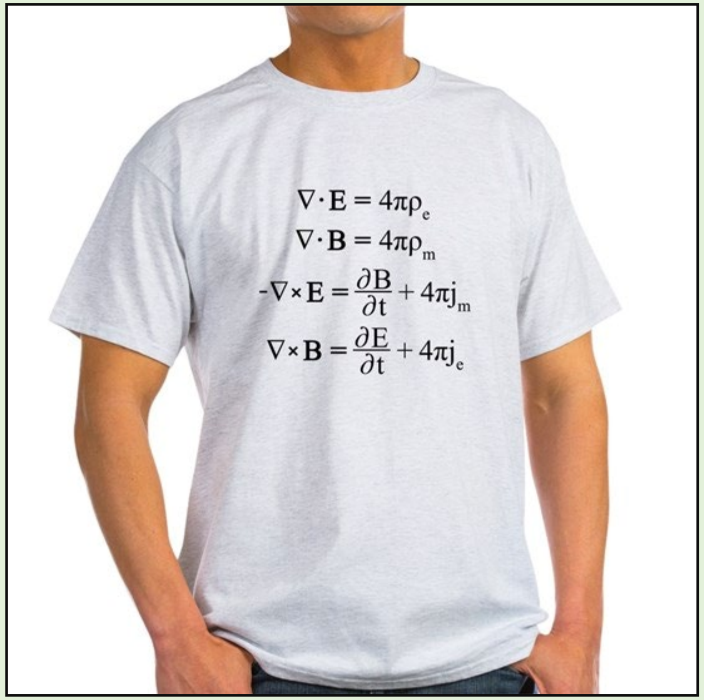
Lecture 1

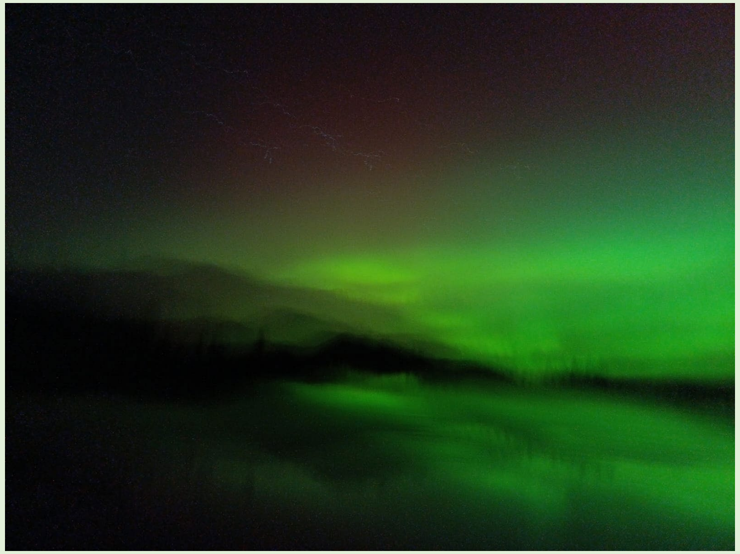
Please join iClickers – we have something to discuss!

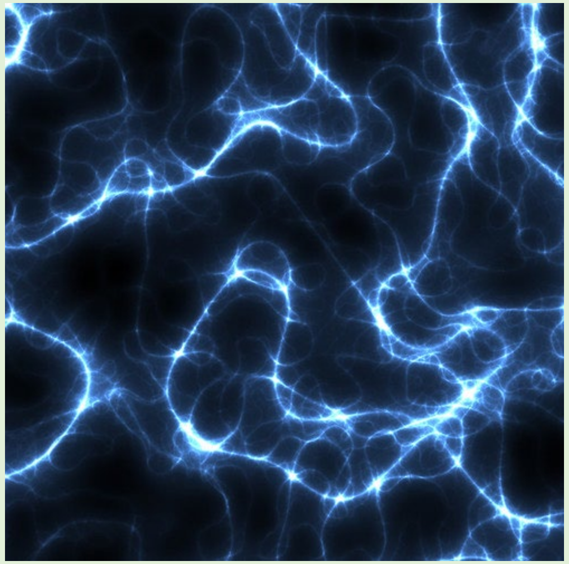
Intro. Coordinate systems. Charge density.

Welcome to PHYS 301

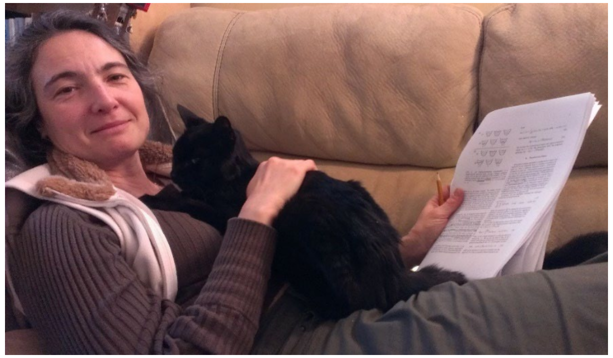
Electricity and Magnetism







Instructor: Dr. Marina Litinskaya (left in this picture)

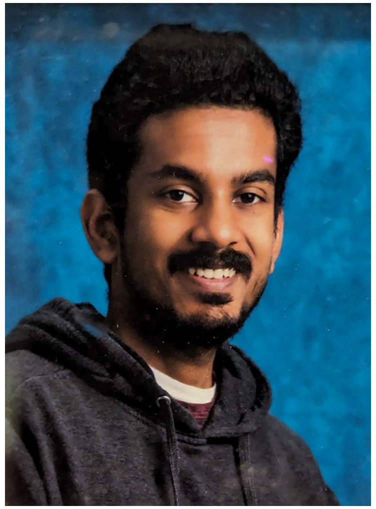


Research interests: Optics and spectroscopy of atoms, molecules and nanostructures (theory)

- Same said in a simple way: I use light to learn about what's going on inside gases and solids, and to manipulate various processes in atoms and molecules.

Teaching: PHYS 100, 131, 118, 158, 159, 170, 301, 333

Teaching Assistants:



Saran Vijayan

Daniel Stedman

Tutorials

Andree Coschizza

Daniel Alvarez

Piazza & Office hours

Does this sound familiar?

- Gauss's law
- Electric and magnetic field
- Coulomb's law
- Poisson and Laplace equations
- Electric and magnetic energy
- Electric potential
- Dielectrics
- Separation of variables
- Multipole moments
- Ampere's law
- Magnetics
- Boundary value problems
- Lorentz force
- Vector potential
- Magnetic induction
- Electromagnetic waves
- Faraday's law
- Maxwell's equations

Law	Integral form (1st yr)	Differential form (3rd yr)	Meaning
Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$	$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$	The net electric flux through a closed surface is proportional to the net charge inside it.
Gauss's law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	$\text{div}\,\vec{B} = 0$	There are no magnetic monopoles, therefore the net magnetic flux through any closed surface is always zero.
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_m}{dt}$	$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	The electromotive force (electric field) induced in a closed circuit is determined by the rate of change of the magnetic flux it encloses.
Ampère-Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{ext} + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt}$	$\text{curl}\,\vec{B} = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$	The magnetic field induced around a closed loop is proportional to the electric current through the loop plus the rate of change of electric flux it encloses.

