Lecture 20

Magnetic potential (two more examples).

Magnetism in matter.

Auxiliary field **H**.

Example: Vector potential of a solenoid

Q: A solenoid is multiple current loops (current I, radius R) stacked vertically, with n turns per unit length. Find the vector potential inside and outside the solenoid.

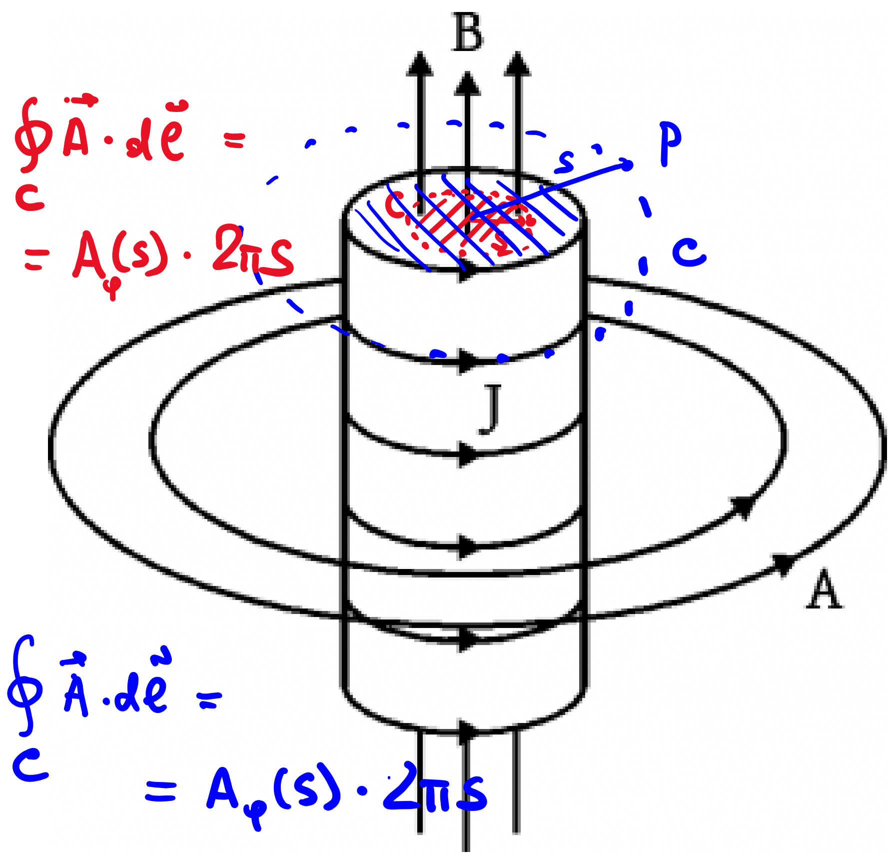
Note that it is a somewhat unusual problem: here the magnetic field is straight, and vector potential is curly

We know what the vector potential should look like, and we also know the magnetic field inside the solenoid:

$$\mathbf{A}(s) = A_\varphi(s)\,\hat{\boldsymbol{\varphi}} \qquad \mathbf{B}(s) = \mu_0 n I\,\hat{\mathbf{z}} \quad (s < R)$$

We can use relationship between magnetic flux and vector potential which we derived about a week ago:

$$\oint_C \mathbf{A}\cdot d\mathbf{l} = \int_A \nabla\times\mathbf{A}\cdot d\mathbf{a} = \int_A \mathbf{B}\cdot d\mathbf{a}$$



Example: Vector potential of a solenoid

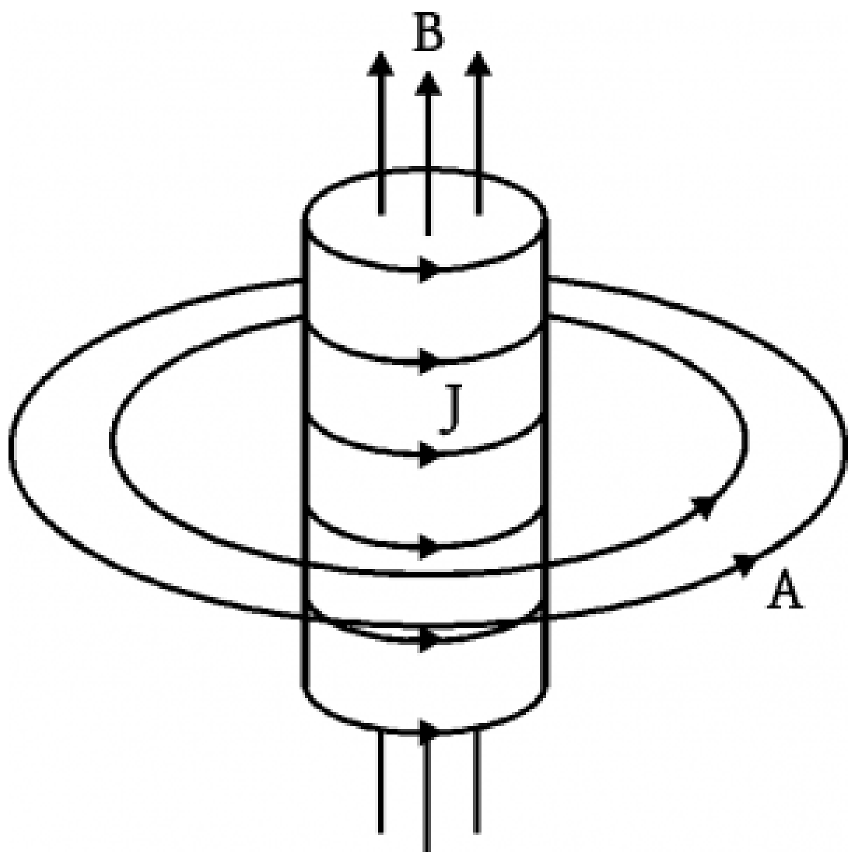
Q: A solenoid is multiple current loops (current I, radius R) stacked vertically, with n turns per unit length. Find the vector potential inside and outside the solenoid.

