PHYS 301: Electricity and Magnetism

Midterm 2

November 13th 2025

Duration: 60 mins

NAME: _____

Student Number: | | **Signature_____**

Please print your Student Number legibly in this box – we need it for proper scanning and uploading your exam!

- This exam consists of 3 questions, which add up to 30 pts.
- Part marks will be awarded for partially correct solutions. Make sure your work is clear and easy to read; don't skip steps. Include diagrams or brief explanations, if useful.

Please turn off and remove from the desk all cell phones, tablets and other communications devices!

Please note: you are not required to write this exam in series. Consider reading the entire exam first and beginning with what you feel most comfortable

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

• having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including cell phones), or other memory aid devices, other than those authorized by the examiners;

• speaking or communicating with other candidates; and

• purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

5. Please **don't detach any pages**, including "extra space" pages, from the exam booklet.

6. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

7. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

You can use extra pages at the end of the exam booklet. If you want them to be marked, write "see extra page" in the exam booklet, next to the question that you want us to mark on these extra pages.

Formulas that might (or might not) be useful

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x)\,dx = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\,dx = 2$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x)\,x\,dx = 2$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\;x\,dx = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x)\;x^2\,dx = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\,x^2\,dx = \frac{1}{2}(\pi^2 - 8)$$

$$\int_{-\pi}^{\pi} \sin(x)\,dx = 0$$

$$\int_{-\pi}^{\pi} \cos(x)\,dx = 0$$

$$\int_{-\pi}^{\pi} \sin(x)\,x\,dx = 2\pi$$

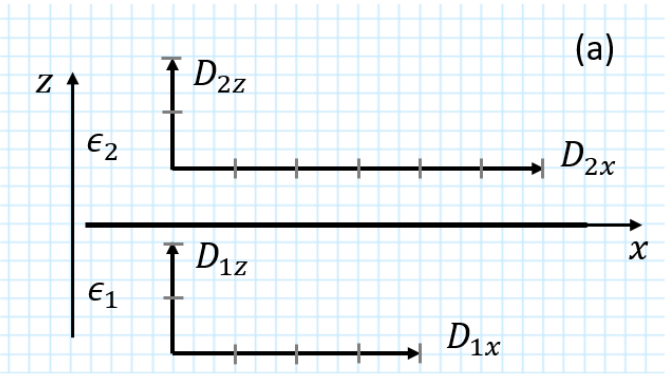
$$\int_{-\pi}^{\pi} \cos(x)\,x\,dx = 0$$

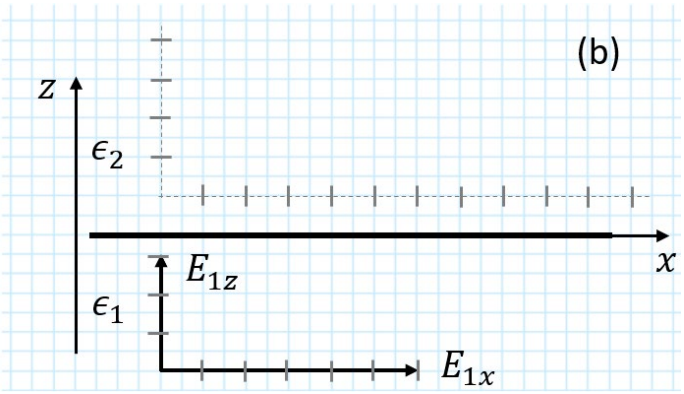
$$\int_{-\pi}^{\pi} \sin(x)\,x^2\,dx = 0$$

$$\int_{-\pi}^{\pi} \cos(x)\,x^2 dx = -4\pi$$

Problem 1 [4 pts] Panel (a) shows a sketch of an interface between two linear dielectrics with permeabilities ϵ_1 and ϵ_2. The vectors show the components of electric displacement vector right above the interface (medium 2) and right below the interface (medium 1). You can use the squares of the grid to estimate their relative size.

Panel (b) shows components of electric field right below that same interface. The size of the squares does not map directly onto panel (a) (remember, $\mathbf{D}$ and $\mathbf{E}$ have different units!). We will use this panel to discuss **electric field** above and below the interface.



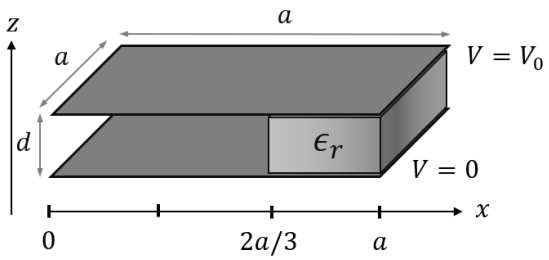


a) [1 pt] What can you say about free surface charge density: is it positive, negative or zero? **Explain**.

b) [1 pt] What is the permeability ϵ_2 if $\epsilon_1 = 2\varepsilon_0$? Show your work.

c) [2 pts] Sketch the components of electric field right above the interface (to scale).

Problem 2 [13 pts]. A parallel plate capacitor with plate area A and plate separation d is filled with a linear dielectric with relative permittivity ϵ_r. The dielectric occupies 1/3 of the volume of the capacitor, top to bottom. A potential difference, V_0, is applied to the plates of the capacitor, as shown.



a) [7 pts] Neglecting edge effects, find $\mathbf{E}$, $\mathbf{D}$ and $\mathbf{P}$ in each region: above and below the capacitor, inside the capacitor in the air and inside the capacitor in the dielectric. Give your answers in terms of V_0, d and ϵ_r. Summarize your answers (**including the direction**) in the provided table. Show your logic / calculations in the space below it.

b) [3 pts] Find **free** and **bound** surface charge densities (**magnitudes and signs**) on the metallic plates and on the dielectric slab, and explain how they are distributed (in words, or using a sketch of the system).

c) [1 pt] Find bound volume charge density inside the dielectric.

Field	Above the capacitor	Below the capacitor	Inside it, in the air	Inside it, in the dielectric

Now, for parts d) and e) only, let us include edge effects.

d) [1 pt] Sketch electric field lines everywhere in space.

e) [1 pt] As which power of r does $E(\mathbf{r})$ decay at a large distance from the capacitor? **Note**: you can answer this question even if you are not sure in your work for parts a)-c).

Problem 3 [13 pts]. A thin insulating rod placed at the origin of a coordinate system, along the x-axis, from $-a$ to a, is charged by a linear charge density $\lambda(x)$. Consider three scenarios (λ_0 is a constant):

(1) $\lambda_1(x) = \lambda_0 \cos(\pi x/2a)$ (2) $\lambda_2(x) = \lambda_0 \sin(\pi x/a)$ (3) $\lambda_3(x) = \lambda_0 \cos(\pi x/a)$.

a) [3 pts] For each case, sketch the charge distribution.

b) [10 pts] For each charge distribution, state what is the leading term $V_l(\mathbf{r})$ in the multipole expansion of its potential, and find it for an arbitrary observation point far away from the rod. You also need to provide arguments explaining why you think that all lower-l multipoles are zero (in words, or using a calculation). **Note**: some formulas on the front cover of the exam might be useful.

Extra page. If you want your work on it to be marked, indicate this clearly next to the question you are solving.

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