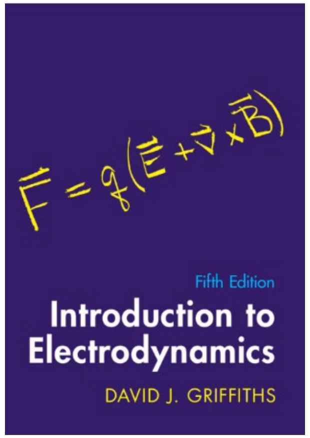
3^{rd} year E&M vs 1^{st} year E&M

- There is a very steep transition from the high-school level of knowledge (that practically everyone understands, better or worse) to scientific knowledge (which nobody except those who have your future degree understands). It is not about the number of facts that you know; it is rather about learning certain language and developing certain vision of the Universe.

- People who did not take physics courses at a university level won't have any idea why it is so important if the Hamiltonian of a system depends on time or if it does not, or how translational invariance in some direction results in conservation of the relevant component of a field. You can of course explain to them what all this is about, but to do that you will need to switch to the "regular" language, as some people do in popular books. We will be doing this transition, and it will feel difficult.

- Practice shows that the difference between 3^{rd} and 1^{st} year E&M is often discovered after the first midterm. Please try to discover it earlier…

How to make this transition easier?



- Start with reading all what is posted in "Course information" module (course rules, deadlines, missed work policy, what you should know, extra problems…)

- Reading: Come prepared, and review after lectures

UBC bookstore, obligatory

Week 1. Introduction. Curvilinear coordinate systems. Dirac delta-function.

Reading:

- Vector Algebra (1.1),
- Working in curvilinear (i.e. cylindrical and spherical) coordinate systems (1.4)
- Dirac Delta function (1.5).

Review (pre-lecture):

- Cartesian, cylindrical and spherical coordinate systems
- grad, div, curl

This is what you should know as a result of the lectures (read it before / after lectures as your unique learning style suggests)

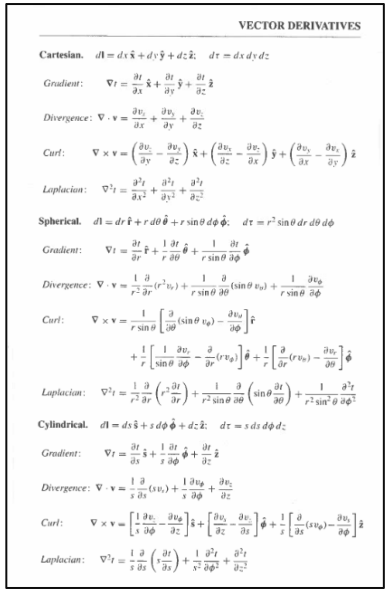
This is what you should know before lectures

- Prepare to work on your own, extra – in addition to lectures / tutorials Solve one extra (not "Exercises", they repeat lectures!) problem from Griffiths each day. Okay, each two days, if you are a very busy person.

What we have learned from PHYS 301 2024

TAs' feedback: common issues

- Math: regular integrals, sometimes derivatives, sometimes algebra.
- Dimension analysis:
  - When you add two quantities together, they have to have the same dimension. We cannot add, for example, a distance to a dimensionless number. Exponents must be dimensionless. If your dimensions are wrong, your answer is wrong.
- Cover of Griffiths book is your friend (posted here as "formulas")



VECTOR DERIVATIVES

Cartesian. $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}; \quad d\tau = dx\,dy\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial x}\hat{\mathbf{x}} + \frac{\partial t}{\partial y}\hat{\mathbf{y}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$

Spherical. $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}; \quad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$

Gradient: $\nabla t = \frac{\partial t}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

Curl: $\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$

Laplacian: $\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

Cylindrical. $d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}; \quad d\tau = s\,ds\,d\phi\,dz$

Gradient: $\nabla t = \frac{\partial t}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\mathbf{z}}$

Divergence: $\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

Curl: $\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\mathbf{z}}$

Laplacian: $\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

Students feedback: what could be improved

- HW sets are long => weekly homework?
  - It will remains bi-weekly, but please start early!
- "It was difficult to come to class, since I am not a morning person"
  - I know! But I think you still should…

Record or not record?

- I surveyed 2024 students about their thoughts on lecture recordings. 50% of respondents said that lecture recordings helped them learn, 50% said that due to recordings they switched to online mode, and this likely negatively impacted their performance.

- Duolingo effect:

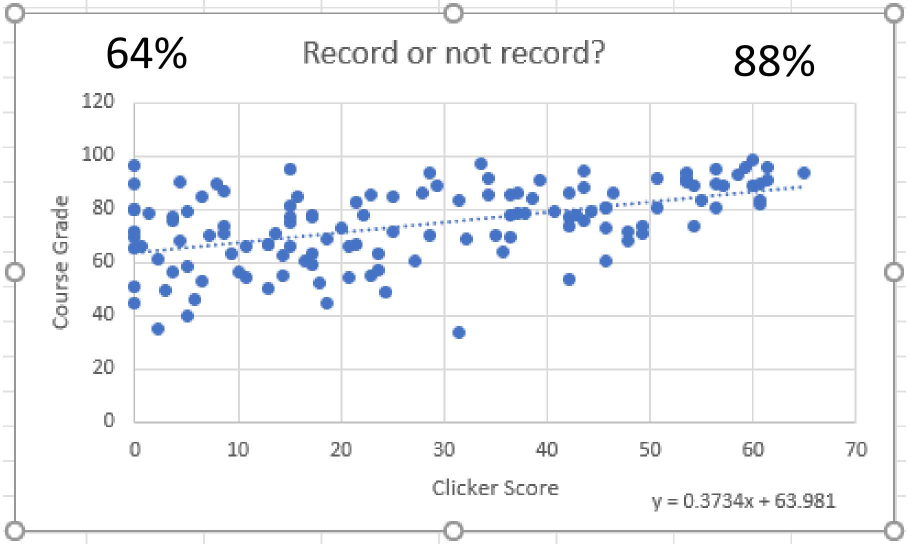
  Assume you want to learn a language. You do want that. You decide to learn it 15 mins each day. You have a textbook and a notebook. What are the chances that you will keep this promise for 3 months in a row?

