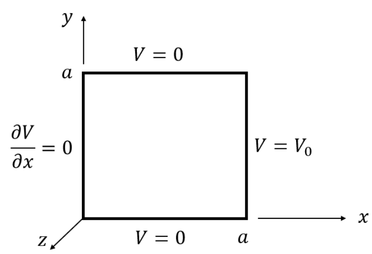
# Physics 301 - Homework #5

Due 23 November 2025

### 1. Potential in a square pipe

A rectangular square pipe (the sides of length $a$) runs parallel to the z-axis, unrestricted throughout the whole interval $-\infty < z < +\infty$. Each of the four sides of the pipe is insulated from the others at the corners, and the four sides of the pipe are maintained at the potentials shown in the figure.




a) Find the potential $V(x, y, z)$ at all points inside this pipe.

b) State in words what the boundary condition on the left wall means - what does it tell you?

c) Sketch the E-field lines inside the pipe. Hint: one way to approach this is to plot the leading term in the potential expansion that you found in part a), and find its equipotential lines; this will help you to figure out the direction of electric field lines and their density.

### 2. Dielectric shells with potential specified at the surfaces

Two concentric dielectric spherical shells have radii $a$ and $b$, with $a < b$. The potential on the inner shell is held at a constant positive value $V(a, \theta) = V_a$, while the outer shell is held to $V(b, \theta) = V_b \cos\theta$, where $V_b$ is a constant. Find the potential in the region $a < r < b$.

### 3. Potential of a rod immersed in a uniform electric field

A very long dielectric cylinder of radius $a$ and relative dielectric constant $\epsilon_r$ is placed in an electric field of magnitude $E_0$ perpendicular to its axis.

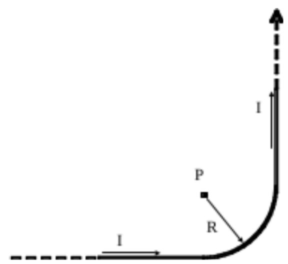
a) Derive general expression for the solution of Laplace equations using separation of variables in cylindrical coordinates.

b) Use your results from part a) to find potential everywhere in space (inside the cylinder and outside it).

c) What is the electric field inside the cylinder? What is the bound volume charge density?

**4. B field of a wire making a 90° turn**

An infinitely long wire makes a 90° turn of radius $R$, centred at the point $P$, as shown in the figure below. Using Biot-Savart law, find the magnetic field at $P$ if a steady current $I$ flows through the wire. **Not for marks, but might be useful**: Can you think of a quick "sanity check" that allows you to verify your answer?

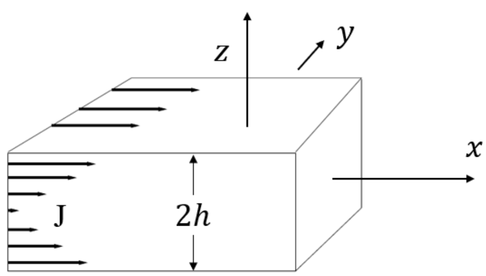



**5. B field of a slab**

a) Consider a thin infinite sheet with uniform surface current density $K_0\hat{\mathbf{x}}$ in the xy-plane at $z = 0$. Find $\mathbf{B}(x, y, z)$ both above and below the sheet using Ampere's law. This problem is solved in the Griffiths' book, but work it out here by explicitly showing Amperian loops and stating all your assumptions.

b) Now let's add a second sheet of current. Let's place it at $z = +a$, and let the current have the same magnitude, but it will be running in the opposite direction, $-K_0\hat{\mathbf{x}}$. Use the superposition principle to find $\mathbf{B}$ between the two sheets, and in the outside region (above and below the sheets).

c) Now instead of the two sheets we have a thick **slab** of current (see figure). It is unrestricted along $y$ and $x$, but has a finite thickness $2h$ along $z$ (it lays between $z = -h$ and $z = +h$). The current is still flowing in the $+x$ direction, and its **volume** density is $\mathbf{J}(\mathbf{z}) = J_0|z|\hat{\mathbf{x}}$: it is uniform in the $x$ and $y$-dimensions, but $\mathbf{J}$ depends on height linearly inside the slab, with a minimum at $z = 0$. Outside the slab the volume current density is of course zero.




Find the B-field (magnitude and direction) everywhere in space (above, below, and inside the slab).

**6. Vector potential.**

a) A very long cylinder of radius $R$ centered on the z-axis carries a uniformly distributed over its cross-section current $I$ in the $+z$-direction. Find the vector potential (i) inside and (ii) outside the cylinder from current distribution. Choose $\mathbf{A} = 0$ at the edge of the cylinder, and work in the Coulomb gauge.

b) Find magnetic field inside and outside the cylinder from (i) your vector potential and (ii) from Ampere's law. Make sure that both approaches give the same answer.