Physics 301 - Homework #2

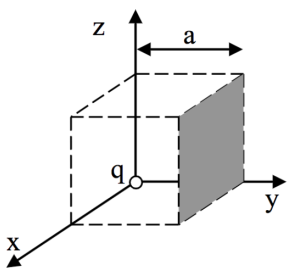
Due 5 Oct 2025

1. Flux due to a point charge

A charge q at the origin is located at the back corner of an imaginary cube of edge length a. What is the electric flux through the shaded region (the plane at $y = a$)? Work out the answer in two ways:

a) Using Gauss' law (set up a proper Gaussian surface to make it work).

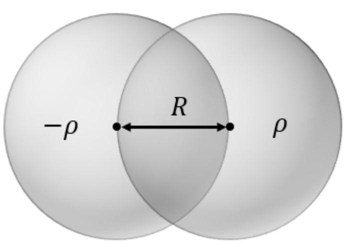
b) By direct integration. Here you might find it useful to work in Cartesian coordinates.



2. Electric field of a sphere & Superposition

a) As a warm-up, find the electric field due to a sphere charged with a uniform volume charge density ρ. The radius of the sphere is R. Sketch the electric field as a function of the radial coordinate. Is the electric field continuous at the boundary of the sphere? Explain.

b) Now consider two overlapping spheres, each with radius R and the same but opposite volume charge densities. Their centers are separated by a distance R.



Compute and sketch the electric field as a function of radial coordinate everywhere on the axis connecting the centers of these spheres. Label characteristic field values.

c) Prove that $\mathbf{E}(\mathbf{r}) = \text{const}$ everywhere in the area of overlap (not only on the axis connecting the centers of the spheres).

Note: The best way to approach this problem is to use the principle of superposition. Remember that $\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r})$ literally means the following: if you have two charged objects, and you know the field produced by the first *in the absence of the second*, and you also know the field produced by the second *in the absence of the first*, you are almost there: you simply need to add these two fields as vectors.

3. Electric field near a hydrogen atom

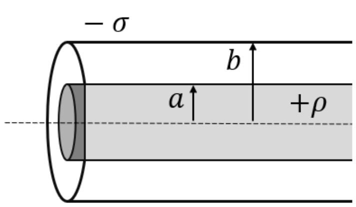
Quantum mechanics tells us that an electron is not a point-like particle but is spread over space. The electron's ground-state contribution to the charge density of a hydrogen atom may be written as

$$\rho(r) = \rho_0 \exp(-2r/a_0),$$

where a_0 is the Bohr radius. (Note: Choose ρ_0 so that the electron's total charge is $-e$.) Add a point-like proton of charge $+e$ at the origin and compute the electric field in this model of the hydrogen atom, as a function of radial distance. Sketch and briefly discuss your result.

4. Coaxial cable

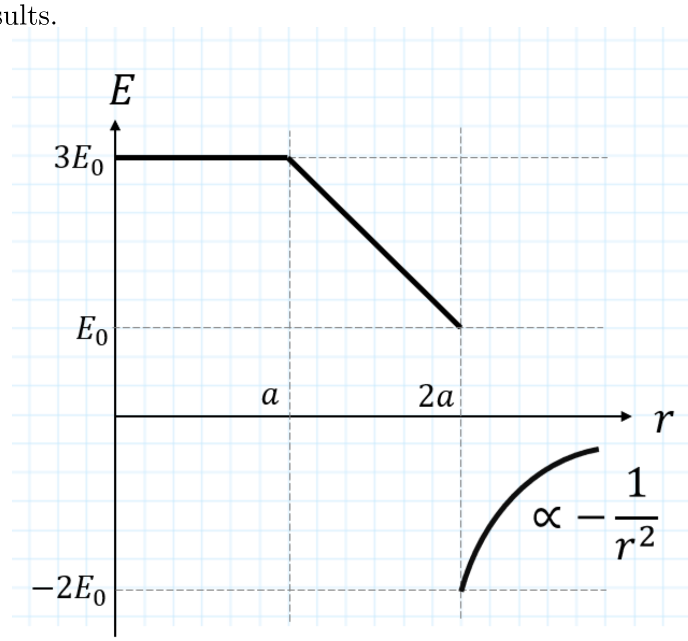
A coaxial cable consists of the inner electrode (radius a, volume charge density ρ) and an outer cylindrical shell (radius b, surface charge density $-\sigma$). The cable is overall electrically neutral. Assume that the outer shell is infinitely thin, volume charge density ρ is uniform, and the length of the coaxial cable greatly exceeds its radial scales.



Find electric field everywhere in space and sketch it. Is your result consistent with what you know about boundary conditions for electric field? Explain.

5. Electric field and potential

This graph shows the radial component of electric field in spherical coordinates. The field has no other components. Compute electric potential at all points in space. Assume $V(r = \infty) = 0$. Graph your results.



6. An infinite charged wire

Consider an infinite line charge along the z-axis with a uniform linear charge density λ.

a) Find the electric field $\mathbf{E}(\mathbf{r})$ at a distance s from the charged line. Do it in two ways: i) integrating over the charge distribution, and ii) using Gauss's law.

b) Find the electric potential $V(\mathbf{r})$ at a distance s from the charged line. Do it in three ways: i) integrating over the charge distribution, ii) integrating electric field from part a), iii) from Laplace's and/or Poisson's equation(s). If you find that your potential diverges, it does not necessarily mean that you made a mistake. Try to compute it using as a reference point (i.e. a point where $V = 0$) a point at a certain finite distance, a, from the charged line.

c) Find the divergence of the electric field of the infinite line charge for $s > 0$.

d) Calculate the electric flux through a Gaussian cylinder of length L, and radius R, centered on the z-axis.

e) Given the results above, what is the divergence of this electric field everywhere in space?

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Was your answer for the divergence of this electric field for $s > 0$ (part c) what you expected it to be from how the field looks like? Does the story in parts c)-e) remind you of something that we did in class? If yes, comment on what it is. If not, check out the lecture notes!