Choices suggested by 2024 students:

A. Record / post right after lecture

B. Record / post before midterms & by individual request (sickness)

C. Don't record



Midterm dates

- MT1: Wednesday Oct 15 (Week 7) at 18:00 pm;
- MT2: Thursday Nov 13 (Week 11) during class time.

Traditionally, PHYS 301 midterms are run during lecture time. However, this year the Truth and Reconciliation Day falls onto a Tuesday, which takes away 80 mins of lecture time for Tu/Thu classes. Choices: (1) rushing through some of the obligatory material, (2) leaving it for your self-study, (3) stick to the regular lecture pace by shifting one of the midterms to the evening.

- If you have a conflict with proposed MT-1 time, please email me asap.

- Notetaking: you will be allowed to bring in your notes (hand-written or printed) to the examss, but not electronic devices.

Office hours

- Note: if you don't have questions about PHYS 301, most likely it means that something goes wrong with your approach…

Me, Hebb 112

A. Wed, 11:00 – 12:00
B. Wed, 12:00 – 13:00
C. Wed, 13:00 – 14:00
D. Wed, 14:00 – 15:00
E. Wed, 15:00 – 16:00

12:30 – 13:30 +

TAs, Hebb 112

A. Thu, 11:00 – 12:00
B. Thu, 12:00 – 13:00
C. Thu, 13:00 – 14:00
D. Fri, 10:00 – 11:00
E. Fri, 11:00 – 12:00

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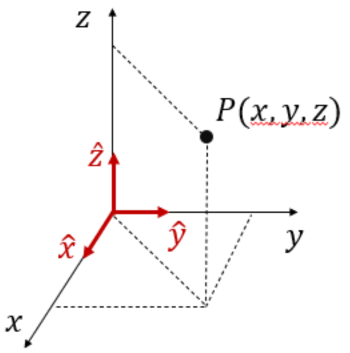
*) You are always welcome to ask questions after lecture, or book an appointment by email.

Coordinate systems

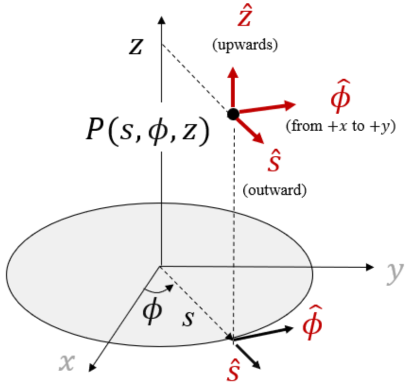
(Ch. 1.4)

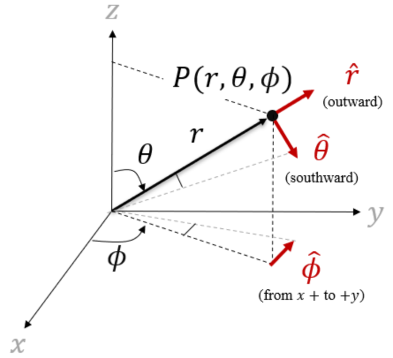


A.

B.

C.





- Cartesian
- Cylindrical
- Spherical

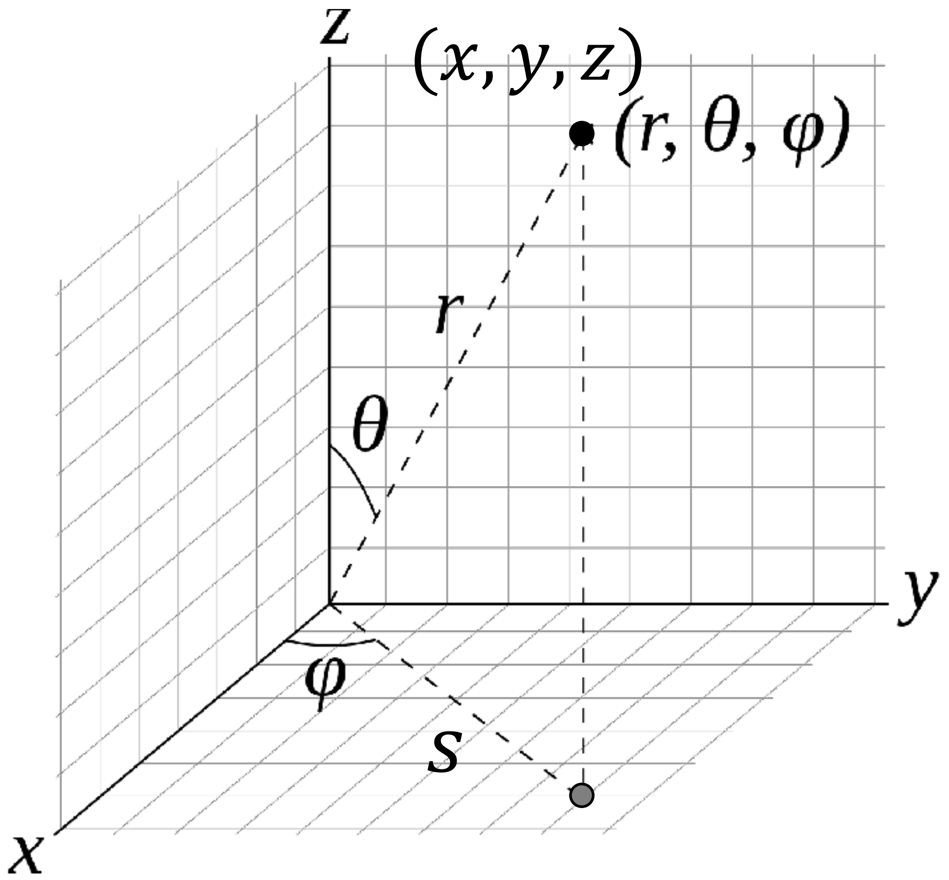
Coordinate systems: Overview

The idea is that to specify a location of a point in 3D space you need to define it with 3 coordinates:

- (x, y, z) - rectangular/Cartesian
- (r, θ, φ) - spherical
- (s, φ, z) - cylindrical

Choice is dictated by the symmetry of the problem:

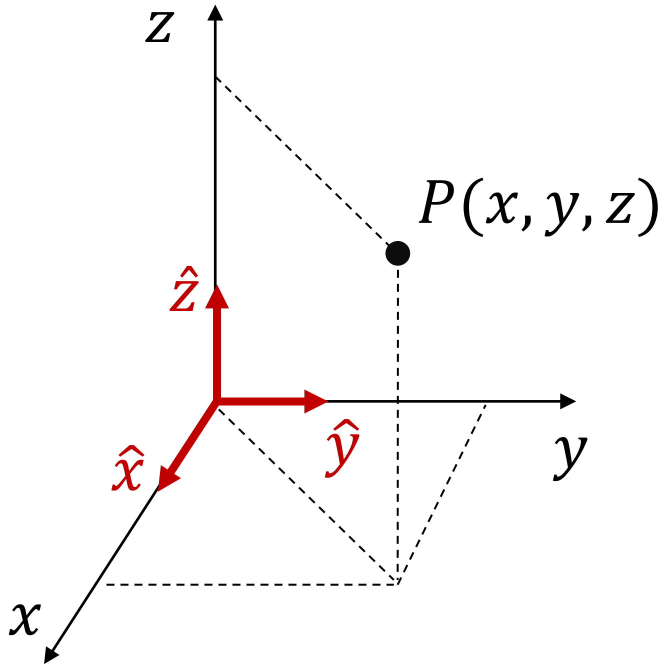
- Rectangular/Cartesian: sheets, planes
- Spherical: spheres, spherical shells
- Cylindrical: wires/cables along z direction

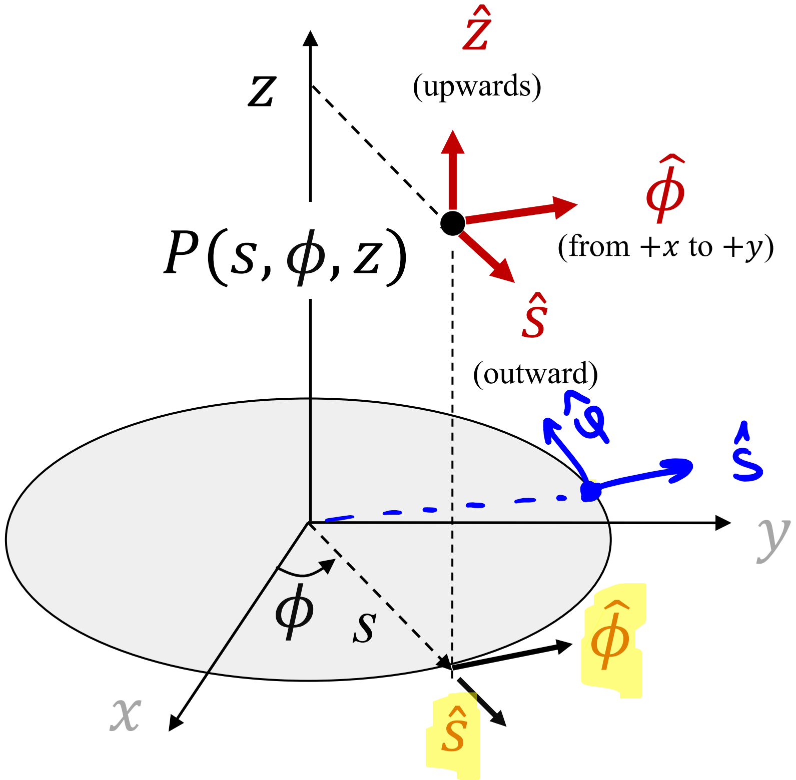


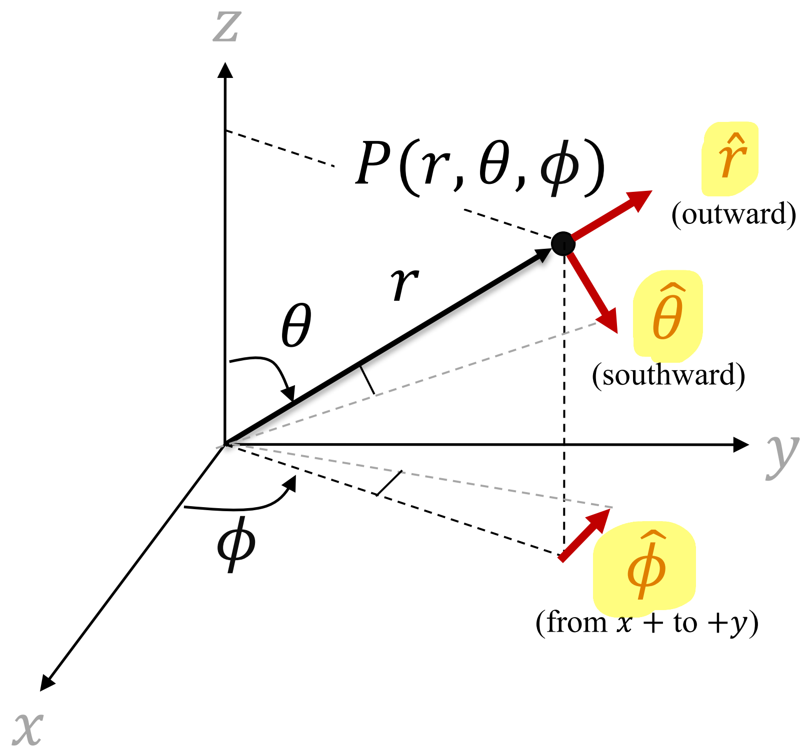
$\varphi \Leftrightarrow \phi$

We can “translate” between different coordinate systems

- Cartesian
- Cylindrical
- Spherical coordinates







* Some authors use ρ or r instead of s

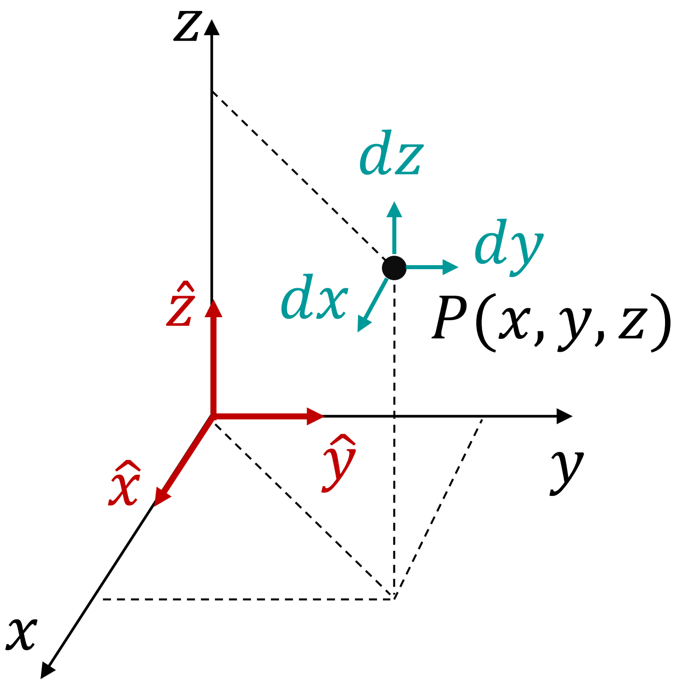
* In math: $\phi \leftrightarrow \theta$

