]. A dielectric spherical shell is charged with a negative surface charge density, $-\sigma$, on its inner surface. In addition, a point charge $+2q$ is fixed at the center of the cavity in the middle of the shell, and $2q = 2(4\pi a^2)|\sigma|$. The outer surface of the shell is uncharged. The dielectric is linear with a dielectric permittivity ϵ. The inner radius of the shell is a, the outer radius is b.



Consider three regions: ① $(r < a)$, ② $(a < r < b)$ and ③ $(r > b)$, see figure.

a) [7 pts] In each region, find the fields $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{P}$.

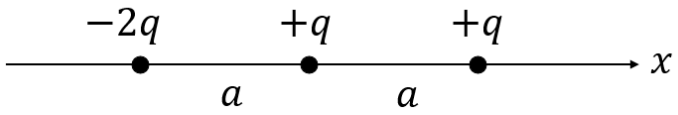
b) [1 pts] Find the bound volume charge density.

c) [2 pts] Find the bound surface charge density on each of the two surfaces.

d) [1 pt] Find the induced electric field inside the dielectric, $\mathbf{E}_{\text{ind}}$.

e) [1 pt] Find the dielectric constant of the dielectric, ϵ_r, if the magnitude of the bound surface charge density at $r = a$ is half the free charge density on the inner surface of the shell.

Problem 3 [12 pts]. Three charges, $-2q$, $+q$ and $+q$ are sitting along a line, separated by equal intervals a, as shown:



Your goal is to find multipole moments and corresponding terms in the potential expansion for this system.

a) [2 pts] What is the monopole moment, and the corresponding term in the potential expansion, for this system?
b) [3 pts] Chose the origin of the coordinate system at the location of the middle charge. Compute the dipole moment $\mathbf{p}$ of this set of charges, and the dipole term in the potential expansion. Write down the answer for $V_1(\mathbf{r})$ using spherical coordinates.
c) [3 pts] In the same coordinate system, compute the quadrupole tensor for this system of charges.
d) [3 pts] Compute the quadrupole term in the potential expansion and write it down in spherical coordinates**.**
e) [1 pts] Can you find a point about which the dipole moment is zero? If yes, find its coordinates. If not, explain why.

$$\mathbf{p} = \sum_a q_a \mathbf{r}_a \qquad V_1(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} \qquad Q_{ij} = \sum_a \frac{q_a}{2}(3r_{a,i}r_{a,j} - r_a^2\delta_{ij}) \qquad V_2(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{ij}\frac{Q_{ij}r_i r_j}{r^5}$$

Extra page. If you want your work on it to be marked, indicate this clearly next to the question you are solving.

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