# Lecture 8

Conductors

# Announcement

- HW-1 is graded, and the grades are on Gradescope

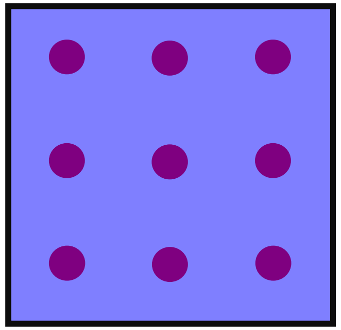
- Solutions: Canvas / Homework.
  - Even if you are happy with your mark, please have a look: there I discuss typical misconceptions, and try to give a variety of approaches.

- Check feedback from our TAs on Piazza: there they tell about the patterns which they have seen. Even if you don’t recognize yourself in this feedback, it is better to know about potential traps other people met.

# Conductors in Electrostatics

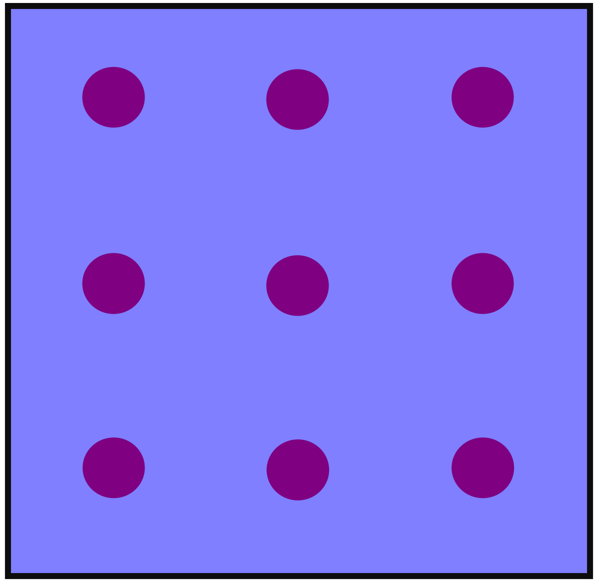
(Ch. 2.5.1-2)

- Properties of conductors
- Field inside a conductor
- Screening
- Charge distribution on their surfaces



# Conductors

- Examples: Aluminum, gold, brass, copper, steel…

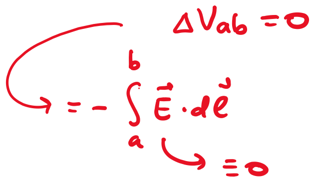


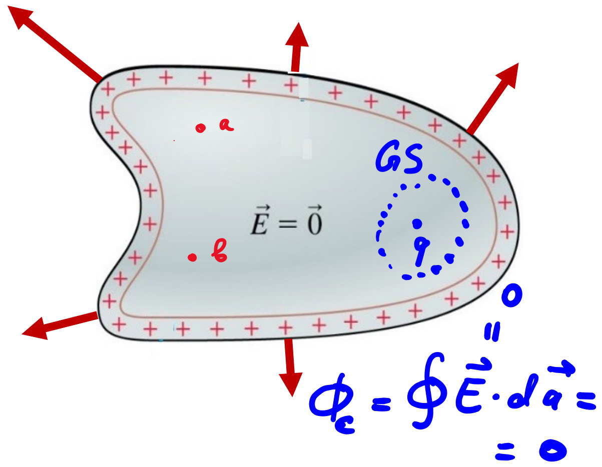
- Ions (+) are fixed (they are heavy!)
  - “Ionic lattice”
- Electrons (−) are mobile (they are light!)
  - “Sea of electrons”

$$m_p \sim 2000\, m_e$$

# Conductors in Electrostatics

**Properties**:




- $\mathbf{E} = 0$ inside the material. Mobile charges rearrange in response to any applied field, until the field inside is 0.

"electrostatic equilibrium": no motion of charges => no force on them

- Inside any conductor, $\rho = 0$. Any excess charge resides on the conductor's surface.

Gauss's law & $E = 0$ inside

- $\mathbf{E}$ is normal (perpendicular) to the surface just outside the conductor.

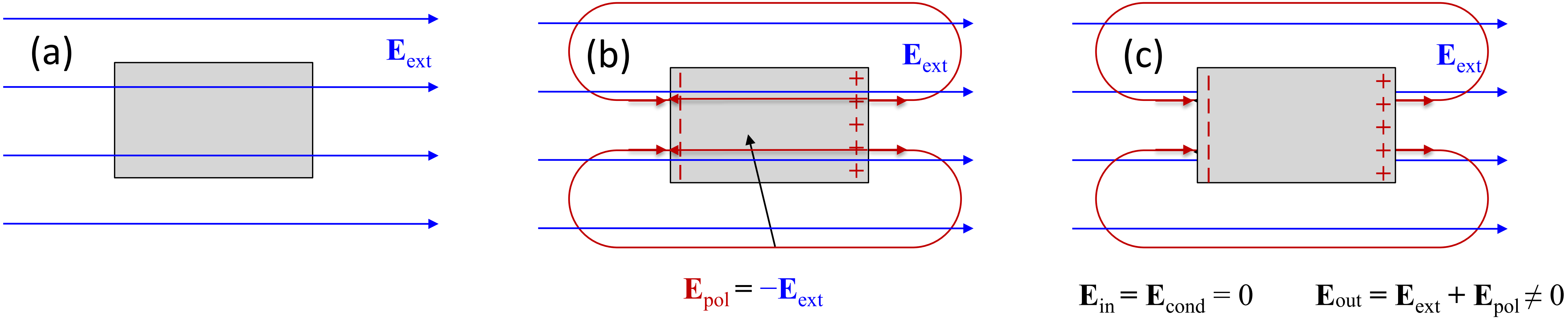
continuity of tangential field component: $E_{\mathrm{t,out}} = E_{\mathrm{t,in}} = 0$

- A conductor is an equipotential ($V = \mathrm{const.}$).

from $V_b - V_a = -\int_a^b \mathbf{E}\cdot d\mathbf{l} = 0$ for any two points $a$ and $b$ on the conductor, including its surface.