- Unit vectors: $\hat{x}, \hat{y}, \hat{z}$
- They all are fixed in space

- Unit vectors: $\hat{s}, \hat{\phi}, \hat{z}$
- $\hat{s} = \hat{s}(\phi),\ \hat{\phi} = \hat{\phi}(\phi)$
- $\hat{z}$ is fixed in space

- Unit vectors: $\hat{r}, \hat{\theta}, \hat{\phi}$
- $\hat{r} = \hat{r}(\theta,\phi),\ \ \hat{\theta} = \hat{\theta}(\theta,\phi),\ \ \hat{\phi} = \hat{\phi}(\theta,\phi)$
- Note that $\hat{\theta} \perp \hat{\phi}$

Cartesian coordinates

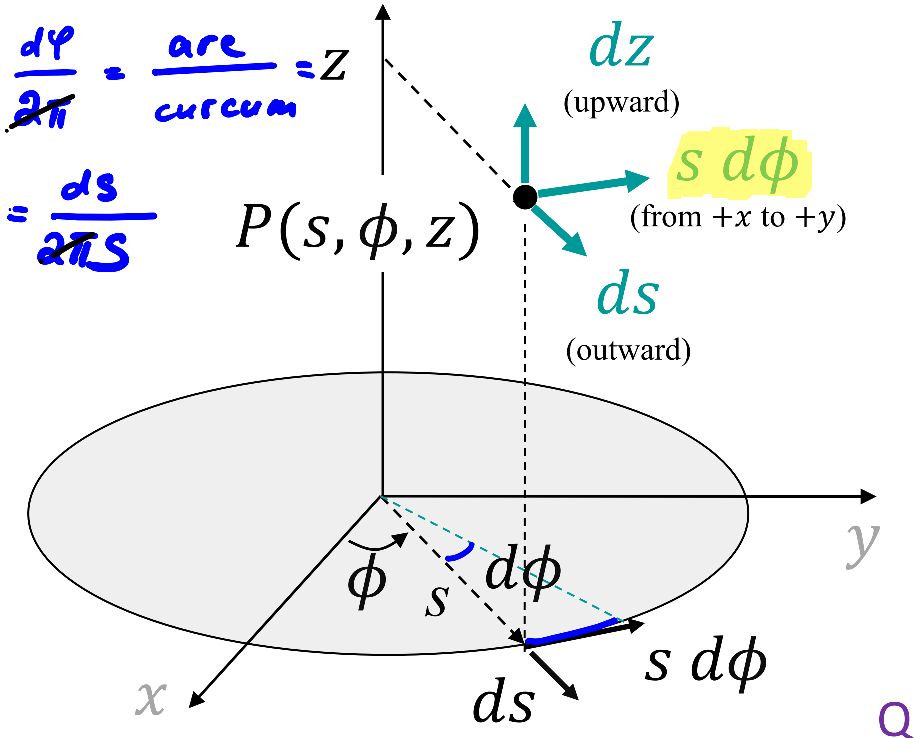


- Each point is specified by three coordinates: (x, y, z)
- Infinitesimal displacements along the x, y, z axes: dx, dy, dz
- Volume element: $d\tau = dx\, dy\, dz$
- Area element: $da = dx\, dy$ or $da = dy\, dz$ or $da = dx\, dz$
- Line element: dx or dy or dz

Q: Show that volume of a cube with an edge a is $V_{\text{cube}} = a^3$.

$$V = \int_{\text{cube}} d\tau = \int_0^a dx \int_0^a dy \int_0^a dz = a^3$$

Cylindrical coordinates



- Each point is specified by three coordinates: (s, ϕ, z)

$$0 < s < \infty, \qquad 0 < \phi < 2\pi, \qquad -\infty < z < \infty$$

$$\hat{s} = \hat{s}(\phi), \qquad \hat{\phi} = \hat{\phi}(\phi)$$

- Infinitesimal displacements along the unit vectors are:
  ds along $\hat{s}$; $s\,d\phi$ along $\hat{\phi}$; dz along $\hat{z}$;

- Volume element: $d\tau = ds\, s d\phi\, dz = s\, ds\, d\phi\, dz$

Q: What is the area element of the side surface of a cylinder of radius R?

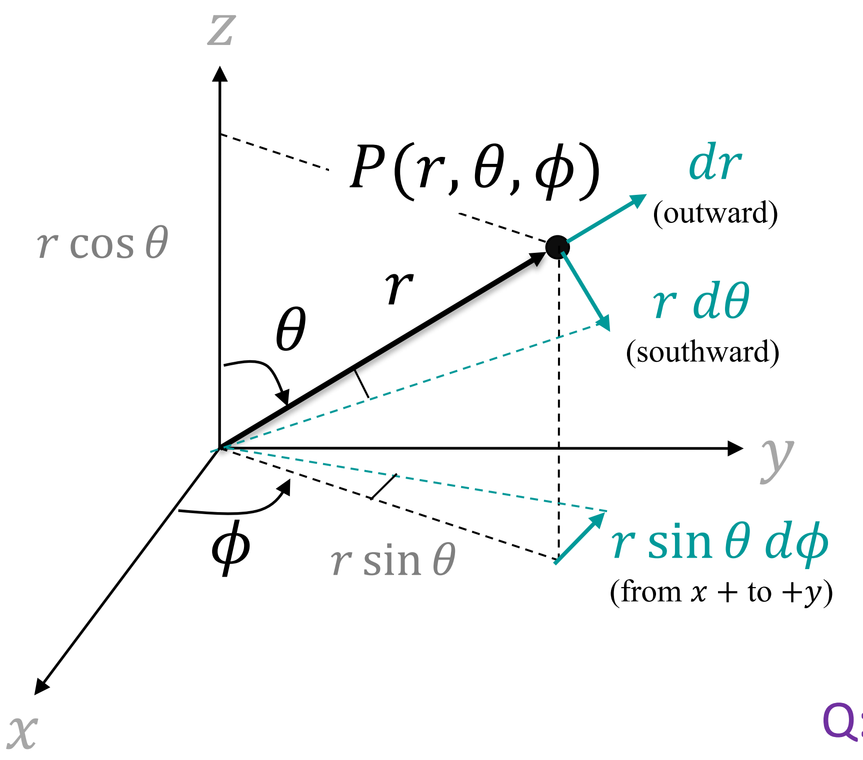
along $\hat{z}$ along $\hat{\varphi}$ @ $s=R$

$$da = (dz)\cdot(R\,d\varphi) = R\,dz\,d\varphi$$

Q: Show that volume of a cylinder of radius R and height h is $V_{\text{cyl}} = \pi R^2 h$.

