Physics 301 - Homework #2 - Solutions

1. Flux due to a point charge

The **E** field due to the point charge is radial and spherically symmetric, therefore the flux through each of the cube faces in the coordinate planes, (x, y), (x, z), and (y, z), is zero since the field is perpendicular to the unit normals of those faces. The flux through each of the opposite faces must be equal by symmetry. To get the value of the flux, construct a Gaussian surface composed of 8 such cubes, one in each octant of the coordinate system. This Gaussian cube has edge length $2a$ and the total flux through it must be the same as the flux through a Gaussian sphere of any radius, centred on the charge,

$$\Phi = \oint_S \mathbf{E} \cdot d\mathbf{a} = \frac{q}{\epsilon_0}. \tag{1}$$

Since there are 24 such faces in our Gaussian cube, the flux through any one of them is

$$\boxed{\Phi = \frac{q}{24\epsilon_0}.} \tag{2}$$

Let us now verify this by direct integration. It will be convenient to express the **E** field in Cartesian coordinates,

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{r^2} = \frac{q}{4\pi\epsilon_0}\frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{r^3}, \tag{3}$$

where $r \equiv \sqrt{x^2 + y^2 + z^2}$. The face of interest lies at $y = a$, has a unit normal $\hat{\mathbf{n}} = \hat{\mathbf{y}}$, and an area element $d\boldsymbol{a} = \hat{\mathbf{y}}\,dx\,dz$. The flux integral then becomes

$$\Phi = \int_0^a dx \int_0^a dz\, E_y(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\int_0^a dx \int_0^a dz\, \frac{a}{(x^2 + z^2 + a^2)^{3/2}}. \tag{4}$$

From Wolfram Alpha, the z integral has the form

$$\int \frac{dz}{(b^2 + z^2)^{3/2}} = \frac{z}{b^2\sqrt{b^2 + z^2}} \qquad (b^2 \equiv a^2 + x^2). \tag{5}$$

(This can be obtained using the substitution $z = b\tan\theta$, and integrating with respect to θ.) Then

$$\Phi = \frac{qa}{4\pi\epsilon_0}\int_0^a dx\, \frac{z}{(a^2 + x^2)\sqrt{a^2 + x^2 + z^2}}\Bigg|_0^a = \frac{qa^2}{4\pi\epsilon_0}\int_0^a dx\, \frac{1}{(a^2 + x^2)\sqrt{2a^2 + x^2}}. \tag{6}$$

From Wolfram Alpha, the x integral is found to be

$$\int_0^a \frac{dx}{(a^2 + x^2)\sqrt{2a^2 + x^2}} = \frac{1}{a^2}\tan^{-1}\left(\frac{x}{\sqrt{2a^2 + x^2}}\right)\Bigg|_0^a = \frac{1}{a^2}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6a^2}. \tag{7}$$

This gives the expected result,

$$\boxed{\Phi = \frac{q}{24\epsilon_0}.} \tag{8}$$

2. Electric field of a sphere & Superposition

a) Since electric field in this problem must have spherical symmetry, $\mathbf{E}(\mathbf{r}) = E_r(r)\hat{\mathbf{r}}$, we can use Gauss's law. To find the field outside the sphere at a distance r from its center, we draw a concentric Gaussian surface with a radius r. The total charge enclosed by this Gaussian surface is $Q_{enc} = \rho V_{\text{sphere}} = \rho(4\pi R^3/3)$, while the flux through it is $E(r)\, 4\pi r^2$. We get:

$$E_{\text{out}}(r) = \frac{\rho R^3}{3\epsilon_0}\frac{1}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, \tag{9}$$

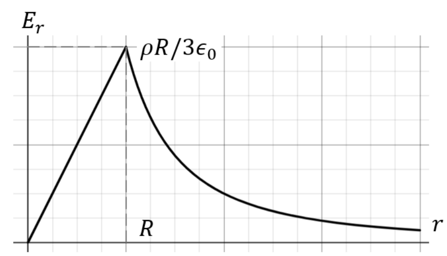
with

$$\frac{\rho R^3}{3\epsilon_0} = \frac{Q}{4\pi\epsilon_0}. \tag{10}$$

To find electric field inside the sphere, at a distance r from its center, we again draw a Gaussian sphere of radius r. The difference with the previous situation is that now the amount of enclosed charge depends on the distance r: $Q_{enc} = \rho(4\pi r^3/3)$. Therefore, $E(r)\, 4\pi r^2 = 4\pi\rho R^3/3\epsilon_0$ we get:

$$E_{\text{in}}(r) = \frac{\rho r}{3\epsilon_0} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{R^3}. \tag{11}$$