# Conductor in External Field: $E_{in} = 0$

(a) A neutral metal block is placed in an external E field. What happens to the charges and to the field?



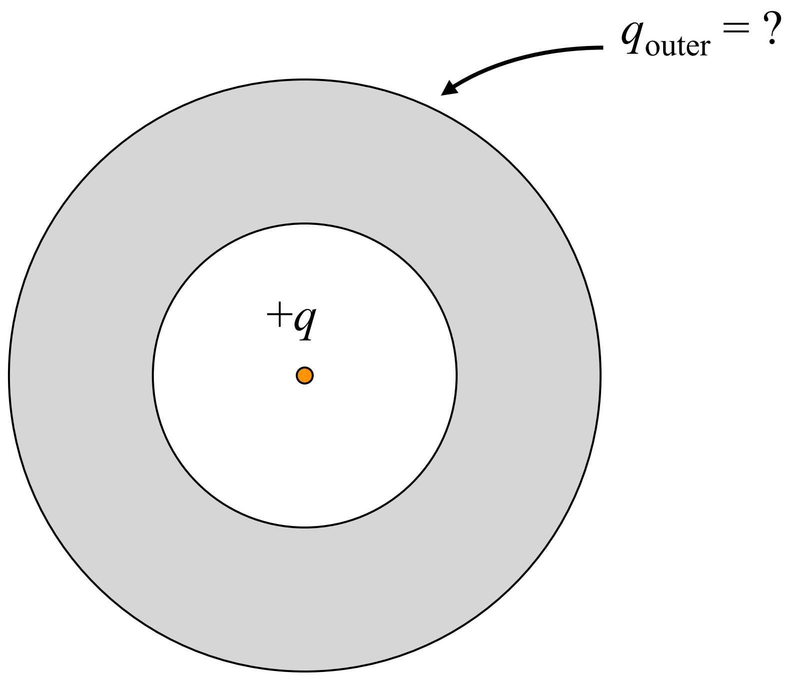

(b) The charges experience electric force, and they are free to move =>
they redistribute to the ends of the block (positive to the right, negative to the left)

They create "additional" E field ("polarization field"). Their motion continues until…

(c) …the induced "polarization" field inside the block cancels the external field entirely: New equilibrium is reached.

# Hollow Conducting Sphere – 1

Q: A **neutral** copper sphere has a concentric spherical cavity in the center. A charge $+q$ is placed in the center of the hollow sphere. What is the total charge induced on the *outside* surface of the conductor? (Assume electrostatic equilibrium.)




A. Zero

B. $+q$

C. $-q$

D. $0 < q_{\text{outer}} < q$

E. $-q < q_{\text{outer}} < 0$

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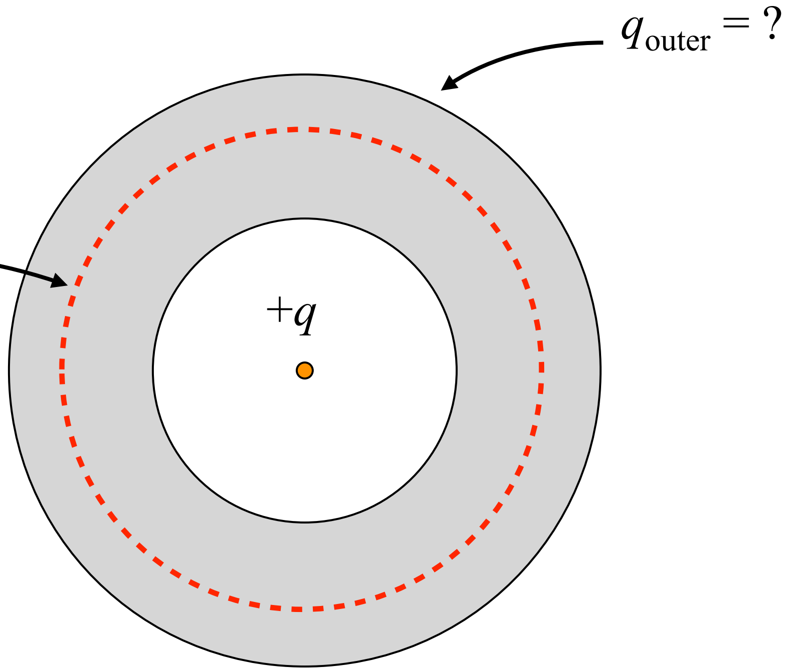
Hint: think about what is on the *inside* surface.




$\oint \vec{E} \cdot d\vec{a} = 0$

$\Phi = 0$

$\rightarrow q_{\text{enc}} = 0$

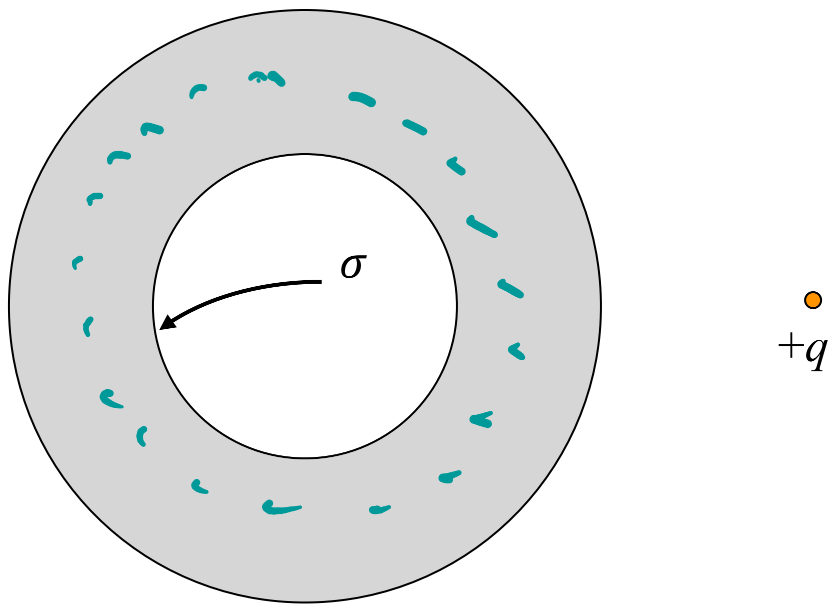
$\rightarrow q_{\text{inner}} = -q$

$\rightarrow q_{\text{outer}} = +q$

A. Zero
B. $+q$
C. $-q$
D. $0 < q_{\text{outer}} < q$
E. $-q < q_{\text{outer}} < 0$

# Hollow Conducting Sphere – 2

Q: A point charge $+q$ is placed near a neutral copper sphere with a concentric spherical cavity inside. In electrostatic equilibrium, what can we say about the surface charge density $\sigma$ on the *interior* of the conductor?