$$V = \int_{cyl} d\tau = \iiint_{cyl} s\,ds\,d\varphi\,dz = \int_0^R s\,ds \int_0^{2\pi} d\varphi \int_0^h dz = \frac{R^2}{2}\cdot 2\pi \cdot h = \pi R^2 h$$

Spherical coordinates



- Each point is specified by three coordinates: (r, θ, ϕ)

$$0 < r < \infty, \qquad 0 < \theta < \pi, \qquad 0 < \phi < 2\pi$$

$$\hat{r} = \hat{r}(\theta,\phi), \qquad \hat{\theta} = \hat{\theta}(\theta,\phi), \qquad \hat{\phi} = \hat{\phi}(\theta,\phi)$$

- Infinitesimal displacements along the unit vectors are:
  ~~dr~~ along $\hat{r}$; $r\sin\theta\,d\phi$ along $\hat{\phi}$; $r\,d\theta$ along $\hat{\theta}$; @ $r = R$

- Volume element: $d\tau = dr\, r\sin\theta\, d\phi\, r d\theta = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$= \frac{4\pi R^3}{3}$$

Q: What is the area element of the surface of a sphere of radius R?

$$da = (R\sin\theta\, d\phi)(R\, d\theta)$$

$$= R^2 \sin\theta\, d\theta\, d\phi$$

Q: Show that volume of a sphere of radius R is $V_{\text{sphere}} = 4\pi R^3/3$.

$$V = \int_{\text{sphere}} d\tau = \int_{\text{spher}} dr \cdot r\sin\theta\, d\phi \cdot r\, d\theta = \int_0^R dr\, r^2 \underbrace{\int_0^\pi d\theta \sin\theta}_{\pi} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} = \frac{R^3}{3} \overbrace{(-\cos\theta)\Big|_0^\pi}^{2} \cdot 2\pi$$

Warning

- Cartesian coordinate system:

$$\mathbf{r}_1 = (x_1, y_1, z_1) \qquad r_1 \equiv |\mathbf{r}_1| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

$$\mathbf{r}_2 = (x_2, y_2, z_2) \qquad \mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2 = (x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$$

- Cylindrical coordinate system:

$$\mathbf{r}_1 = (s_1, \theta_1, z_1) \qquad r_1 \equiv |\mathbf{r}_1| \neq \sqrt{s_1^2 + \theta_1^2 + z_1^2}$$

$$\mathbf{r}_2 = (s_2, \theta_2, z_2) \qquad \mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2 \neq (s_1 + s_2,\ \theta_1 + \theta_2,\ z_1 + z_2)$$

- Spherical coordinate system: it does not work the "Cartesian" way either!

Converting between coordinate systems

- Assume that we have some vector field $\mathbf{v}(\mathbf{r})$ expressed in some coordinate system, $\hat{e}_1, \hat{e}_2, \hat{e}_3$:

$$\mathbf{v}(\mathbf{r}) = v_1^e(\mathbf{r})\,\hat{e}_1 + v_2^e(\mathbf{r})\,\hat{e}_2 + v_3^e(\mathbf{r})\,\hat{e}_3 \qquad (1)$$

and we want to express it in some other coordinate system, $\hat{u}_1, \hat{u}_2, \hat{u}_3$:

$$\mathbf{v}(\mathbf{r}) = v_1^u(\mathbf{r})\,\hat{u}_1 + v_2^u(\mathbf{r})\,\hat{u}_2 + v_3^u(\mathbf{r})\,\hat{u}_3 \qquad (2)$$

- Idea: let's express $\hat{e}_1, \hat{e}_2, \hat{e}_3$ in terms of $\hat{u}_1, \hat{u}_2, \hat{u}_3$ and use Eq.(1)!

- Cartesian ⬄ Spherical [Griffiths, (1.64)]

$$\begin{aligned}
\hat{\mathbf{r}} &= \sin\theta\cos\phi\,\hat{\mathbf{x}} + \sin\theta\sin\phi\,\hat{\mathbf{y}} + \cos\theta\,\hat{\mathbf{z}}, \\
\hat{\boldsymbol{\theta}} &= \cos\theta\cos\phi\,\hat{\mathbf{x}} + \cos\theta\sin\phi\,\hat{\mathbf{y}} - \sin\theta\,\hat{\mathbf{z}}, \\
\hat{\boldsymbol{\phi}} &= -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}},
\end{aligned}$$

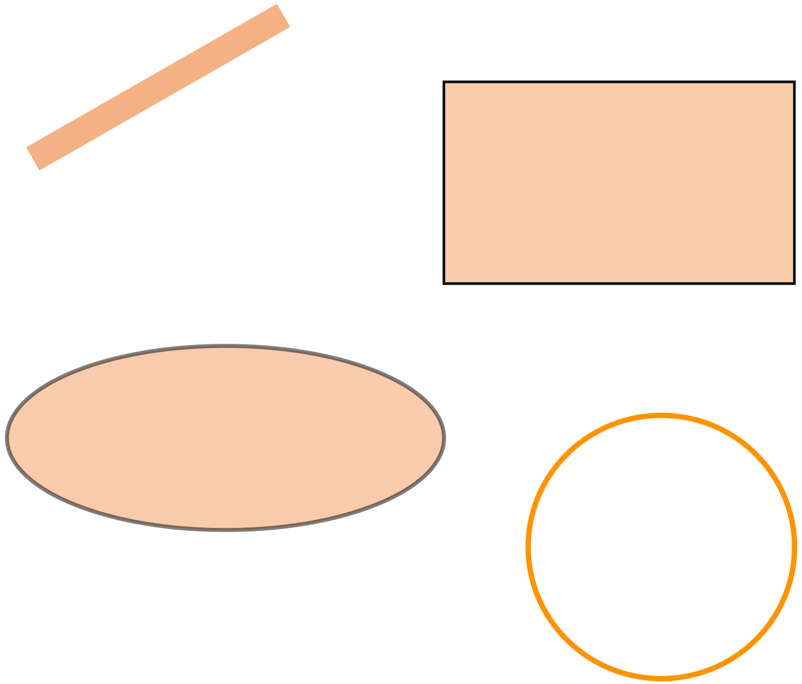
- Cartesian ⬄ Cylindrical [Griffiths, (1.75)]

$$\begin{aligned}
\hat{\mathbf{s}} &= \cos\phi\,\hat{\mathbf{x}} + \sin\phi\,\hat{\mathbf{y}}, \\
\hat{\boldsymbol{\phi}} &= -\sin\phi\,\hat{\mathbf{x}} + \cos\phi\,\hat{\mathbf{y}} \\
\hat{\mathbf{z}} &= \hat{\mathbf{z}}.
\end{aligned}$$

Practice: in HW1

Charge density integrals

$$Q = ?$$



- Line of charge
- Sheet of charge
- Disk of charge
- Spherical shell of charge
- Uniform vs non-uniform charge distribution

Charge density integrals

Our first E&M topic is computing the total charge of a particular charge distribution. This will involve defining the charge distribution and integrating over it. The steps typically proceed as follows:

(a) Establish the dimensionality of your charge density:

In general, these could be functions of position.

