PHYS 301: Electricity and Magnetism

Midterm 2

March 19th 2025

Duration: 50 min

NAME: _____

Student Number: | | **Signature_____**

Please print your Student Number legibly in this box – we need it for proper scanning and uploading your exam!

- This exam consists of 2 questions, which add up to 26 pts.
- Part marks will be awarded for partially correct solutions. Make sure your work is clear and easy to read; don't skip steps. Include diagrams or brief explanations, if useful.

Please turn off and remove from the desk all cell phones, tablets and other communications devices!

Please note: you are not required to write this exam in series. Consider reading the entire exam first and beginning with what you feel most comfortable

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including cell phones), or other memory aid devices, other than those authorized by the examiners;
- speaking or communicating with other candidates; and
- purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

You can use extra pages at the end of the exam booklet. If you want them to be marked, write "see extra page" in the exam booklet, next to the question that you want us to mark on these extra pages.

Mathematical expressions that might (or might not) be useful

$$\int_0^\pi \sin^2 x\, dx \;=\; \int_0^\pi \cos^2 x\, dx \;=\; \frac{\pi}{2}$$

$$\int_0^\pi \sin x \cos x\, dx = 0$$

$$\int_0^\pi \sin x \cos^2 x\, dx = \frac{2}{3}$$

$$\int_0^\pi \sin^2 x \cos x\; dx = 0$$

$$\int_0^\pi \sin^2 x\; \cos^2 x\; dx = \frac{\pi}{8}$$

$$\int_0^\pi \sin x \cos^3 x\; dx = \int_0^\pi \sin^3 x\; \cos x\; dx = 0$$

$$Q \equiv \int_V \rho(\mathbf{r}')\, d\tau' \qquad V_0(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad V_0(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r}\int_V \rho(\mathbf{r}')\, d\tau'$$

$$\mathbf{p} \equiv \int_V \rho(\mathbf{r}')\, \mathbf{r}'\, d\tau' \qquad V_1(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} \qquad V_1(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^2}\int_V \rho(\mathbf{r}')\, r' \cos\theta' d\tau'$$

$$Q_{ij} \equiv \int_V \frac{\rho(\mathbf{r}')}{2}(3r'_i r'_j - r'^2\delta_{ij})\, d\tau' \qquad V_2(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{ij}\frac{Q_{ij}\hat{r}_i\hat{r}_j}{r^3} \qquad V_2(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\int_V \rho(\mathbf{r}')\,\frac{r'^2}{2}\left(3\cos^2\theta' - 1\right)\, d\tau'$$

Extra page. If you want your work on it to be marked, indicate this clearly next to the question you are solving.

Problem 1 [14 pts]. A thin infinite sheet of charge with surface charge density $+\sigma_0$ is sitting at $z = 0$. It is surrounded by two identical infinite neutral linear **dielectric** slabs, of thickness t each, with dielectric constants ϵ_r. The gaps between the charge and the dielectrics are d (see figure).

a) [4 pts] Find $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{P}$ in all regions.

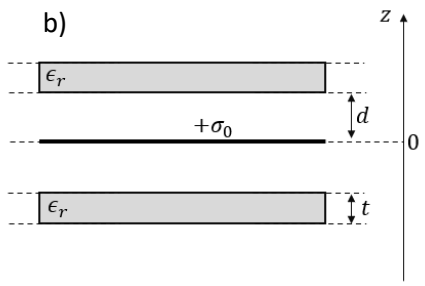
b) [2 pts] Find bound surface and volume charge densities in the dielectric. Show their location and their signs in the figure "b)" at the right.

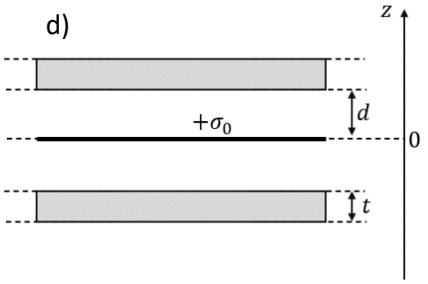
c) [3 pts] Plot $E_z(z)$ in all regions, including $z < 0$. If electric field has discontinuities at some points, explain why they happen.

Now assume that the slabs are made of a neutral metal, not of a dielectric.

d) [2 pts] Find surface and volume charge densities in the metal, show their location and their signs in the figure "d)" at the right.

e) [3 pts] Find and plot $E_z(z)$ in all regions, including $z < 0$.





Problem 2 [12 pts]. A sphere of radius R is charged with a surface charge density $\sigma = \sigma(\theta)$, which depends only on the polar angle θ. The origin of the coordinate system is at the center of the sphere. Your goal is to calculate the potential of the sphere at a large distance $\mathbf{r}$ from it; $r \gg R$ is assumed for the rest of the problem. You need to find the monopole term in the potential expansion (it might be zero or non-zero), **and the next non-zero term after it**, too. If this next term is a dipole, find it for $\mathbf{r}$ with arbitrary direction; if it is a quadrupole, find it only for $\mathbf{r} = (0,0,\mathrm{z})$.

a) [6 pts] Assume that $\sigma = \sigma_0 \cos\theta$, where σ_0 is a constant. Calculate $V(\mathbf{r})$ as explained above.

b) [6 pts] Assume that $\sigma = \sigma_0 \sin\theta$, where σ_0 is a constant. Calculate $V(\mathbf{r})$ as explained above.

You can use integrals and formulas at the first page of the exam.

Advice [not for marks]: Start with guessing what the answer should be. Use the symmetry of the charge distribution.

Extra page. If you want your work on it to be marked, indicate this clearly next to the question you are solving.

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