A. Zero everywhere

B. Non-zero, but with zero net charge on the interior surface

C. Non-zero, with non-zero net charge on the interior surface.

# Hollow Conducting Sphere – 2

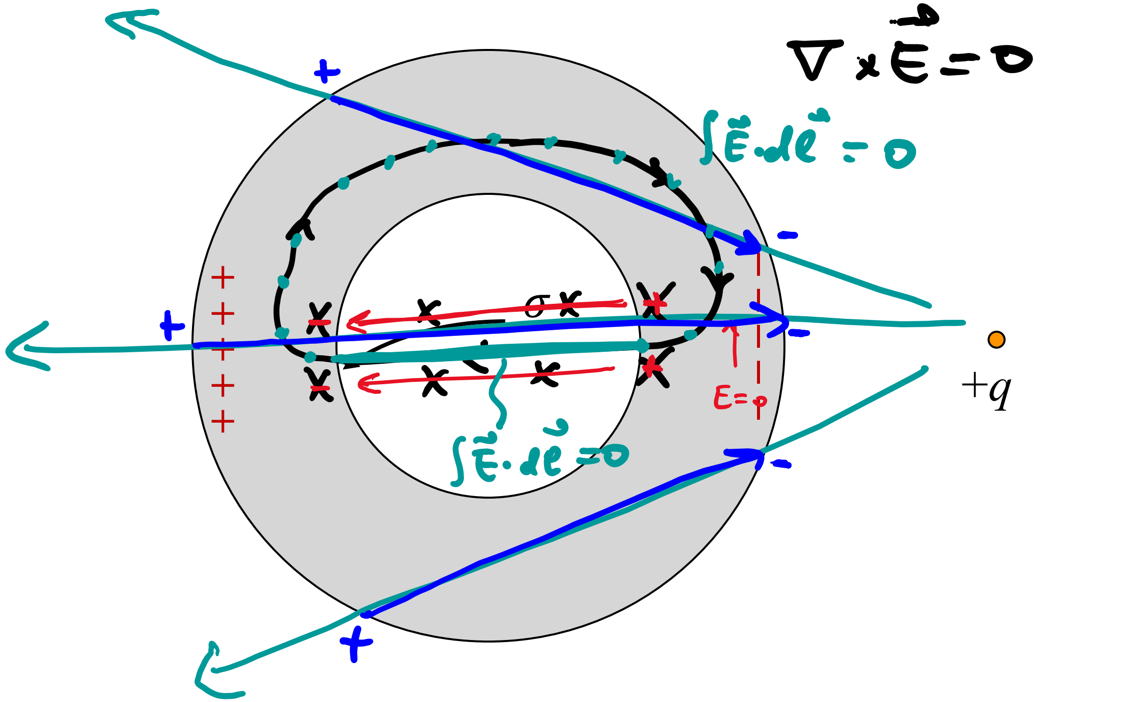
Q: A point charge $+q$ is placed near a neutral copper sphere with a concentric spherical cavity inside. In electrostatic equilibrium, what can we say about the surface charge density $\sigma$ on the *interior* of the conductor?

- *Inside:* Imagine not a shell but solid sphere. What's E-field inside?

What changes if you chop out a piece out of this neutral, $E = 0$ environment?

Nothing! => $\sigma_{\text{inner}} = 0$

- *Outside* will polarize




A. Zero everywhere

B. Non-zero, but with zero net charge on the interior surface

C. Non-zero, with non-zero net charge on the interior surface.

Griffiths:

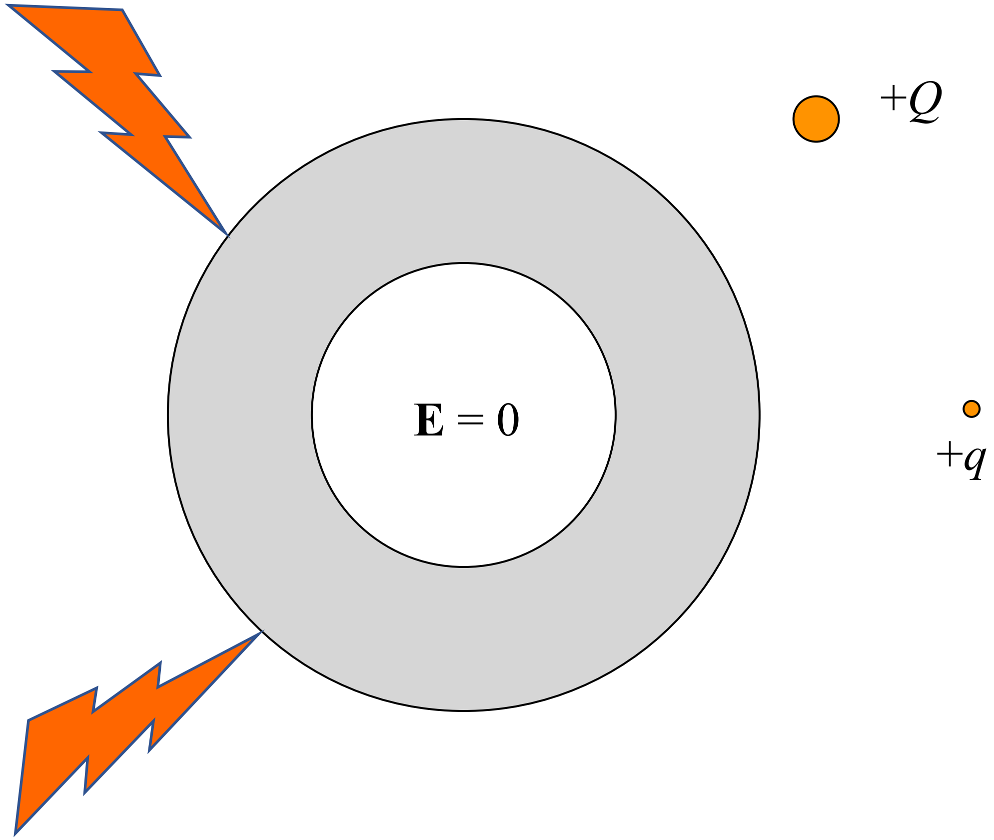
Assume B is correct => $\mathbf{E} \neq 0$ inside the cavity, but $E = 0$ inside the metal => $\oint \mathbf{E} \cdot d\mathbf{l} \neq 0$ => wrong!