Inside: $$\oint_C \mathbf{A}\cdot d\mathbf{l} = A_\varphi 2\pi s = \int_A \mathbf{B}\cdot d\mathbf{a} = \mu_0 n I \pi s^2$$

Outside: $$\oint_C \mathbf{A}\cdot d\mathbf{l} = A_\varphi 2\pi s = \int_A \mathbf{B}\cdot d\mathbf{a} = \mu_0 n I \pi R^2$$

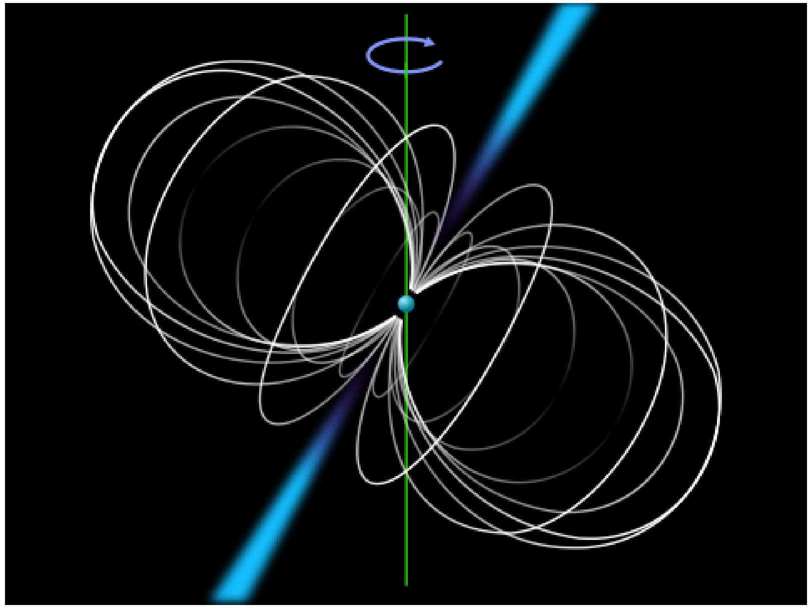
so that, inside ($s < R$): $$\mathbf{A}(s) = \frac{\mu_0 n I}{2}\, s\,\hat{\boldsymbol{\varphi}}$$

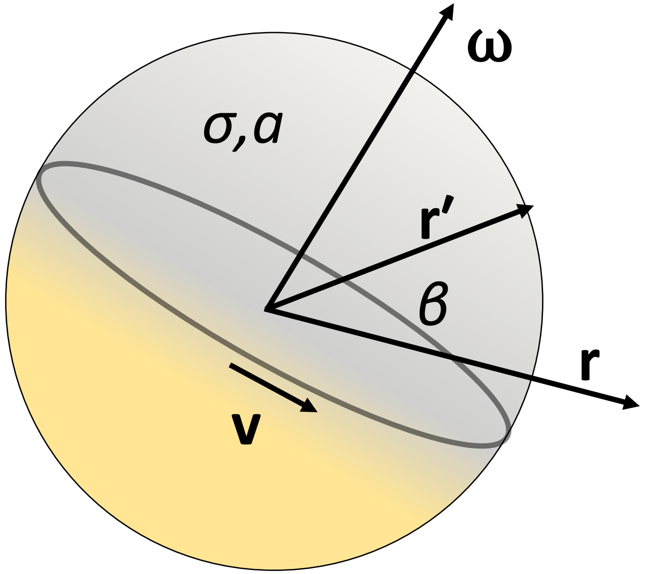
and outside ($s > R$): $$\mathbf{A}(s) = \frac{\mu_0 n I}{2}\,\frac{R^2}{s}\,\hat{\boldsymbol{\varphi}}$$



Example: Field of a pulsar

A pulsar is a rotating neutron star with a surface charge that forms a surface current and a dipolar magnetic field.





Griffiths Ex. 5.11

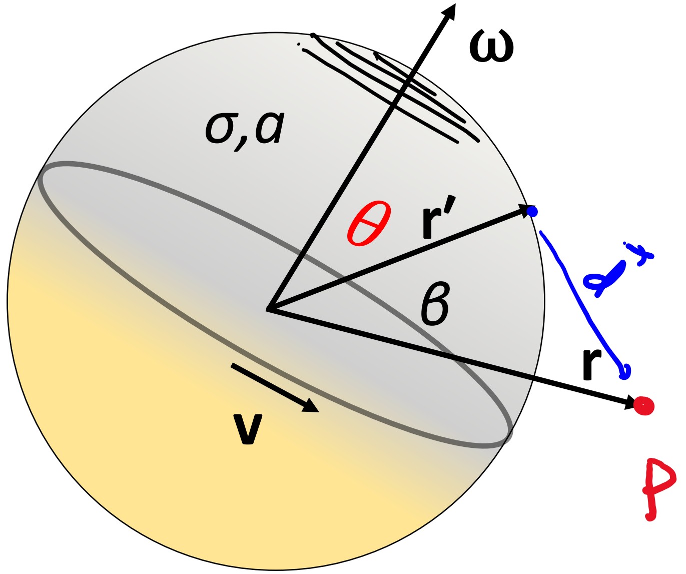
A spherical shell of radius a carries a uniform surface charge σ and spins with angular velocity ω.

1. Find an expression for the surface current $\mathbf{K}$.

2. Find an expression for the vector potential $\mathbf{A}$.

Example: Field of a pulsar

A pulsar is a rotating neutron star with a surface charge that forms a surface current and a dipolar magnetic field. Find surface current $\mathbf{K}$ and vector potential $\mathbf{A}$.