1D: λ (charge per unit length)
2D: σ (charge per unit area)
3D: ρ (charge per unit volume)

(b) Define the tiny (differential) element of your space, appropriate to the dimensionality:

1D: dl 2D: da 3D: $d\tau$

(c) Amount of charge per tiny space element is given by: dq

($\mathbf{r}$ = position vector expressed in appropriate coordinates)

1D: $\lambda(\mathbf{r})\,dl$ 2D: $\sigma(\mathbf{r})da$ 3D: $\rho(\mathbf{r})d\tau$

(d) Add the tiny charges in all these tiny space elements = evaluate the integral with the appropriate limits:

1D: $Q = \int_a^b \lambda(\mathbf{r})\,dl$ 2D: $Q = \int_A \sigma(\mathbf{r})\,da$ 3D: $Q = \int_V \rho(\mathbf{r})\,d\tau$

Example 1: Line of charge

A line of charge of length a carries uniform linear charge density λ.

a) Draw the line of charge and a coordinate system.

b) Write a definite integral to find the total charge, Q.

c) Suppose the line moves up by distance b. Modify your integral appropriately, and show that the total charge is unchanged.

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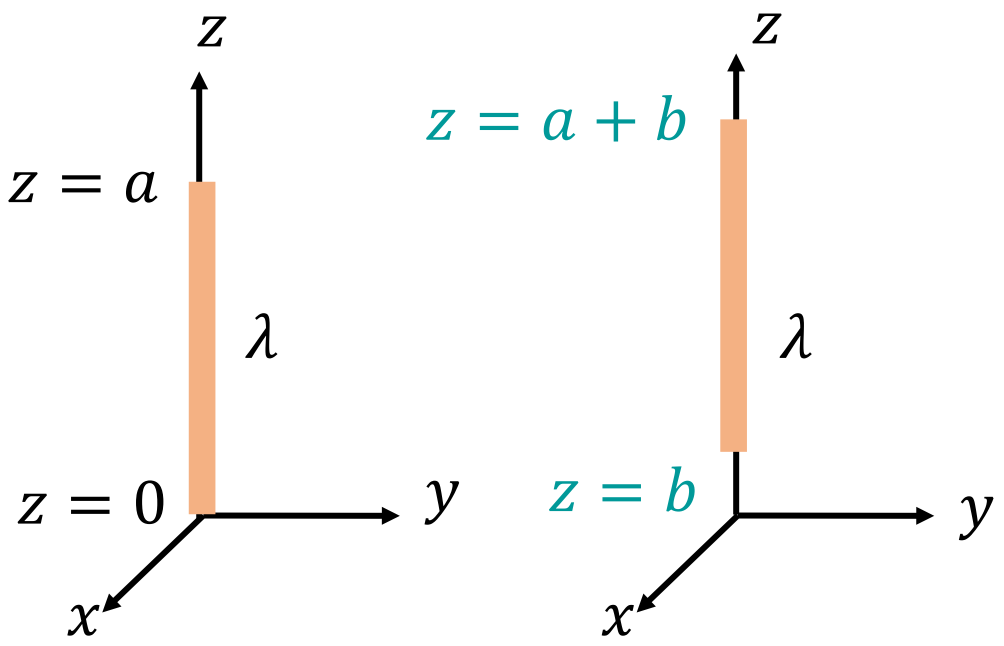

$$dq = \lambda\, dz$$

$$Q = \int_{\text{rod}} dq = \int_0^a \lambda\, dz = \lambda \int_0^a dz = \lambda a$$

$$Q = \int_{\text{rod}} dq = \int_b^{a+b} \lambda\, dz = \lambda(a + b - b) = \lambda a$$

Example 2: Sheet of charge with non-uniform charge distribution

You have a rectangular sheet of charge which runs from $x = 0$ to $x = a$ and $y = 0$ to $y = b$. The sheet carries charge distribution: $\sigma(x, y) = A \sin\left(\frac{\pi \mathrm{x}}{a}\right) \sin\left(\frac{\pi y}{b}\right)$.

a) Is A in the equation for σ area of your system, or something else?

b) Sketch the system and chose a coordinate system.

c) Compute total amount of charge in this system.

A. $Q = A$

B. $Q = A\frac{4\pi^2}{ab}$

C. $Q = A\frac{4ab}{\pi^2}$

D. $Q = \frac{4A}{\pi^2}$

E. $Q = \frac{4A}{ab}$

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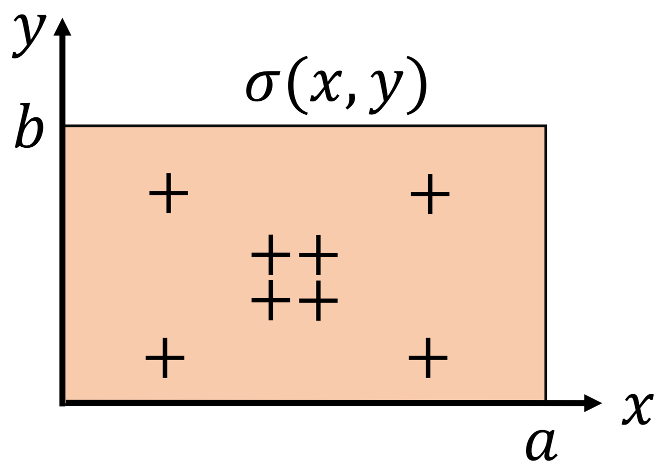
$$[\sigma] = \frac{\text{charge}}{\text{area}} = [A]$$



$$dq = \sigma(x, y)\, dx\, dy$$

$$Q = \int_{\text{sheet}} dq = \int_0^a dx \int_0^b dy\ A \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

$$= A \int_0^a dx \sin\left(\frac{\pi x}{a}\right) \int_0^b dy \sin\left(\frac{\pi y}{b}\right) = A \frac{4ab}{\pi^2}$$

A. $Q = A$

B. $Q = A \frac{4\pi^2}{ab}$

C. $Q = A \frac{4ab}{\pi^2}$

D. $Q = \frac{4A}{\pi^2}$

E. $Q = \frac{4A}{ab}$

Example 3: Disk of charge with non-uniform charge distribution

A disc of charge of radius a carries surface charge density σ. The surface charge increases quadratically as you move outwards from the center of the disc.

a) Sketch the system and chose a coordinate system. Draw in some charges.

b) Write down the expression for σ. Make sure that your units make sense.

c) Write down the integral for the total charge on the disk and compute Q.