# Faraday's Cage

- A cavity completely enclosed by a conducting surface is called Faraday cage.

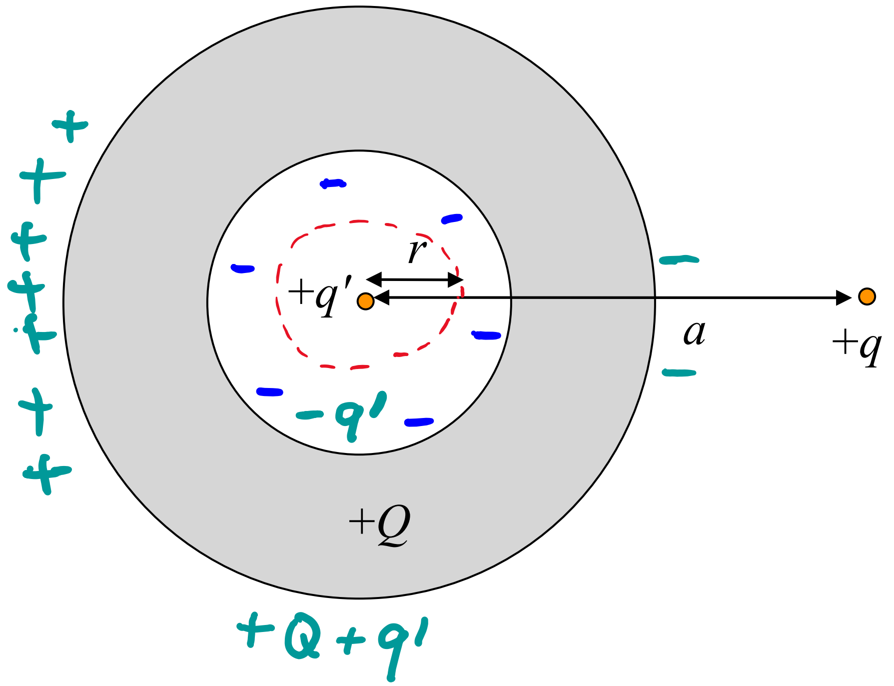
- Since the charge density on the interior is zero (unless there are other charges inside), the electric field inside the cavity is zero.

- The cavity is "screened", or "shielded" from the outside electricity



# Hollow Conducting Sphere – 3

Q: A hollow copper sphere has total charge $+Q$. A point charge $+q$ sits outside, a distance $a$ from the center of the sphere. A charge, $+q'$, is at the center of the sphere. Assume electrostatic equilibrium. What is the magnitude of the E-field a distance $r$ from $+q'$, but, still within the hollow cavity? $(k = 1/4\pi\epsilon_0)$




A. $E = kq'/r^2$

B. $E = k(q' - Q)/r^2$

C. $E = 0$

D. $E = kq/(a-r)^2$

E. Something else

# Hollow Conducting Sphere – 3

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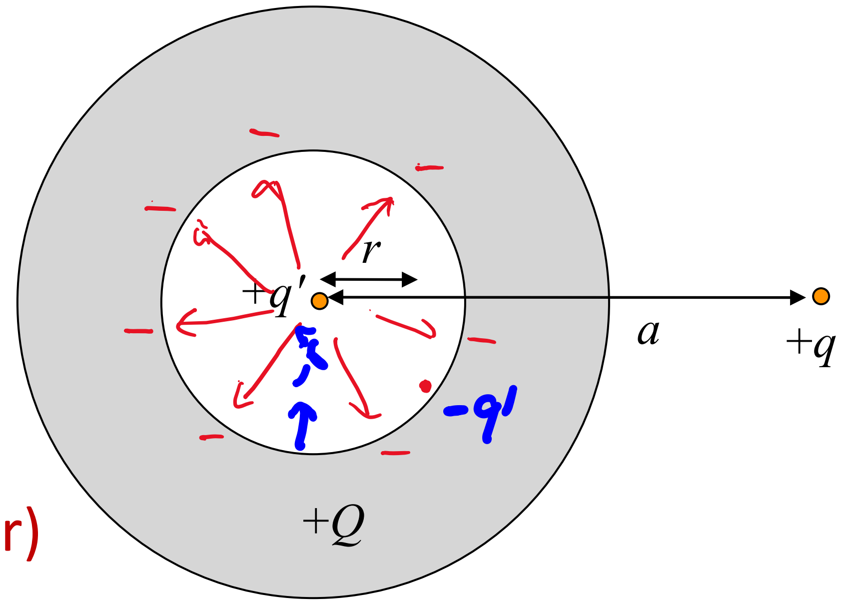
- $+q$ is screened from the interior of the shell => neglect it entirely => the remaining charge distribution is spherically symmetric => We can use Gauss' law!
- Gauss's law: $E_{\text{cavity}}(r)\, 4\pi r^2 = +q'/\epsilon_0$

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- Charge on the inner surface = ? (GS anywhere inside the conductor)