The surface current is:

$$\mathbf{K}(\mathbf{r}') = \sigma \mathbf{v}(\mathbf{r}') = \sigma(\boldsymbol{\omega} \times \mathbf{r}') = \sigma \omega a \sin\theta\, \hat{\boldsymbol{\varphi}}$$

From here you can find vector potential as: (Griffiths Ex. 5.11):

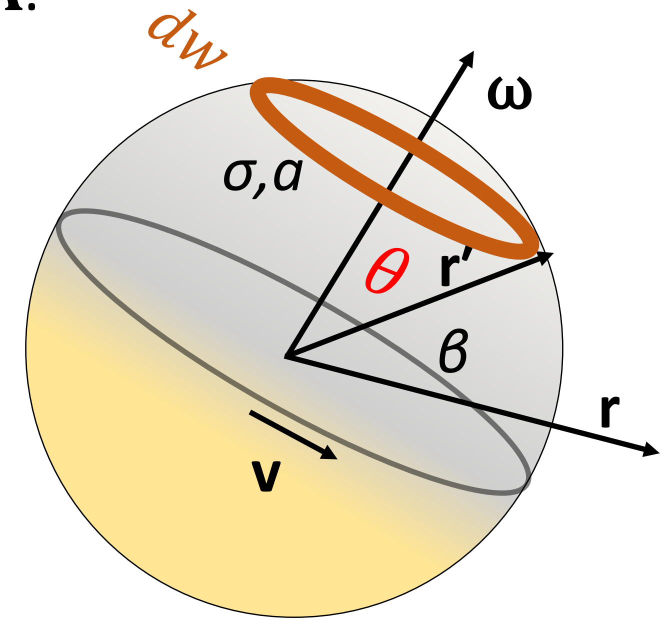
$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{K}(\mathbf{r}')}{\sqrt{r^2 + r'^2 - 2rr' \cos\beta}}\, da'$$

(try it on your own!)

Example: Field of a pulsar

$$\mathbf{K}(\mathbf{r}') = \sigma\omega a \sin\theta\,\hat{\boldsymbol{\varphi}}$$

A pulsar is a rotating neutron star with a surface charge that forms a surface current and a dipolar magnetic field. Find surface current $\mathbf{K}$ and vector potential $\mathbf{A}$.



Alternative approach: In the dipole approximation,

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2} \qquad \mathbf{m} = \frac{1}{2}\int_V \mathbf{r}'\times\mathbf{J}(\mathbf{r}')\,d\tau' = ?$$

To find $\mathbf{m}$, split the current into infinitesimally thin current loops:

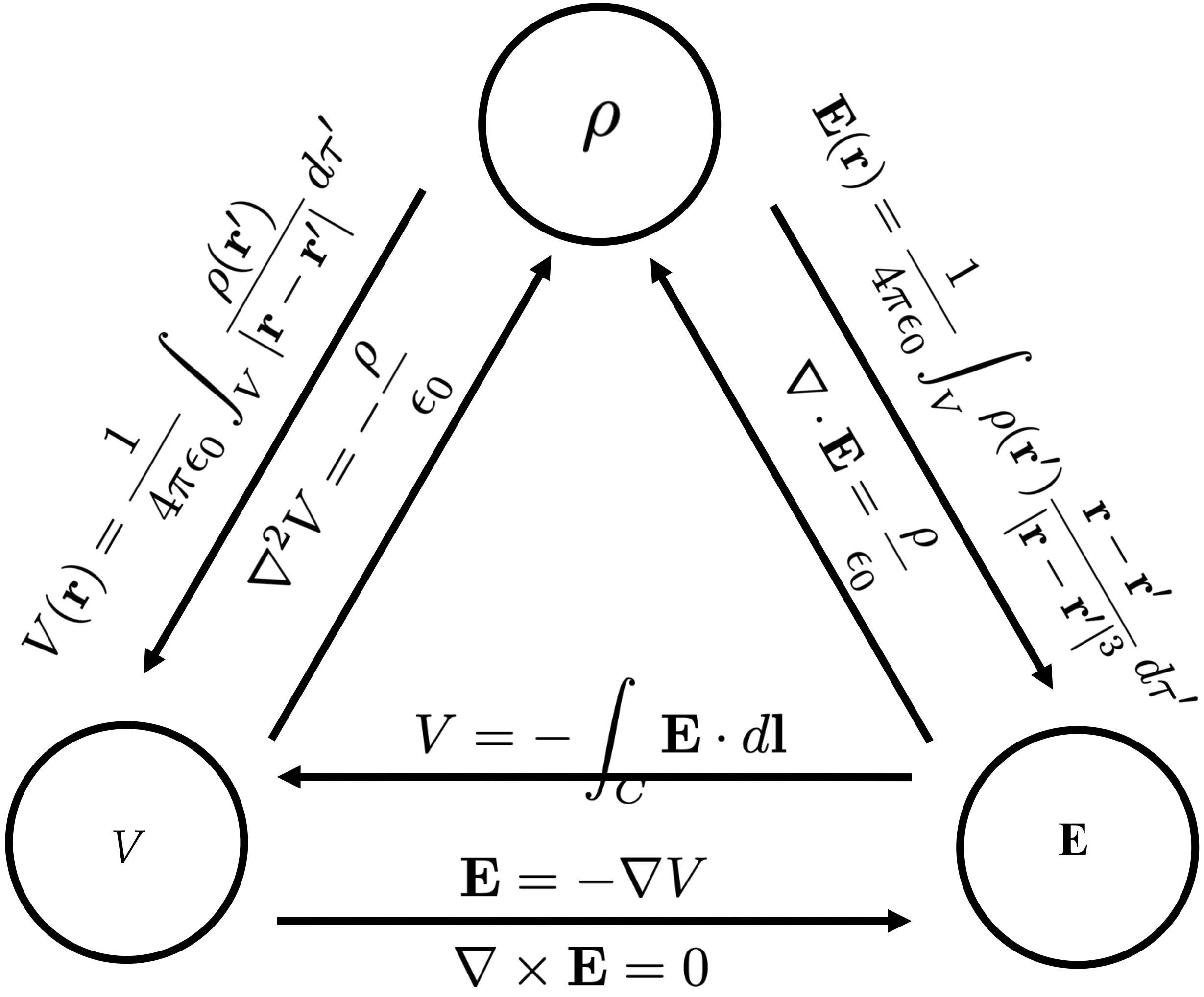
$$dw = a\,d\theta \qquad d\mathrm{J} = K\,dw = (\sigma\omega a\sin\theta)(a\,d\theta)$$

$$d\mathbf{m} = d\mathrm{J}\,A_{\text{loop}}(\theta)\,\hat{\mathbf{z}} \qquad A_{\text{loop}}(\theta) = \pi\,(a\sin\theta)^2 \qquad d\mathbf{m} = \sigma\omega\,\pi\,a^4\sin^3\theta\,d\theta\,\hat{\mathbf{z}}$$