Do this one on your own.

Write $dq = \ldots$

Compute $Q = \int dq$ (proper limits)

Check your answer on the next page

Construct surface charge density which behaves this way, e.g. $\sigma(s) = A s^2$ (some const)

Example 3: Disk of charge with non-uniform charge distribution

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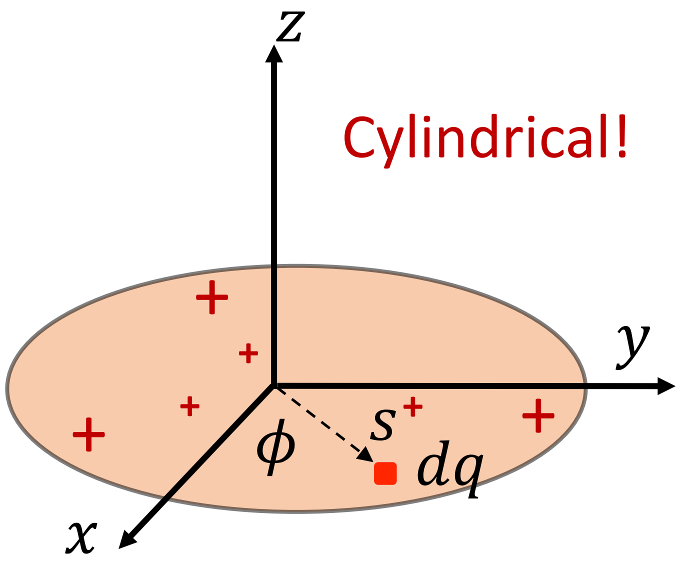
a) Sketch the system and chose a coordinate system. Draw in some charges.

b) Write down the expression for σ. Make sure that your units make sense. $\sigma = \sigma(s) = A\, s^2$

c) Write down the integral for the total charge on the disk and compute Q.



Cylindrical!

$$[\sigma] = \frac{\text{charge}}{\text{area}} \qquad [A] = \frac{\text{charge}}{L^4}$$

$$dq = \sigma(s)\, da \qquad da = ds\, s\, d\phi$$

$$Q = \int_{\text{disk}} dq = \int_0^a ds \int_0^{2\pi} d\phi\ s\,\sigma(s)$$

$$= A \int_0^a ds\, s^3 \int_0^{2\pi} d\phi = 2\pi A \frac{a^4}{4} = A \frac{\pi a^4}{2}$$

A. $Q = A\frac{\pi a^4}{2}$

B. $Q = A\frac{\pi a^4}{4}$

C. $Q = A\frac{a^4}{2\pi}$

D. $Q = A\frac{a^4}{4\pi}$

E. None of them

Example 4: Spherical shell

You have a spherical shell of radius R with a uniform surface charge density σ.

a) Sketch the system and chose a coordinate system.

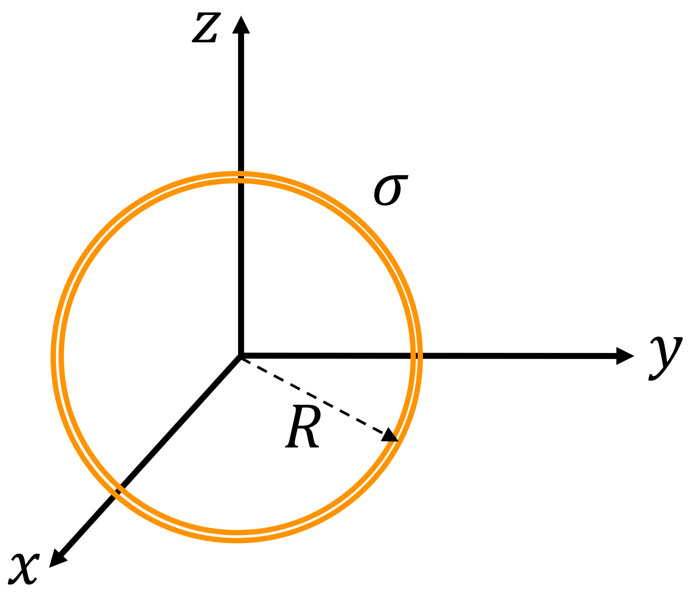
b) Write the expression for total charge and compute it.

c) Sanity checks: Does your answer have the right units? Does it depend on R? Should it?

Do this on your own!

- $dq = \sigma \, da$

What is da for this shell?



A. Q depends on R

B. Q does not depend on R

Answer on the next page.

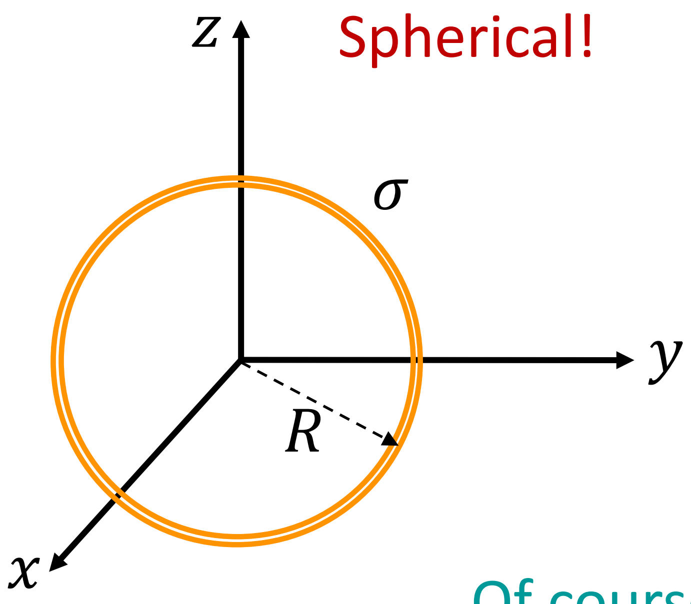
Example 4: Spherical shell

You have a spherical shell of radius R with a uniform surface charge density σ.

a) Sketch the system and chose a coordinate system.

b) Write the expression for total charge and compute it.

c) Sanity checks: Does your answer have the right units? Does it depend on R? Should it?



Spherical!

$$dq = \sigma\, da \qquad da = (R\, d\theta)(R \sin\theta\, d\phi)$$

$$Q = \int_{\text{shell}} dq = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \;(\sigma\, R^2 \sin\theta)$$

$$= \sigma R^2 (2\pi)(2) = 4\pi\sigma R^2$$

Of course! For a uniform density: $Q = \sigma A_{\text{surface}} = \sigma \cdot 4\pi R^2$