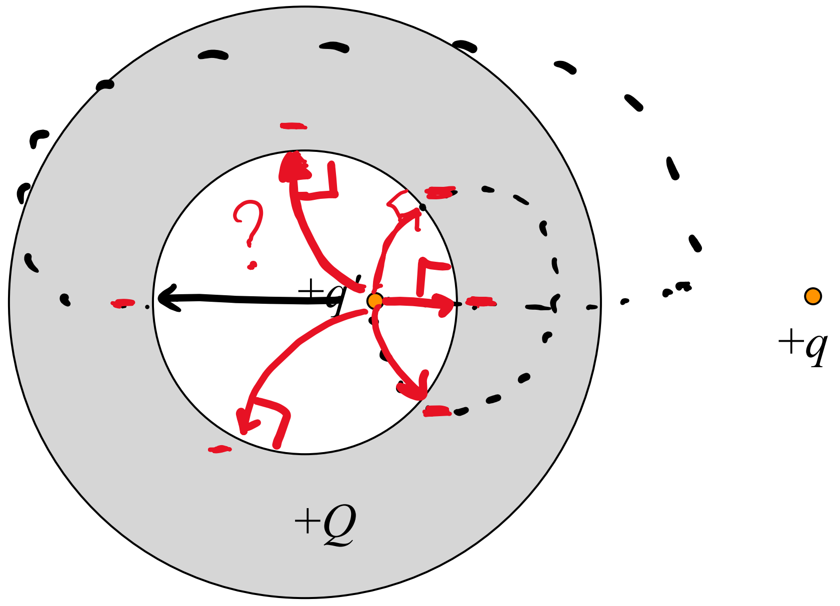
$$\Phi = 0 \rightarrow q_{\text{enc}} = 0 \rightarrow q_{\text{inner}} = -q' \ (\& \ q_{\text{outer}} = Q + q')$$

- Note that:

$$\mathbf{E}(\mathbf{r}) = \frac{kq'}{r^2}\hat{\mathbf{r}} = \frac{q'}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{r^2} \quad \rightarrow \mathbf{E}(\mathbf{R}) = \frac{q'}{4\pi\epsilon_0}\frac{\hat{\mathbf{r}}}{R^2} = \frac{1}{\epsilon_0}\frac{-q'}{4\pi R^2}(-\hat{\mathbf{r}}) = \frac{\sigma_{\text{inner}}}{\epsilon_0}\hat{\mathbf{n}}$$

# Hollow Conducting Sphere – 3

Q: Now suppose we move the inner charge away from the center of the cavity. What is the E-field inside the cavity now?



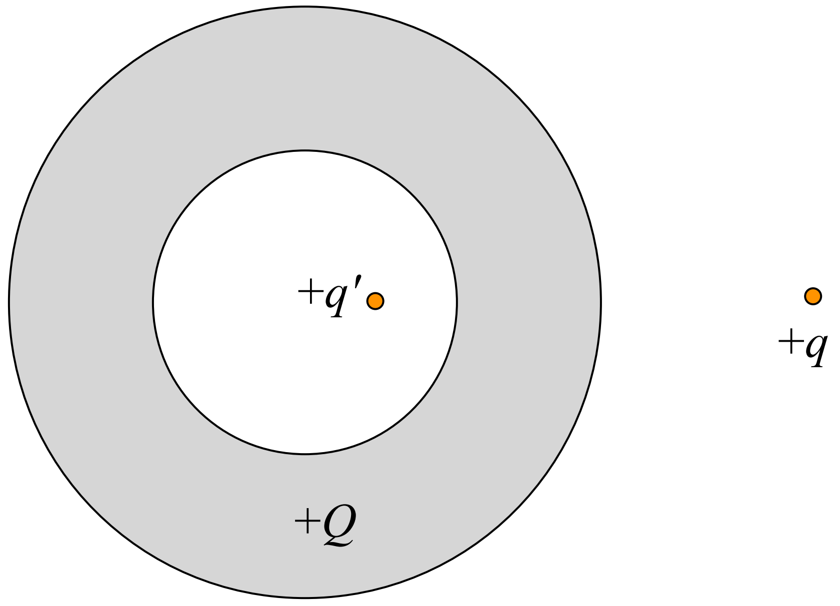

A. A radial Coulomb field up to the cavity wall.

B. A more complicated field that is, in general, hard to calculate.

# Hollow Conducting Sphere – 3

Q: Now suppose we move the inner charge away from the center of the cavity. What is the E-field inside the cavity now?

- The field lines start off radially outward from $+q'$, but they must curve inside the cavity to end up perpendicular to the inner cavity wall.




A. A radial Coulomb field up to the cavity wall.

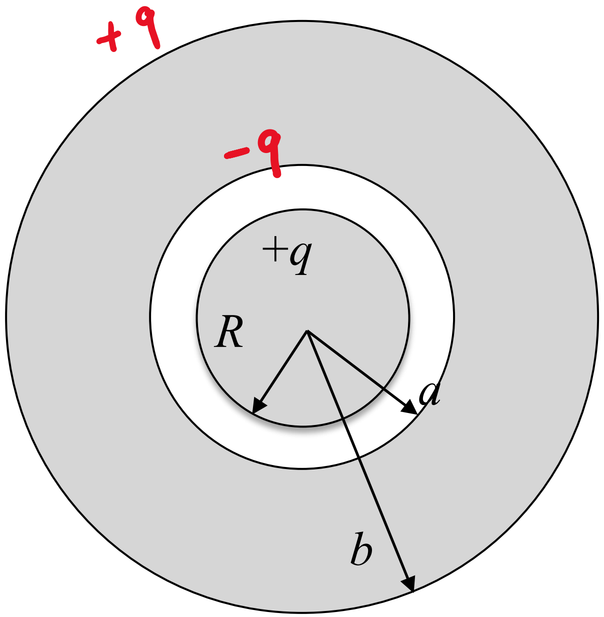
B. A more complicated field that is, in general, hard to calculate.