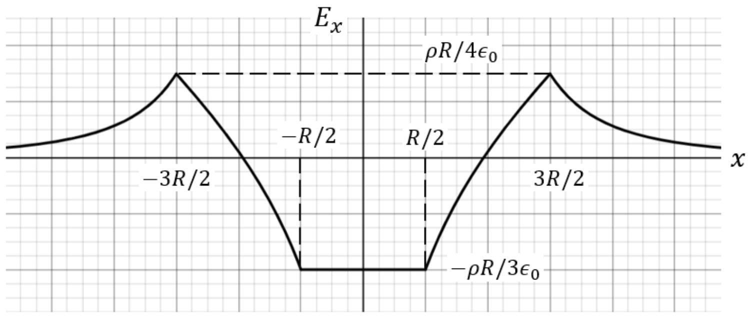
The electric field is continuous at $r = R$ as it should be (there is no surface charge density).



b) Now we can use electric field from part a) along with the principle of superposition for the two spheres. We will split the x-axis into six intervals and compute electric field in each of these intervals:

$$\begin{aligned}
&(1),\ x > 3R/2: && E_1(x) = \frac{\rho}{3\epsilon_0}\left(\frac{R^3}{(x-R/2)^2} - \frac{R^3}{(x+R/2)^2}\right);\\
&(2),\ R/2 < x < 3R/2: && E_2(x) = \frac{\rho}{3\epsilon_0}\left((x-R/2)) - \frac{R^3}{(x+R/2)^2}\right);\\
&(3),\ 0 < x < R/2: && E_3(x) = \frac{\rho}{3\epsilon_0}\Big(-(R/2-x)) - (R/2+x))\Big) = -\frac{\rho R}{3\epsilon_0};\\
&(4),\ -R/2 < x < 0: && E_4(x) = \frac{\rho}{3\epsilon_0}\Big(-(R/2+|x|)) - (R/2-|x|))\Big) = -\frac{\rho R}{3\epsilon_0};\\
&(5),\ -3R/2 < x < -R/2: && E_5(x) = \frac{\rho}{3\epsilon_0}\left(-\frac{R^3}{(R/2+|x|)^2} + (|x|-R/2))\right);\\
&(6),\ x < -3R/2: && E_6(x) = \frac{\rho}{3\epsilon_0}\left(-\frac{R^3}{(|x|+R/2)^2} + \frac{R^3}{(|x|-R/2)^2}\right).
\end{aligned} \tag{12}$$

We see that electric field is a continuous function of the coordinate. This is consistent with the fact that there is no surface charge density.



c) Finally, we want to prove that electric field is constant in the region of the overlap. Let **a** be a vector from the center of the right sphere to the center of the left sphere; $\mathbf{r}_1$ and $\mathbf{r}_2$ be, respectively, vectors from the center of the right / left sphere to an arbitrary observation point in the overlap zone. Then using expression (11) for the field inside the spheres, we get:

$$\boxed{\mathbf{E}(\mathbf{r}) = \frac{\rho}{3\varepsilon_0}\mathbf{r}_1 - \frac{\rho}{3\varepsilon_0}\mathbf{r}_2 = \frac{\rho}{3\varepsilon_0}(\mathbf{r}_1 - \mathbf{r}_2) = \frac{\rho}{3\varepsilon_0}\mathbf{a} = \text{const},} \tag{13}$$

with **a** pointing from the positive to the negative sphere, and with $a = R$.