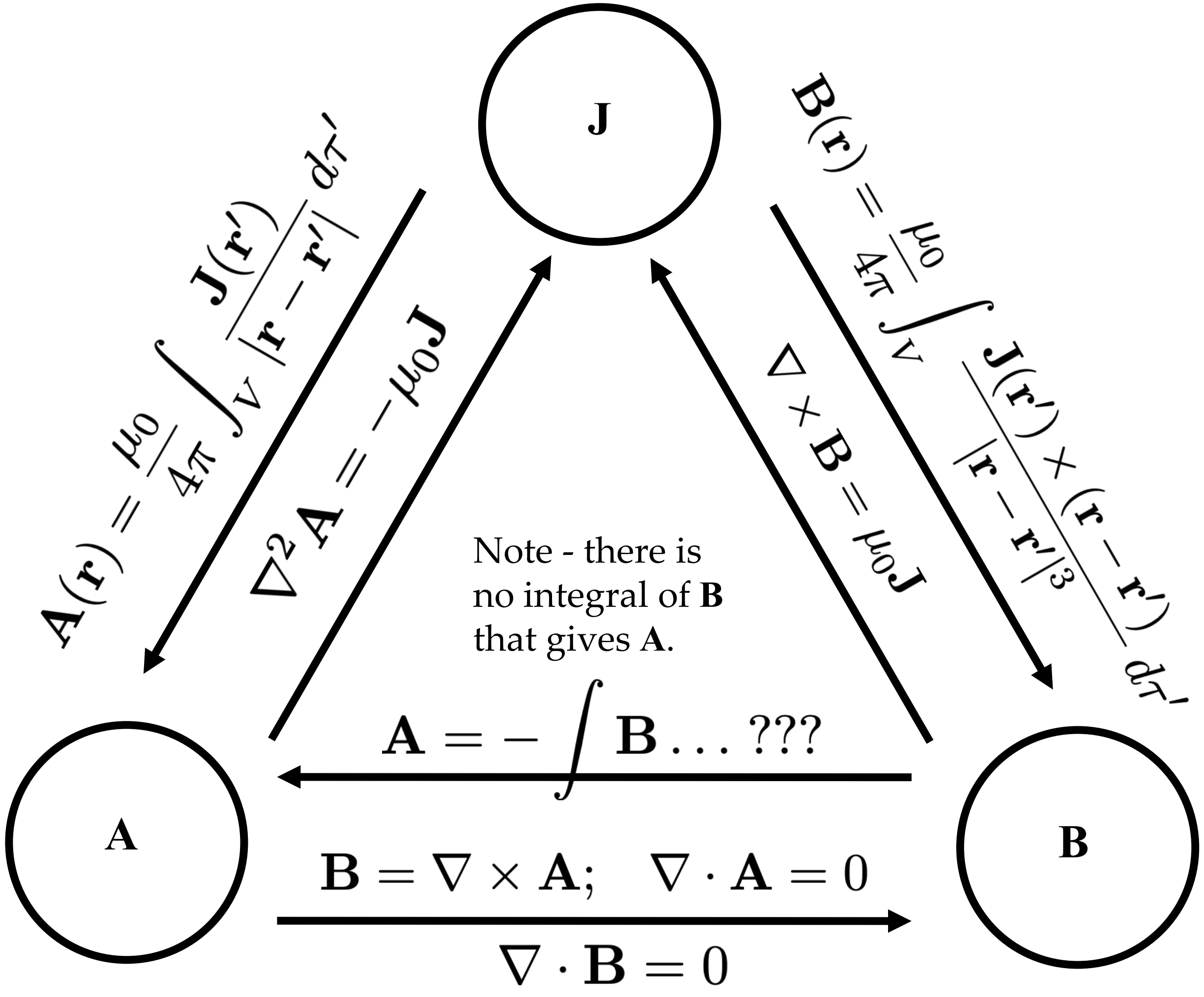
$$\mathbf{m} = \int_0^\pi d\mathbf{m} = \frac{4\pi\sigma\omega a^4}{3}\hat{\mathbf{z}}$$

$$\boxed{\mathbf{A}_{\text{out}}(\mathbf{r}) = \frac{\mu_0 a^4\omega\sigma}{3}\frac{\sin\theta}{r^2}\,\hat{\boldsymbol{\varphi}}}$$

The electrostatic trinity



The magnetostatic trinity



$\vec{E} = -\nabla V = -\frac{dV}{d\vec{r}}$

$dV = -\vec{E}\cdot d\vec{r}$

$V = -\int \vec{E}\cdot d\vec{r}$

Magnetism in matter

(Ch 6.1-2)

Diamagnetic levitation of a frog

The frog, by the way, was unharmed. “We returned it to the biology department,” Geim said.

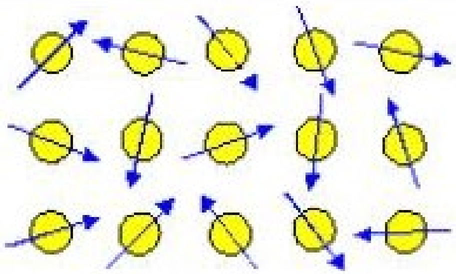
https://news.harvard.edu/gazette/story/2024/04/how-did-you-get-that-frog-to-float/

- Magnetization field
- Paramagnetics, diamagnetics, ferromagnetics
- Surface and bound currents

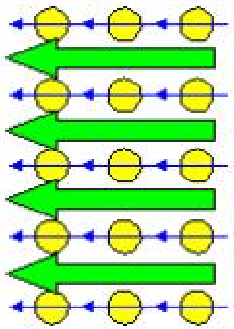
Magnetism in matter

There are three common forms of magnetism in matter: paramagnetism, diamagnetism, and ferromagnetism. They all have to do with the alignment of electron spins in a material.