# Exercise: Conducting Spheres

Q: A metal sphere of radius $R$ carries a charge $q$. It is surrounded by a concentric metal shell with radii $a$ and $b$, which carries no net charge.

a) Find surface charge density at $R, a$, and $b$.

b) Find the potential at the center using infinity as your reference (i.e., zero) point.




$$\sigma = \frac{Q}{A}$$

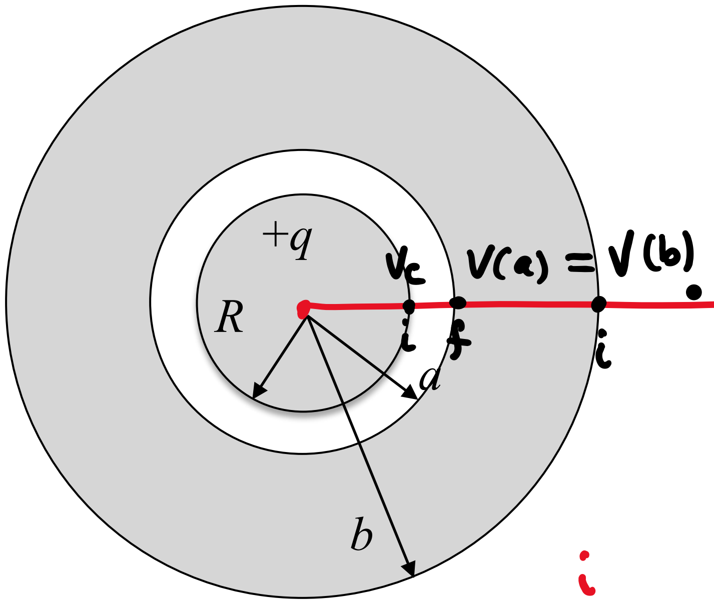
# Exercise: Conducting Spheres

- Charge density inside conductor is zero, so we only have surface charge density:

$$\sigma(R) = \frac{q}{4\pi R^2}$$

- Charge at inner surface $a$ must be $-q$ (by Gauss' law), so:

$$\sigma(a) = -\frac{q}{4\pi a^2}$$




- Charge at outer surface $b$ must be $+q$ (since there is no net charge on the shell), so:

$$\sigma(b) = \frac{q}{4\pi b^2}$$

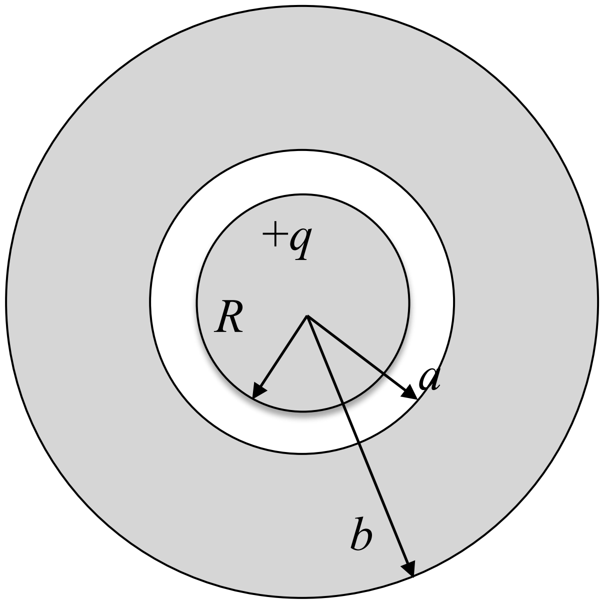
$$\Delta V = V_f - V_i = -\int_a^b \vec{E}\cdot d\vec{l}$$

$V_\infty = 0$

$$k \equiv \frac{1}{4\pi\epsilon_0}$$

# Exercise: Conducting Spheres

- Potential at outer surface $b$: $V(\infty) - V(b) = -\int_b^\infty \frac{kq}{r^2}\,dr \quad \Rightarrow \quad V(b) = \frac{kq}{b}$
- Potential at inner surface $a$ ($V = $ const inside the conductor): $V(a) = \frac{kq}{b}$




- Potential at surface $R$:

$$V(a) - V(R) = -\int_R^a \frac{kq}{r^2}\,dr \quad \Rightarrow \quad V(R) = \frac{kq}{R} - \frac{kq}{a} + \frac{kq}{b}$$

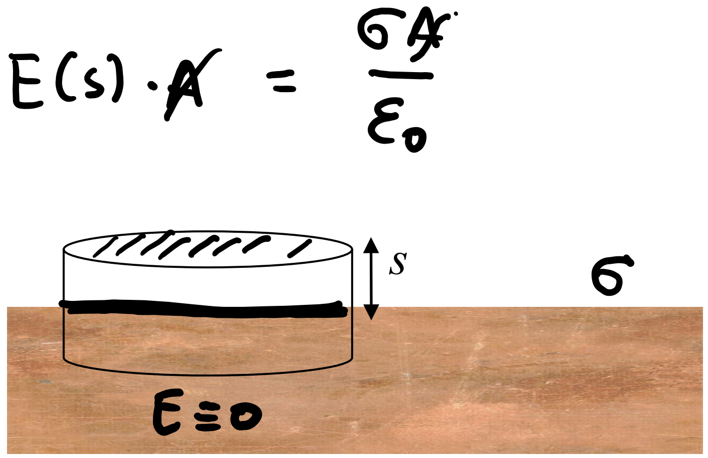
- Potential at $r = 0$:

$$V(0) = V(R) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{a} + \frac{1}{b}\right)$$

- Note: we proceed from a known potential $V(\infty) = 0$ to other regions in space, using $\Delta V = -\int_a^b \mathbf{E}\cdot d\mathbf{l}$ and continuity of potential

# E-field near a surface – 1

Q: We have a large, thick copper plate with uniform surface charge density $\sigma$ on its top surface. Using the Gaussian surface shown below, what is the $\mathbf{E}$ field a small distance $s$ above the conductor surface? (The bottom cap of the Gaussian surface is within the copper.)