3. Electric field near a hydrogen atom

The net charge density of the combined proton plus electron may be written as

$$\rho(\mathbf{r}) = e\,\delta^3(\mathbf{r}) + \rho_e(\mathbf{r}) = e\,\delta^3(\mathbf{r}) + \rho_0\,e^{-2r/a_0}. \tag{14}$$

Let's first determine the value of ρ_0 required to make the electron charge density integrate to $-e$, that is,

$$\int_V \rho_e(\mathbf{r})\,d\tau = -e. \tag{15}$$

Since ρ_e is spherically symmetric, the volume element is $d\tau = 4\pi r^2 dr$, so that

$$\int_V \rho_e(\mathbf{r})\,d\tau = 4\pi\rho_0 \int_0^\infty e^{-2r/a_0}\,r^2 dr = -e. \tag{16}$$

From Wolfram Alpha, the integral is

$$\int_0^\infty e^{-2r/a_0}\,r^2 dr = -\frac{a_0}{4}e^{-2r/a_0}\left(a_0^2 + 2a_0 r + 2r^2\right)\Big|_0^\infty = \frac{a_0^3}{4}. \tag{17}$$

Hence

$$\frac{4\pi\rho_0 a_0^3}{4} = -e \quad \rightarrow \quad \rho_0 = -\frac{e}{\pi a_0^3}. \tag{18}$$

Note that ρ_0 has units of charge/volume, as expected.

By symmetry, the field due to this distribution is radial, thus we can use Gauss' law to solve for the electric field within the hydrogen atom

$$\Phi(r) = \oint_{A_r} \mathbf{E}(r)\cdot d\mathbf{a} = \frac{q_{\text{enc}}(r)}{\epsilon_0}, \tag{19}$$

where A_r represents a Gaussian sphere of radius r centred on the origin, $\mathbf{E}(r)$ is the (radial) electric field at radius r, and $q_{\text{enc}}(r)$ is the charge enclosed within the Gaussian sphere.

The flux integral is straightforward,

$$\oint_{A_r} \mathbf{E}(r) \cdot d\mathbf{a} = E(r)\,\hat{\mathbf{r}} \cdot 4\pi r^2\,\hat{\mathbf{r}} = 4\pi r^2 E(r). \tag{20}$$

The enclosed charge within radius r requires another integral,

$$q_{\text{enc}}(r) = \int_{V_r} \rho(\mathbf{r})\,d\tau = e - \frac{4\pi e}{\pi a_0^3} \int_0^r e^{-2r'/a_0}\, r'^2 dr', \tag{21}$$

where the upper limit is now finite. Using the integral form given above, we have

$$\int_0^r e^{-2r'/a_0}\, r'^2 dr' = \frac{a_0^3}{4}\left[1 - \exp(-2r/a_0)\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right)\right]. \tag{22}$$

Therefore, the enclosed charge within radius r is

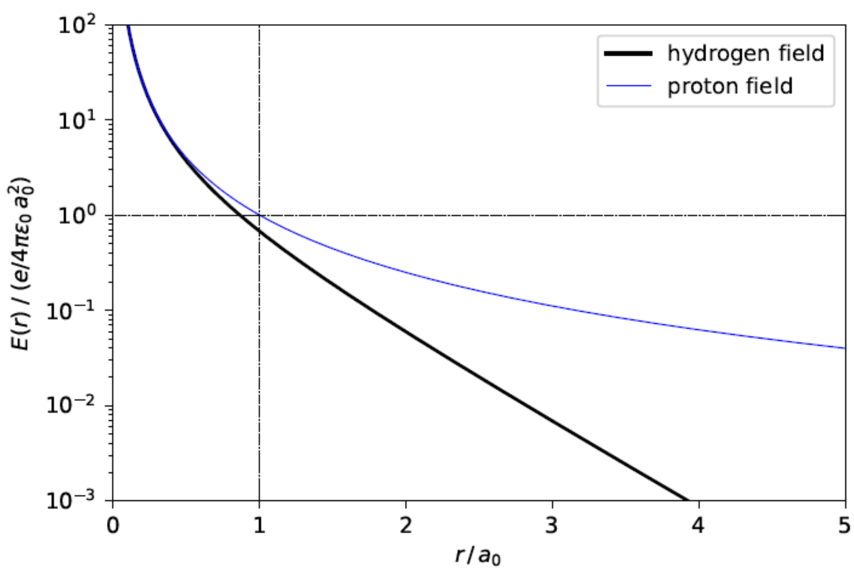
$$q_{\text{enc}}(r) = e - e\left[1 - \exp(-2r/a_0)\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right)\right] = e \cdot \exp(-2r/a_0)\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right). \tag{23}$$