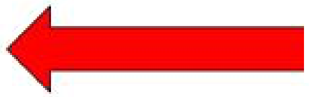
Electrons have charge and spin, and thus a magnetic dipole moment. (This is fundamentally a quantum phenomenon.) Their moments can be aligned by a $\mathbf{B}$ field.



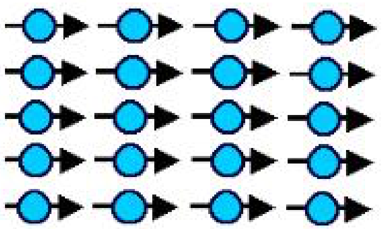
random



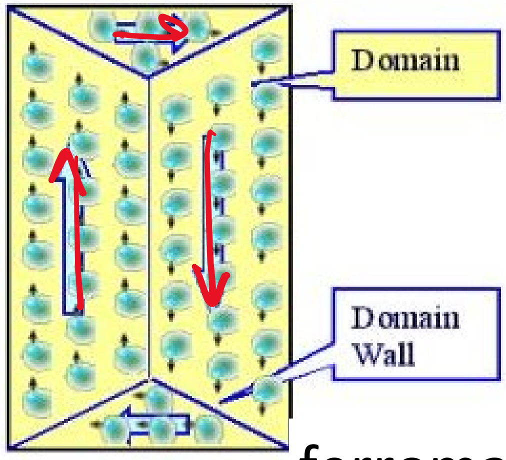
paramagnetism



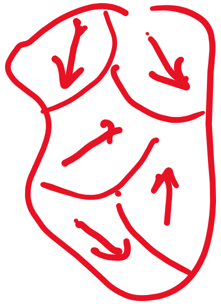
$\mathbf{B}_{ext}$



diamagnetism

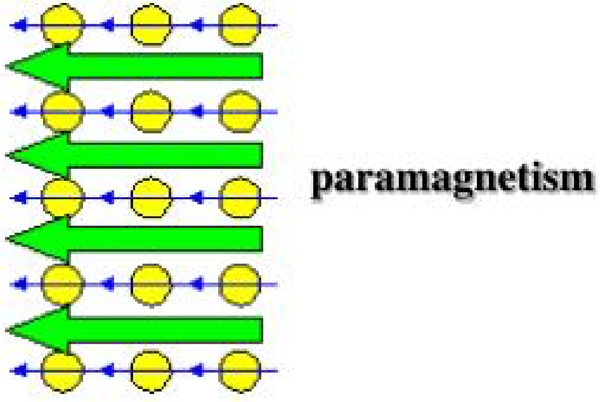


ferromagnetism



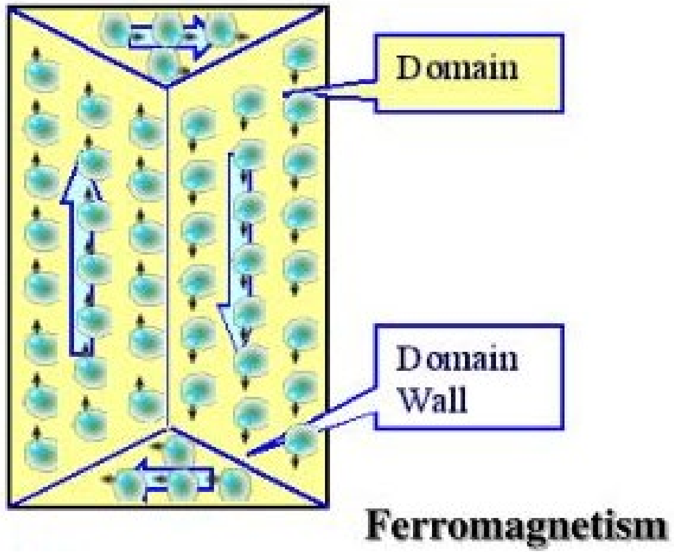
Paramagnetism and ferromagnetism

Paramagnetism - Analogous to electric polarization. Produced by the alignment of the electron's *spin* dipole moments in an external $\mathbf{B}$ field. Magnetic dipoles orient parallel to the $\mathbf{B}$ field and strengthen in proportion to the applied field strength.



Mainly occurs in atoms and molecules with odd numbers of electrons subject to magnetic torque. Overall, much weaker than ferromagnetism. Disappears when external $\mathbf{B}$ field is gone.

Ferromagnetism (Fe, Co, Ni) - Moments align with each other over large areas ("domains") which grow when a $\mathbf{B}$ field is applied.



This is the only strong form of magnetism in common materials. Ferromagnets become paramagnets above the Curie temperature (e.g., 1037 K for Fe).

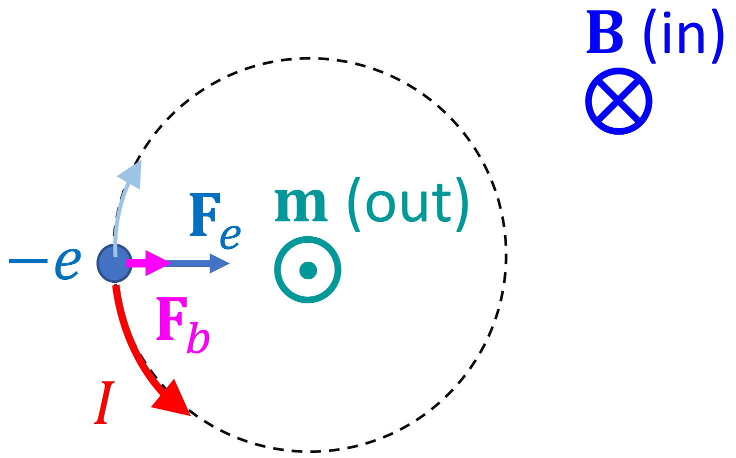
Diamagnetism

All materials are diamagnetic to some degree. It arises from interaction of induced atomic magnetic dipoles with the $\mathbf{B}$ field.