A. $E = 0$

B. $E = \sigma/4\epsilon_0$

C. $E = \sigma/2\epsilon_0$

D. $E = \sigma/\epsilon_0$

E. $E = (1/4\pi\epsilon_0)(\sigma/s^2)$

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$$|\mathbf{E}|_{\text{above}} - |\mathbf{E}|_{\text{below}} = \sigma/\epsilon_0$$

$$|\mathbf{E}|_{\text{below}} = 0$$

A. $E = 0$

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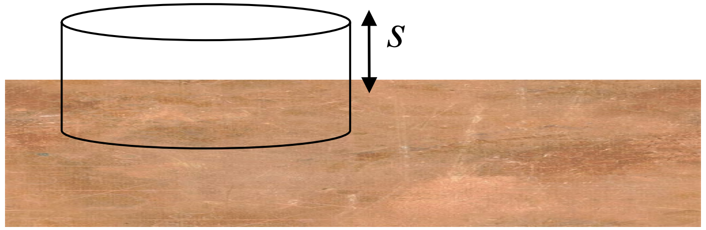
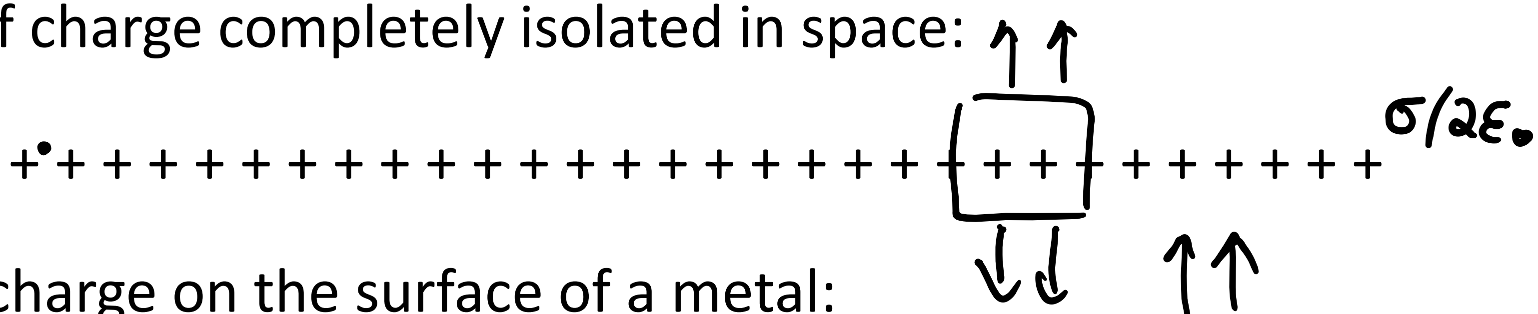


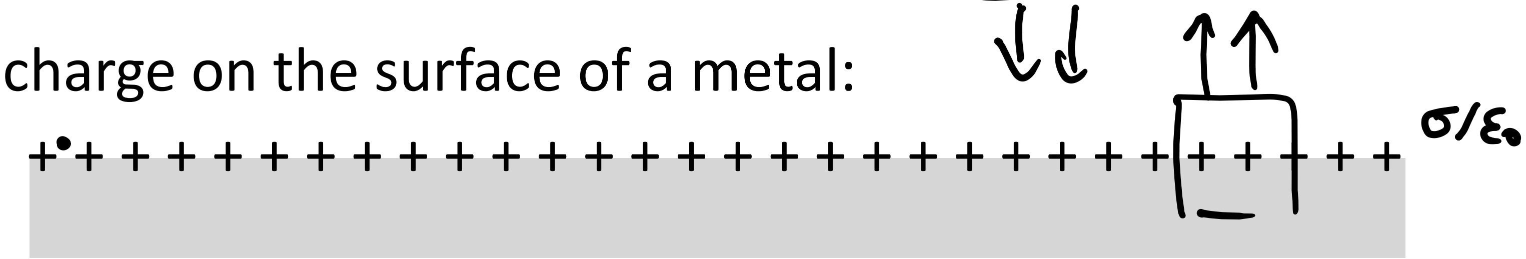
# E-field near a surface – 2

Q: Now Consider two systems, both with very large (~infinite) planes of charge, with the same uniform charge per unit area $\sigma$:

I. A plane of charge completely isolated in space:




II. A plane of charge on the surface of a metal:




Which system has a larger electric field *above* the plane?

A. I

B. II

C. They have the same E-field strength

# E-field near a surface – 2

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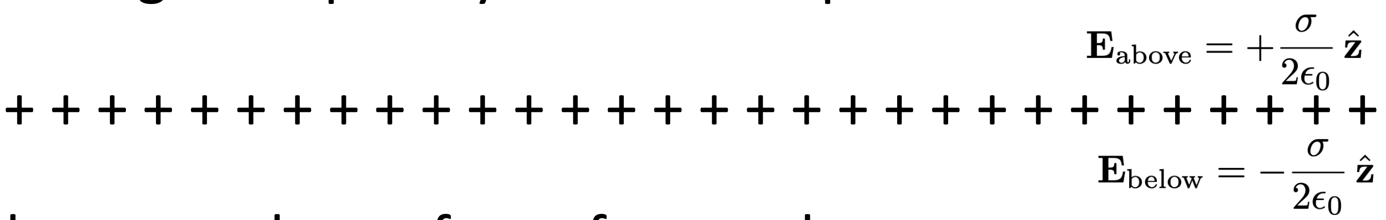
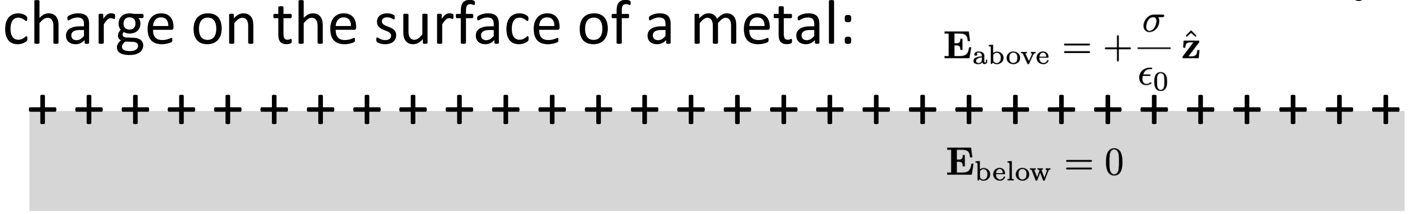



II. A plane of charge on the surface of a metal:

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$$|\mathbf{E}|_{\text{above}} - |\mathbf{E}|_{\text{below}} = \sigma/\epsilon_0$$

# E-field of a Capacitor

Q: Given a pair of very large, flat, conducting capacitor plates with surface charge densities $\pm\sigma$, what is the $\mathbf{E}$ field in the region between the plates?