Finally, applying Gauss' law gives,

$$\boxed{E(r) = \frac{e}{4\pi\epsilon_0}\frac{1}{r^2}\exp(-2r/a_0)\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right).} \tag{24}$$

Let's plot this field and compare it to the field of a bare proton. The electric field strength for a model of the hydrogen atom is shown in black. The $1/r^2$ field of a bare proton (blue) is shown for comparison. The exponential suppression of the field due to the electron's charge screening becomes clearly evident beyond the Bohr radius, a_0.

Note that for $r \ll a_0$, this reduces to the field of a point charge e (the proton), while for r greater than the Bohr radius, a_0, the field is exponentially suppressed by the charge screening effect of the electron cloud.



4. Coaxial cable

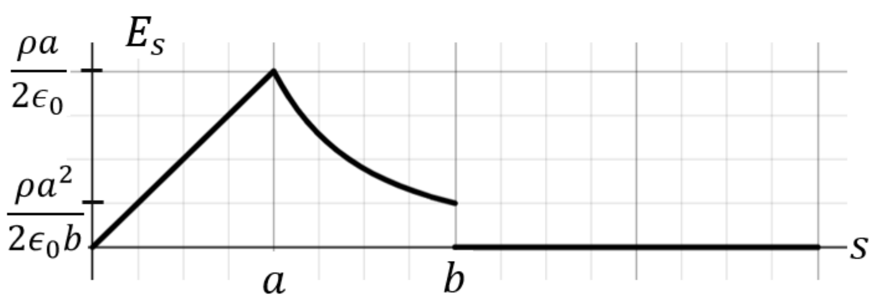
By symmetry, $\mathbf{E}(\mathbf{r}) = E_s(s)\hat{\mathbf{s}}$. We use Gauss's law, $\oint_{GS} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{a} = Q_{enc}/\epsilon_0$, with the Gaussian surface being a coaxial cylinder with radius s (passing through the observation point, as always) and length L. The flux through it is

$$\oint_{GS} \mathbf{E}(\mathbf{r}) \cdot d\mathbf{a} = 2\pi s \, L \, E_s(s). \tag{25}$$

The amount of enclosed charge depends on the location of the observation point. If $s < a$, then $Q_{enc} = \rho \, \pi s^2 \, L$. For $a < s < b$, $Q_{enc} = \rho \, \pi R^2 \, L$, since going away from the inner cylinder does not add any extra charge. Finally, for $s > b$ $Q_{enc} = 0$, since the cable is overall electrically neutral. Hence:

$$\begin{aligned} &(1),\ 0 < s < a: \quad E_s(s) = \frac{\rho s}{2\epsilon_0}; \\ &(2),\ a < s < b: \quad E_s(s) = \frac{\rho}{2\epsilon_0}\frac{a^2}{s}; \\ &(3),\ b < s: \qquad E_s(s) = 0. \end{aligned} \tag{26}$$

The plot of $E_s(s)$ is shown below. Note that the electric field is continuous at $s = a$, where we have volume charge density, and has a jump at $s = b$ due to the surface charge density σ. Marvellous – everything is as it should be!



5. Electric field and potential

We start with finding functional form for electric field in the three regions shown in the graph. The third interval is a function of the form $-\text{const}/r^2$, which passes through a point $(2a, -2E_0)$, so we conclude that

$$E_3(r) = -2E_0 \frac{(2a)^2}{r^2} = -\frac{8E_0 a^2}{r^2}. \tag{27}$$

Furthermore, in the second interval the electric field is a linear function with a (negative) slope $-2E_0/a$, and it intersects the vertical axis at $E = 5E_0$. Hence,

$$E_2(r) = -2E_0 \frac{r}{a} + 5E_0. \tag{28}$$

Now we can calculate electric potential everywhere in space. Since the equation

$$\Delta V = V(r_f) - V(r_i) = -\int_i^f \mathbf{E} \cdot d\mathbf{r} \tag{29}$$

only provides information about potential *difference*, we need to have a point where potential is known. Since $V(r=\infty)=0$ is our reference point, we start with the region $r>2a$:

$$V(\infty) - V(r) = -\int_r^\infty \left(-\frac{8E_0a^2}{r^2}\right) dr \tag{30}$$