Electrons *orbiting* around the nucleus speed up in a $\mathbf{B}$ field. This alters the orbital dipole moment from electrons in a direction anti-parallel to $\mathbf{B}$ (contrary to paramagnetism).

Diamagnetism is weaker than paramagnetism, so it is best to observe it in atoms with even numbers of electrons (when all electron's spins are paired).

- Assume $\mathbf{B}$ is into the page
- Centripetal force $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_b$ increases
- $\mathbf{F} \uparrow \Rightarrow$ electron speeds up ($I \uparrow$)
- $\mathbf{m} \uparrow$ (anti-parallel to $\mathbf{B}$)



Magnetization field $\mathbf{M}$

In analogy to the polarization field $\mathbf{P}$, for any type of magnetism, we can define the magnetization field $\mathbf{M}$:

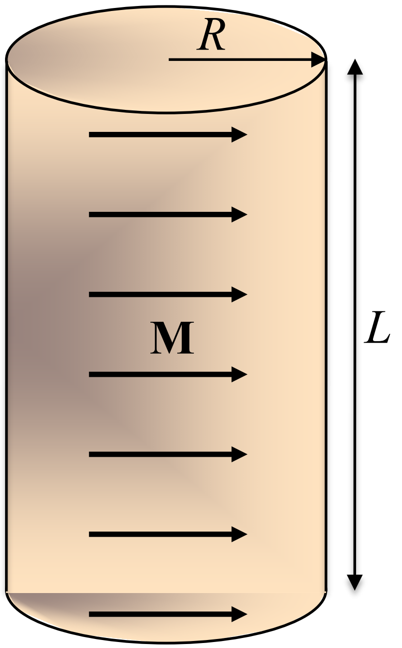
$\mathbf{M}$ = magnetic dipole moment/unit volume

so that $\mathbf{m} = \int_V \mathbf{M}(\mathbf{r}')\, d\tau'$ is the magnetic dipole moment of the object.

Magnetization field $\mathbf{M}$

Q: A solid cylinder has uniform magnetization $\mathbf{M}$ throughout the volume in the x direction as shown.

What's the magnitude of the total magnetic dipole moment of the cylinder?



A. $|\mathbf{M}|\pi R^2 L$

B. $|\mathbf{M}|2\pi RL$

C. $|\mathbf{M}|\pi R^2$

D. $|\mathbf{M}|2\pi R$

E. none of the above

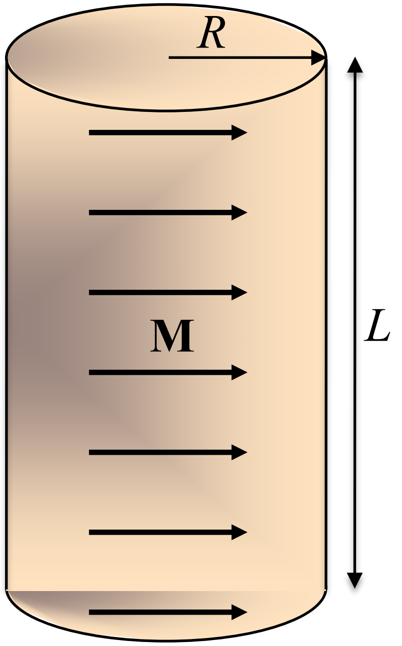
Magnetization field $\mathbf{M}$

Q: A solid cylinder has uniform magnetization $\mathbf{M}$ throughout the volume in the x direction as shown.

What's the magnitude of the total magnetic dipole moment of the cylinder?

$\mathbf{M}$ = magnetic dipole moment/unit volume

A. $|\mathbf{M}|\pi R^2 L$
B. $|\mathbf{M}|2\pi RL$
C. $|\mathbf{M}|\pi R^2$
D. $|\mathbf{M}|2\pi R$
E. none of the above



Bound currents – 1

(Griffiths 6.2)

We can express the vector potential due to a magnetization field as:

$\vec{M} \times \nabla g$

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2} \rightarrow \frac{\mu_0}{4\pi}\int_V \mathbf{M}(\mathbf{r}') \times \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\, d\tau'$$

$\frac{\vec{M} \times \hat{d}}{d}\, d\tau$

Now rewrite the last factor as a gradient:

$$\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} = \nabla' \frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

and use the product rule for a scalar field times a vector field:

$$\nabla \times (g\mathbf{M}) = g(\nabla \times \mathbf{M}) - \mathbf{M} \times (\nabla g)$$

to rewrite the integral as:

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_V \left(\frac{\nabla' \times \mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} - \nabla' \times \frac{\mathbf{M}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \right) d\tau'$$

Bound currents – 2

(Griffiths 6.2)

The second term can be rewritten as a surface integral using: $\int_V (\nabla \times \mathbf{v})\, d\tau = -\oint_S \mathbf{v} \times d\mathbf{a}$

(see Griffiths problem 1.61b)

so that:

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi} \left(\int_V \overbrace{\frac{\nabla' \times \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}}^{J_b} d\tau' + \oint_A \overbrace{\frac{\mathbf{M}(\mathbf{r}') \times \hat{\mathbf{n}}}{|\mathbf{r} - \mathbf{r}'|}}^{K_b} da \right)$$

$$\vec{A} = \int \frac{\text{smth}}{d}\, d\tau'$$

Now, we can define volume and surface "bound currents":

$$\boxed{\begin{aligned} \mathbf{J}_b &\equiv \nabla \times \mathbf{M} \\ \mathbf{K}_b &\equiv \mathbf{M} \times \hat{\mathbf{n}} \end{aligned}}$$

