Physics 301 - Homework #6

Due 5 December 2025

1. Spinning disk

A thin disk of radius R, centered on the origin in the x-y plane, carries a uniform surface charge σ. It spins about the z axis with angular frequency ω.

a) Find the magnetic moment, $\mathbf{m}$, of the spinning disk.

b) Find **exact** expression for the $\mathbf{B}$ field along the z axis. After that, find its leading term at distances much larger than the radius of the disk.

c) Compare it to the dipole approximation for the $\mathbf{B}$ field which you can get using your result from a). Do the two answers match?

2. Conducting slab

A conducting slab parallel to the x-y plane extends from $z = -a$ to $z = +a$, and carries a uniform free current density $\mathbf{J}_f = J_0\,\hat{\mathbf{x}}$. The magnetic susceptibility of the slab is zero, and the surrounding medium is linear with susceptibility χ_m. Find $\mathbf{B}$ and the bound currents, $\mathbf{J}_b$ and $\mathbf{K}_b$, everywhere in space.

3. Frozen Magnetization

A long cylinder of radius R carries a built-in ("frozen") non-uniform magnetization $\mathbf{M} = ks^2\hat{\boldsymbol{\varphi}}$, where k is a constant and s is the distance from the axis.

a) Find the magnetic field $\mathbf{B}$ due to $\mathbf{M}$ for the points inside and outside the cylinder.

b) Find the auxiliary field $\mathbf{H}$ due to $\mathbf{M}$ for the points inside and outside the cylinder.

c) Write down boundary conditions for $\mathbf{B}$ and $\mathbf{H}$ at $s = R$. Check that your fields satisfy them.

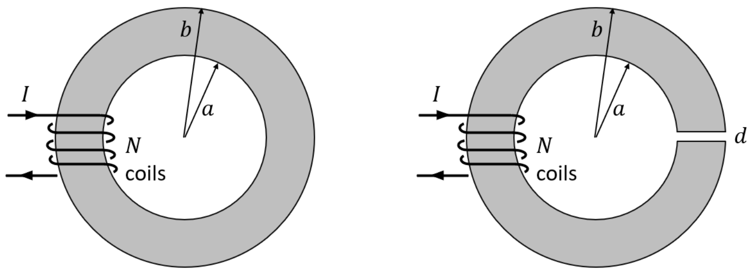
d) Is this material a linear magnetic? Explain.

4. Fields in a gap

A toroidal piece of "soft iron" (iron that is roughly a linear magnetic, but has a very large permeability μ characteristic of ferromagnetic materials) has a wire wrapped N times around a section of the toroid, which carries a steady current I. The toroid has a constant cross-sectional area A, and the inner and outer radii a and b (left image).

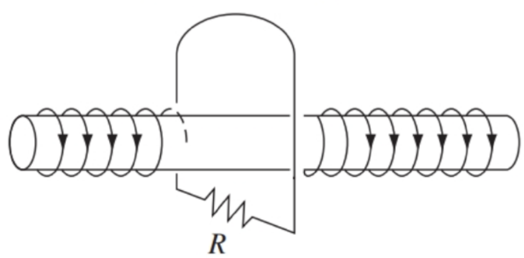
a) Find the magnetic field inside the toroid on the left. We don't need a strict derivation, but clear arguments about the symmetry and the magnitude of the field. Hint: can you argue that the fact that the coil embraces the toroid only partially does not have a great impact on $\mathbf{B}$ because $\mu \gg 1$?

b) Now a very thin gap of width d is cut in the toroid as shown (right image). Find the $\mathbf{B}$ and $\mathbf{H}$ fields in this gap. Assume that fringe fields are negligible, that $\mu \gg \mu_0$ and $d \ll a, b$.



5. A solenoid and a loop

A long solenoid of radius a, carrying n turns of wire per unit length, is looped by a wire with resistance R, as shown.



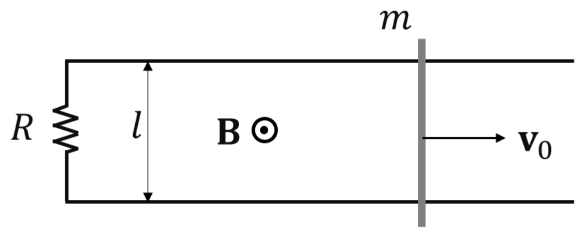
a) If the current in the solenoid is increasing at a constant rate, $dI/dt = k$, what current flows in the loop, and which way (left or right) does it pass through the resistor (as viewed in the figure)?

b) Now assume that the current I in the solenoid is kept constant, but the solenoid is pulled out of the loop towards the left, far away from the loop. How much charge passes through the resistor while the solenoid is being pulled out?

(see the next page)

6. Sliding bar.

A metal bar with mass m is sliding to the right without friction on two parallel conducting rails a distance l apart, as shown below. The rails are connected by a resistor R at the left. The bar, the rails and the resistor form a circuit which is immersed into a uniform magnetic field $\mathbf{B}$ pointing out of the page. At $t = 0$ the bar is moving to the right with speed v_0.



a) Find the emf in the circuit as an integral of force per unit charge. Show the contribution of each piece of the circuit.

b) Now calculate the emf in the circuit from the flux of the $\mathbf{B}$ field. Does it agree with your previous answer? If not, why not?

c) Find the magnitude and direction of the current through the resistor using the emf from part b).

d) Determine the motion of the bar after $t = 0$, that is, find an expression for $v(t)$. Does your answer make sense? If you find that the speed is increasing, explain where the bar gets the energy from. If you find that the speed is decreasing, explain where the energy goes.