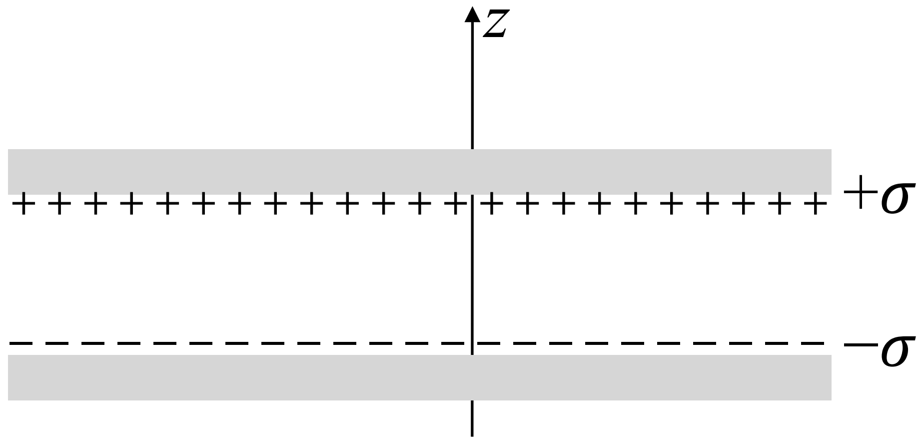
A. $(+\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

B. $(-\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

C. $(+2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

D. $(-2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

E. None of the above



# E-field of a Capacitor

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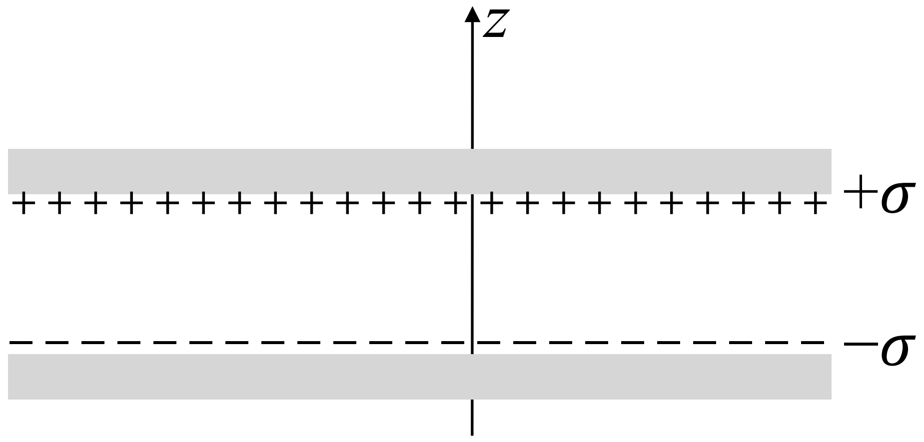
- Points from + to − => in negative-z direction
- The magnitude of the field near a surface of a conductor always is $\sigma/\epsilon_0$!

A. $(+\sigma/\epsilon_0)\,\hat{\mathbf{z}}$
B. $(-\sigma/\epsilon_0)\,\hat{\mathbf{z}}$
C. $(+2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$
D. $(-2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$
E. None of the above



# E-field of a Capacitor

We have just learned that a conductor with a surface charge density $\sigma$ produces electric field with a magnitude $\sigma/\epsilon_0$ above its surface. Okay.

Now we are combining two such conductors, with fields having the same magnitudes but pointing in opposite directions. According to superposition principle, the field in the space between them should double => it should be $2\sigma/\epsilon_0$!

What's wrong with this explanation?

A. $(+\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

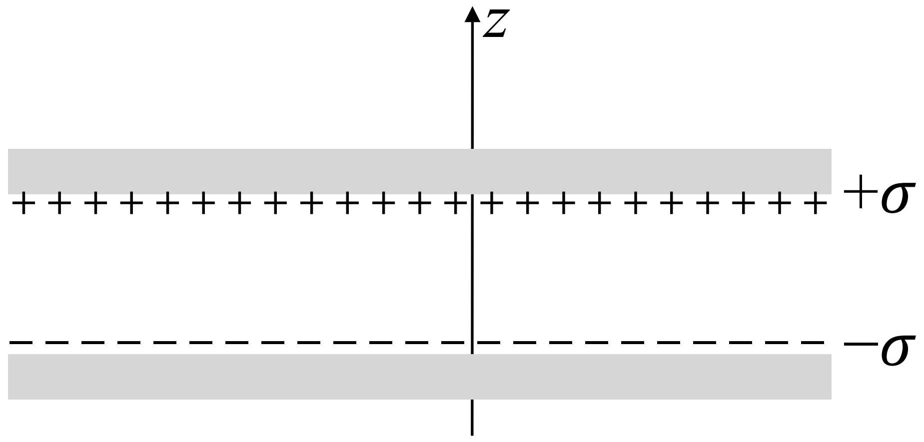
B. $(-\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

C. $(+2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$

D. $(-2\sigma/\epsilon_0)\,\hat{\mathbf{z}}$ ← ?

E. None of the above

Wait a minute !!!



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What's wrong with this explanation?

- If each of these two conductors is left alone, the charges will redistribute over both surfaces

=> each will have surface density $\sigma/2$ =>

=> each will create E-field $E = (\sigma/2)/\epsilon_0$ =>

=> Between the plates, they will add up to $E = \sigma/\epsilon_0$!