(integrating "outwards", i.e. in the direction of increasing r, results in less chances to make a mistake). From here we find:

$$V(r>2a) = -\frac{8E_0a^2}{r}. \tag{31}$$

From (31) we know that $V(2a) = -4E_0a$. Since potential must be continuous, we can now use this value to compute electric potential in the second interval, $a<r<2a$:

$$V(2a) - V(r) = -\int_r^{2a} \left(-2E_0\frac{r}{a} + 5E_0\right) dr, \tag{32}$$

which, together with $V(2a) = -4E_0a$, gives:

$$V(a<r<2a) = 2E_0a - 5E_0r + E_0\frac{r^2}{a}. \tag{33}$$

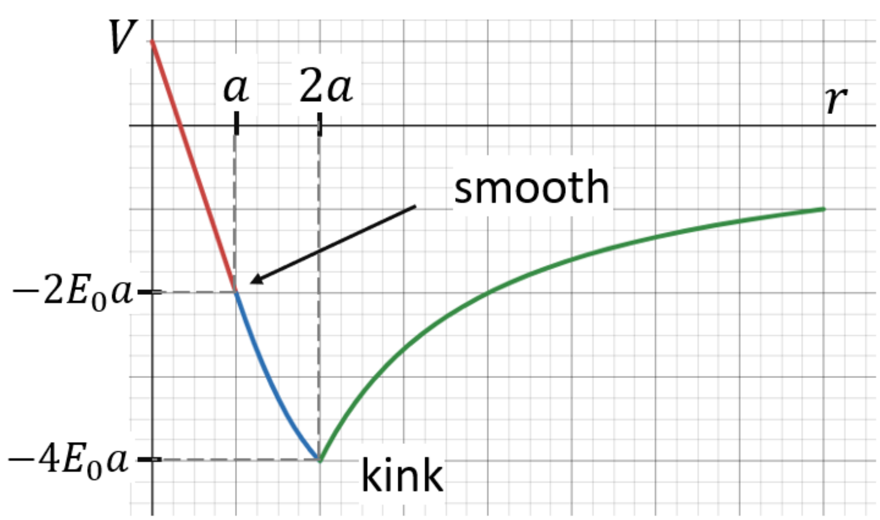
Finally, we can find electric potential everywhere in the region $r<a$, since due to continuity of the potential we know its value at the edge of this interval: $V(a) = -2E_0a$. We have:

$$V(a) - V(r) = -\int_r^a (3E_0)\, dr, \tag{34}$$

which gives

$$V(r<a) = E_0a - 3E_0r. \tag{35}$$

Plotting these expressions in their respective intervals yields a smooth function at $r=a$ (where electric field is continuous), and a kink at $r=2a$ (where electric field has a jump). Potential is continuous, as it should be. Its negative derivative gives us correct values of electric field – done!



6. An infinite charged wire

a) Electric field.

i) Using Gauss's law: by symmetry, $\mathbf{E}(\mathbf{r}) = E_s(s)\hat{s}$. Choosing Gaussian surface as a coaxial cylinder with length L and radius s, where s is the radial coordinate of the observation point, we get:

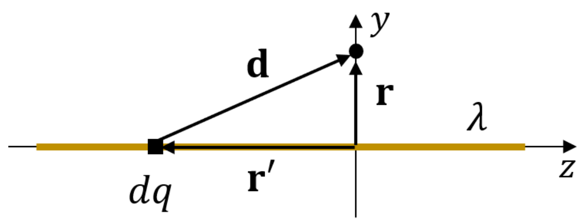
$$E_s\, 2\pi s\, L = \frac{\lambda L}{\epsilon_0} \qquad \rightarrow E_s(s) = \frac{\lambda}{2\pi\epsilon_0 s}. \tag{36}$$

ii) By integration: Since for an infinite line of charge all the points at the same perpendicular distance from it are equivalent, it is enough to consider the point opposite to the origin, at a perpendicular distance s from it. The equation for electric field is:

$$\mathbf{E} = \int \frac{\rho(r^{'})\, d\tau^{'}}{d^2}\hat{\mathbf{d}} \rightarrow \int_{-\infty}^{\infty} \frac{\lambda\, dz}{d^2}\hat{\mathbf{d}}, \tag{37}$$

with $\mathbf{r} = (0, s, 0)$ and $\mathbf{r}^{'} = (0, 0, z)$. Then

$$\mathbf{d} = (0, s, -z), \qquad d = \sqrt{s^2 + z^2}. \tag{38}$$



The y-component of electric field is (all other components are equal to zero by symmetry):

$$E_y = \frac{\lambda s}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz}{(s^2 + z^2)^{3/2}} = \frac{\lambda}{2\pi\epsilon_0 s}. \tag{39}$$

Both ways, we get:

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{\hat{\mathbf{s}}}{s}. \tag{40}$$

b) Electric potential.

i) By integration: Using the same geometry as before and $dV = dq/(4\pi\epsilon_0 d)$, we get:

$$V(s) = \frac{1}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{\lambda dz}{d} = \frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz}{\sqrt{s^2 + z^2}}. \tag{41}$$

This integral diverges, so we replace ∞ by a very large linear size L and get:

$$V(s) = \frac{\lambda}{4\pi\epsilon_0} \lim_{L\rightarrow\infty} \frac{1}{2} \ln \frac{\sqrt{s^2 + z^2} + z}{\sqrt{s^2 + z^2} - z}. \tag{42}$$

We use Taylor's expansion, $\sqrt{s^2 + L^2} - L \approx L + \frac{1}{2}\frac{s^2}{L} - L = s^2/2L$. After simplifying, we get:

$$V(s) = -\frac{\lambda}{2\pi\epsilon_0} \ln \frac{s}{2L}. \tag{43}$$

This expression for the potential diverges. The reason for this is that the electric field decays slowly, as $E \propto 1/s$, which gives a divergent integral. This can be resolved if, instead of doing what we did above (which was basically an attempt to calculate the potential with respect to a reference point at $s = \infty$, where it diverges instead of decaying to zero), to compute a *potential difference* between two points at perpendicular distances a and b. We get:

$$\Delta V_{a\to b} = V(b) - V(a) = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{2L} + \frac{\lambda}{2\pi\epsilon_0}\ln\frac{a}{2L} = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a}, \tag{44}$$

which is a nice converging potential.

Another way to think about it is that, by using $dV = \lambda dz/4\pi\epsilon_0 d$, we have chosen infinity as the reference point for the potential. If we want to count potential not from infinity but from some other point, we have to add an integration constant to Eq.(43). Choosing some finite point a as a reference, we realize that the integration constant we have to add to satisfy the boundary condition $V(a) = 0$ is

$$-\frac{\lambda}{2\pi\epsilon_0}\ln\frac{2L}{a}, \tag{45}$$

which leaves us with

$$V(s) = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{s}{a}. \tag{46}$$

The take-away message is: in cylindrical geometry with an infinite charge distribution, it is in our best interests to set up the reference point for electric potential at a finite distance from its axis. If, for example, we had a charged cylinder, we could set the zero of the potential on its surface. In this wire problem we do not have a natural length scale, so we just have to pick it arbitrarily.

ii) By integrating $\mathbf{E}$: Lesson learned – let's say that the reference point for the potential is $s = a$. Then:

$$V(s) = V(a) - \int_a^s \mathbf{E}\cdot d\mathbf{s}' = V(a) - \int_a^s \frac{\lambda}{2\pi\epsilon_0}\frac{ds'}{s'} = V(a) - \frac{\lambda}{2\pi\epsilon_0}\ln\frac{s}{a}. \tag{47}$$

iii) By solving Laplace's equation (we leave Poisson's equation aside, since we are looking at $s > 0$, where $\rho(\mathbf{r}) \equiv 0$). Writing down Laplacian in cylindrical coordinates, we get:

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) = 0, \tag{48}$$

which reduces to

$$s\frac{\partial V}{\partial s} = C_1, \tag{49}$$

and hence

$$V(s) = C_1 \ln s + C_2, \tag{50}$$

where C_1 and C_2 are two constants. Since we want $V(a) = 0$, we find that $C_2 = -C_1 \ln a$, and hence

$$V(s) = C_1 \ln\frac{s}{a}. \tag{51}$$

Finally, to find C_1 we recall that

$$E_s(s) = \frac{\lambda}{2\pi\epsilon_0 s} = -\nabla V(s) = -\frac{C_1}{s}, \tag{52}$$

from where we conclude that $C_1 = -\lambda/2\pi\epsilon_0$, and hence, if $V(a) = 0$, then, same as before,

$$V(s) = -\frac{\lambda}{2\pi\epsilon_0}\ln\frac{s}{a}. \tag{53}$$

c) The full expression for the divergence in cylindrical coordinates is

$$\nabla \cdot \mathbf{E} = \frac{1}{s}\frac{\partial(sE_s)}{\partial s} + \frac{1}{s}\frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z}. \tag{54}$$

In our problem, $E_s = 2\lambda/(4\pi\epsilon_0 s)$, and $E_\varphi = E_z = 0$, so that

$$\boxed{\nabla \cdot \mathbf{E} = \frac{2\lambda}{4\pi\epsilon_0}\frac{1}{s}\frac{\partial(s/s)}{\partial s} = 0 \qquad (s > 0).} \tag{55}$$

d) Consider a cylindrical Gaussian surface of radius a and length L. By Gauss' law, the flux through this cylinder is $q_{\text{enc}}/\epsilon_0$, where $q_{\text{enc}} = \lambda L$ is the enclosed charge. Hence,

$$\boxed{\Phi = \frac{\lambda L}{\epsilon_0}.} \tag{56}$$

We'll now verify this by direct integration. The flux of $\mathbf{E}$ through the end caps is zero since, $(\mathbf{E} \parallel \hat{\mathbf{s}}) \perp (d\mathbf{a} \parallel \hat{\mathbf{z}})$. For the body of the cylinder, the area element is $d\mathbf{a} = a\, d\varphi\, dz\, \hat{\mathbf{s}}$, so that

$$\Phi = \int_A \mathbf{E}(a) \cdot d\mathbf{a} = a\int_0^{2\pi} d\varphi \int_0^L dz\, E_s(a) = 2\pi a L\, \frac{2\lambda}{4\pi\epsilon_0}\frac{1}{a}, \tag{57}$$

so that

$$\boxed{\Phi = \frac{\lambda L}{\epsilon_0}.} \tag{58}$$

e) Per Maxwell's equation, the divergence of the electric field is $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$. The simplest way to express the charge density in this problem is $\rho(\mathbf{r}) = \lambda\, \delta(x)\delta(y)$, so that,

$$\boxed{\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{\lambda}{\epsilon_0}\, \delta(x)\delta(y).} \tag{59}$$

Note that, since $\delta(x)$ has units of x^{-1}, the expression $\rho(\mathbf{r}) = \lambda\, \delta(x)\delta(y)$ is dimensionally consistent. Note also that this expression produces the expected charge enclosed within our Gaussian cylinder:

$$q_{\text{enc}} = \int_V \rho(\mathbf{r})\, d\tau = \lambda \int_0^L dz \int_{-a}^{a} dy\, \delta(y) \int_{-\sqrt{a^2-y^2}}^{+\sqrt{a^2-y^2}} dx\, \delta(x) = \lambda L, \tag{60}$$

where the last equality follows since both the x and y integrals encompass 0.

Comment: We can write the delta-function in cylindrical coordinates as (see [1] and [2]; note that $1/s$ does not cause problems since it cancels with s while being integrated over $d\tau = ds\ sd\phi\ dz$):

$$\rho(\mathbf{r}) = \frac{\lambda}{2\pi s}\delta(s). \tag{61}$$

Using this and Eq.(40) for electric field, we can rewrite (59) as:

$$\nabla \cdot \left(\frac{\hat{s}}{s}\right) = \frac{\delta(s)}{s}, \tag{62}$$

which reminds us how we found that $\nabla \cdot (\hat{r}/r^2) = 4\pi\delta(\mathbf{r})$. Similarly to what we discussed in the lectures, here, at first, we found zero divergence for a function visible divergent, and it turned out that its divergence is in fact **not** zero, but *zero everywhere but at the location of the charge*, where it is infinite.