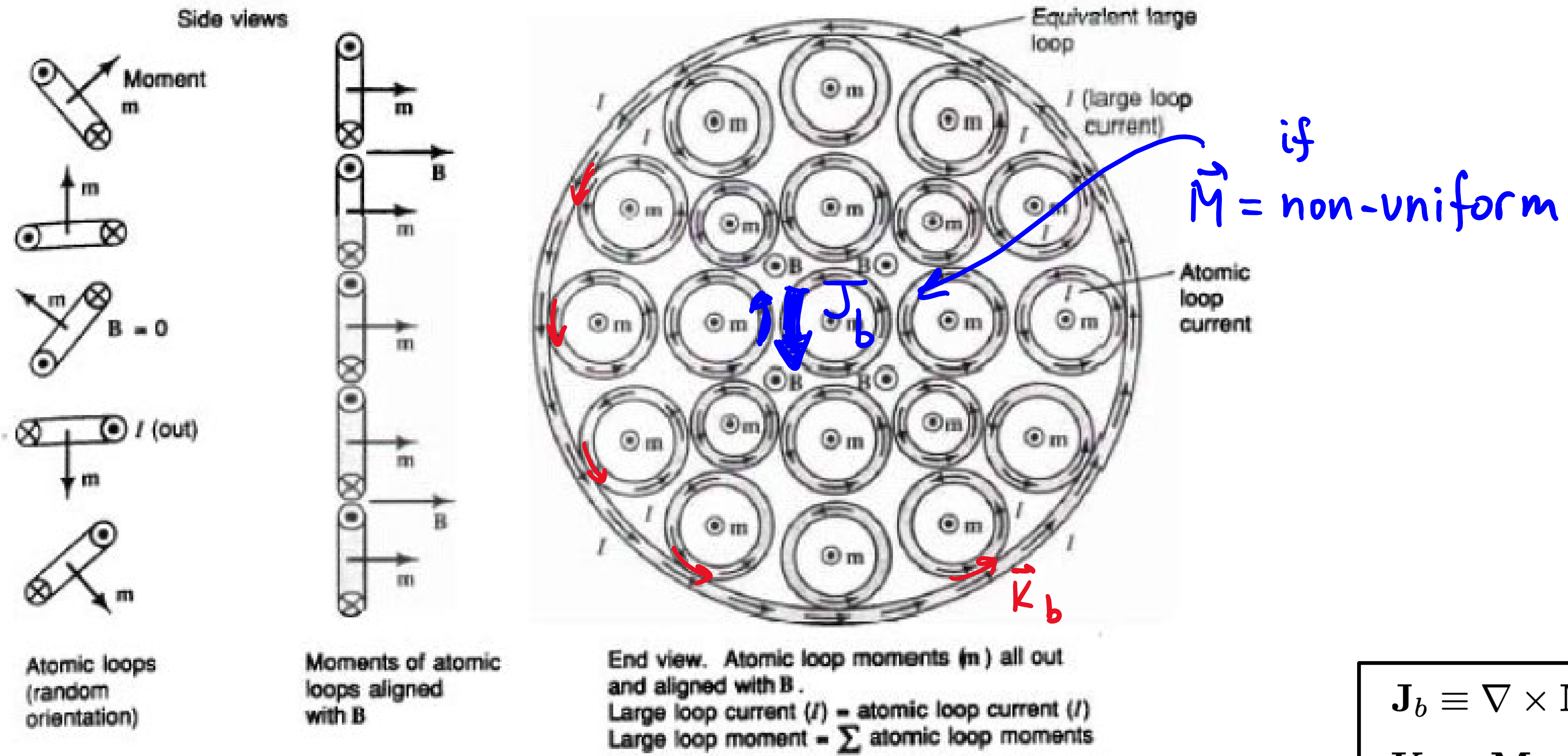
c.f.:

$$\rho_b \equiv -\nabla \cdot \mathbf{P}$$
$$\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}}$$

…and we can express $\mathbf{A}$ in a form the looks like our regular $\mathbf{A}$-$\mathbf{J}$ connection:

$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu_0}{4\pi} \left[\int_V \frac{\mathbf{J}_b(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\tau' + \oint_A \frac{\mathbf{K}_b(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} da' \right]$$

Bound currents – 3



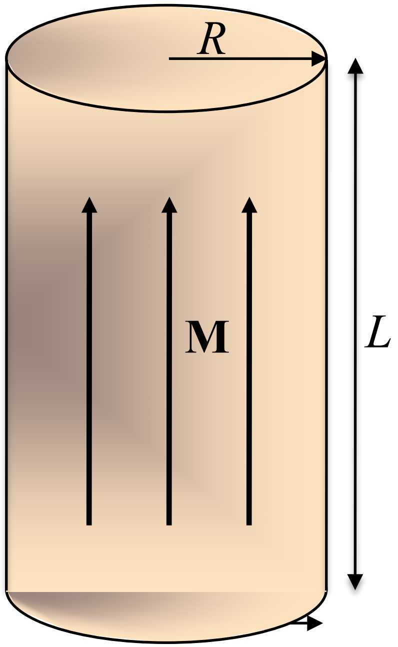
$$\mathbf{J}_b \equiv \nabla \times \mathbf{M}$$

$$\mathbf{K}_b \equiv \mathbf{M} \times \hat{\mathbf{n}}$$

Bound currents – 4

Q: A solid cylinder has uniform magnetization field $\mathbf{M}$ along the z axis as shown.

Where do bound currents appear?



A. Everywhere in the bulk, and on all surfaces

B. In the bulk only, but not on the surfaces

C. The top & bottom surfaces only

D. The cylindrical (side) surface only

E. On all surfaces, but not in the bulk

Bound currents – 4

Q: A solid cylinder has uniform magnetization field $\mathbf{M}$ along the z axis as shown.

