Physics 301 - Homework #4

Due 2 November 2025

1. Conceptual questions

A dielectric is called homogeneous if its material constants (susceptibility and permittivity) do not depend on coordinates (are uniform), and is called inhomogeneous if $\chi_e = \chi_e(\mathbf{r})$, $\epsilon_r = \epsilon_r(\mathbf{r})$.

a) Show that a neutral linear homogeneous dielectric cannot have a bound volume charge.

b) Show that in a linear inhomogeneous dielectric in general $\nabla \times \mathbf{P} \neq 0$.

2. Parallel plate capacitor with dielectric

Consider a parallel plate capacitor with area A, separation d, and charge $\pm Q$ on the two plates. For concreteness, let the z-axis be perpendicular to the plates, and directed from the negative to the positive plate. Neglect edge effects throughout.

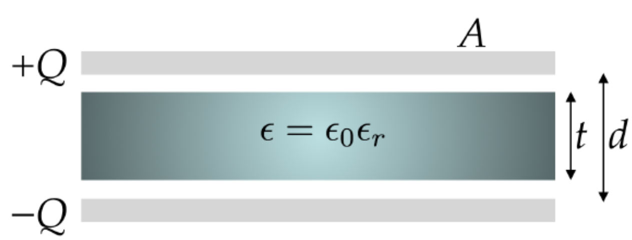
a) Determine $\mathbf{D}$ and $\mathbf{E}$ between the plates, and the voltage drop V across them.

b) Next, insert a slab of dielectric material of thickness $t < d$, area A, and permittivity ϵ midway between the plates. If the plates are isolated (i.e. not connected to any external circuit elements), determine $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{P}$ everywhere between the plates, inside and outside the dielectric, and the voltage drop across the plates. Does $\mathbf{E}$ change, relative to (a), outside the dielectric?

c) What is the bound charge, σ_B and/or ρ_B, induced in the dielectric? What is the electric field due to these charges everywhere between the plates? Call this field $\mathbf{E}_{\text{ind}}$ and define $\mathbf{E} \equiv \mathbf{E}_{\text{ext}} + \mathbf{E}_{\text{ind}}$.

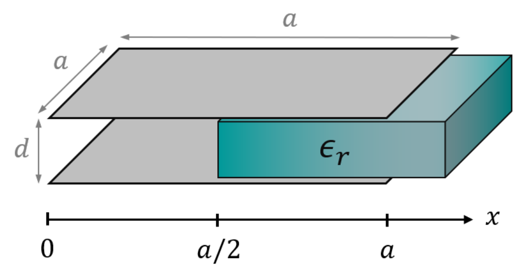
d) Express $\mathbf{E}_{\text{ext}}$ and $\mathbf{E}_{\text{ind}}$ in terms of $\mathbf{D}$ and $\mathbf{P}$ everywhere between the plates.

e) Repeat b)-d) for the case where the capacitor plates are connected to a battery and held at the potential difference you found in a). Here the charge Q may change.



3. Force on a dielectric slab

A dielectric slab with a dielectric constant ϵ_r sits half-way inside a capacitor with square plates (side a) separated by a distance d. The slab fills the capacitor from top to bottom and from front to back. Your goal is to find the force (magnitude and direction) that the electric field of the capacitor exerts on the slab. We will follow the procedure that you used at your tutorial: model this structure as two capacitors connected in parallel, set up their equivalent capacitance C_{eq} treating the distance from the left edge of the capacitor to the edge of the slab as a variable, x, and then compute force as a derivative of its potential energy taken at $x = a/2$.



a) Assume that the charge on the plates of the capacitor is $\pm Q$. Calculate the potential energy stored in the capacitor using $W = Q^2/(2C_{eq})$, and find the force on the slab (magnitude and direction). You can use the equation that we derived in class:

$$\mathbf{F}_{el} = -\nabla W \quad \rightarrow \quad F_{el,x} = -\frac{dW}{dx}. \tag{1}$$

b) Now assume that, instead, you know the voltage across the capacitor's plates, V. Of course we know that charge and voltage are connected as $Q = C_{eq}V$, but what we want in this part is to make use of the alternative expression for capacitor's energy: $W = C_{eq}V^2/2$. Find the force on the slab using it, and then express your result in the same form as in part a) using the connection between charge and voltage (eliminate V). Do you get the same magnitude? The same direction?

c) If you did everything correctly, the answer to at least one of the questions above is "No". Try to figure out the physical reason for this discrepancy. Which of the two methods is correct? Can you fix the incorrect one so that the two methods give the same answer?

Hint: You might need to scrutinize Eq.(1) above.

4. A rod with a non-uniform polarization

A hollow insulating rod with inner radius a and outer radius b has a permanent polarization $\mathbf{P} = k\hat{\mathbf{s}}/s^2$, where $\hat{\mathbf{s}}$ is the radial cylindrical coordinate and k is a constant. There is vacuum for $s < a$ and $s > b$.

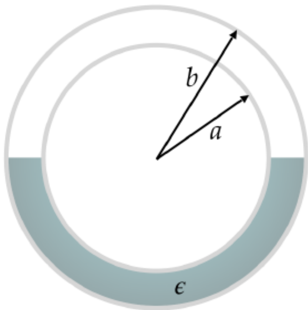
a) Find bound charge densities σ_B and ρ_B.

b) Find $\mathbf{E}$ everywhere in space ($s < a,\ a < s < b,\ s > b$).

c) Find $\mathbf{D}$ everywhere in space ($s < a,\ a < s < b,\ s > b$).

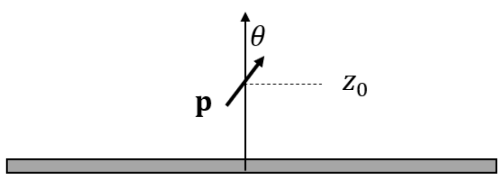
5. Half-filled spherical capacitor

A spherical capacitor consists of a pair of concentric conducing spherical shells of radii a and b. Only the lower half of the volume between the conductors is filled with a dielectric with permittivity ϵ, as shown below. The inner conductor carries a charge $+Q$ and the outer conductor a charge $-Q$. Determine the following quantities everywhere: the fields $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{P}$, the free and bound charge densities, σ_F, σ_B, and ρ_B, and the capacitance, C, of this capacitor. You may neglect edge effects at the equator.



6. Torque on a dipole.

A grounded conducting plane defines the (x,y) plane. A pure dipole with a dipole moment $\mathbf{p}$ is in the (x,z) plane at an angle θ to the z-axis, and its center is at a distance z_0 above the plane. The distance from the dipole to the plane is much larger than the size of the dipole.



a) Find the torque on the dipole.

b) What are the equilibrium values of θ?

Hint: You might (or might not) find it useful to learn about the coordinate-free form of the electric field of a dipole (See problem 3.38 from Griffiths). Even if you don't use it here, it's better to know this formula.