Where do bound currents appear?

$$\mathbf{J}_b \equiv \nabla \times \mathbf{M}$$
$$\mathbf{K}_b \equiv \mathbf{M} \times \hat{\mathbf{n}}$$

$\nabla \times \mathbf{M} = 0 \;\rightarrow\; \mathbf{J}_b = 0$ (in the bulk)

$\mathbf{M} \times \hat{\mathbf{n}} = 0$ (on the caps) $\mathbf{M} \times \hat{\mathbf{n}} \neq 0$ (on the sides)

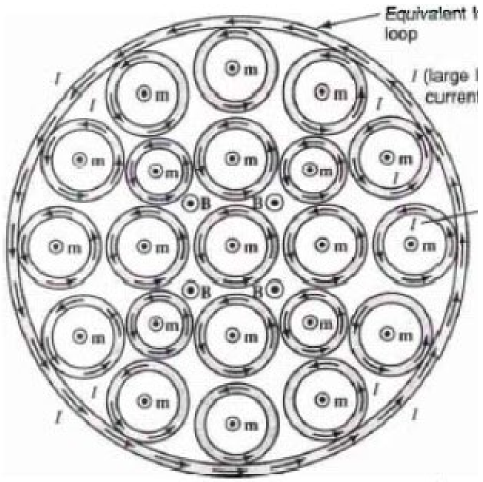
A. Everywhere in the bulk, and on all surfaces

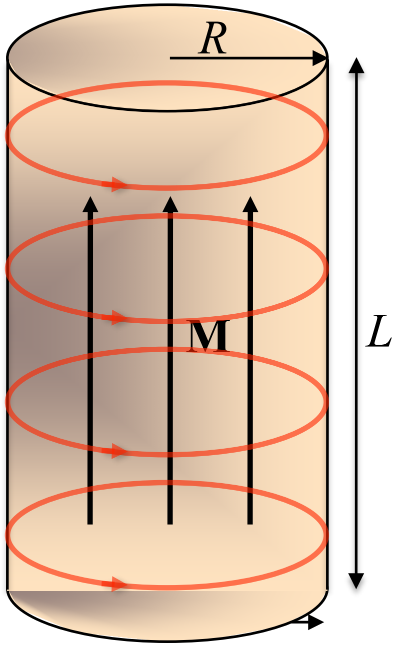
B. In the bulk only, but not on the surfaces

C. The top & bottom surfaces only

D. The cylindrical (side) surface only

E. On all surfaces, but not in the bulk

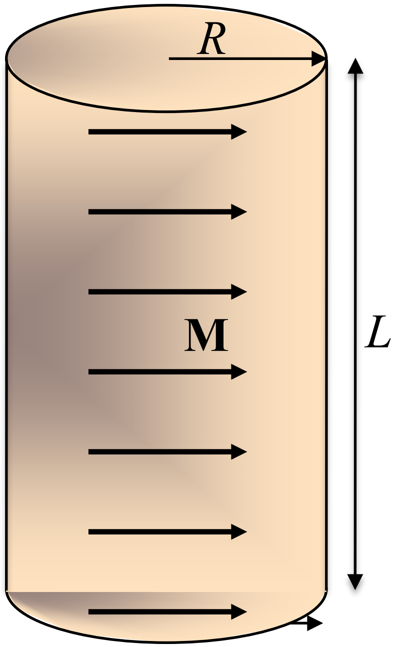




Bound currents – 5

Q: A solid cylinder has uniform magnetization field $\mathbf{M}$ along the x axis as shown.

Where do bound currents appear?



A. Everywhere in the bulk, and on all surfaces

B. In the bulk only, but not on the surfaces

C. The top & bottom surfaces only

D. The cylindrical (side) surface only

E. On all surfaces, but not in the bulk

Bound currents – 5

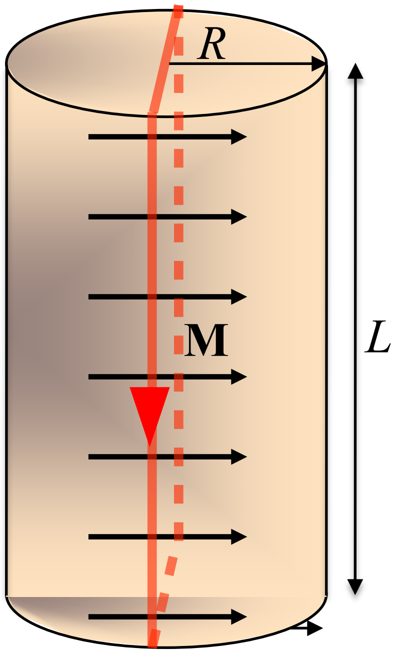
Q: A solid cylinder has uniform magnetization field $\mathbf{M}$ along the x axis as shown.

Where do bound currents appear?

$$\mathbf{J}_b \equiv \nabla \times \mathbf{M}$$

$$\mathbf{K}_b \equiv \mathbf{M} \times \hat{\mathbf{n}}$$

Everywhere on the surface where $\mathbf{M}$ is not parallel to $\hat{\mathbf{n}}$



A. Everywhere in the bulk, and on all surfaces

B. In the bulk only, but not on the surfaces

C. The top & bottom surfaces only

D. The cylindrical (side) surface only

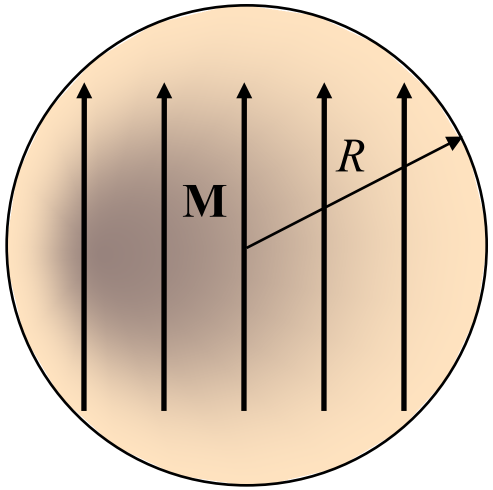
E. On all surfaces, but not in the bulk

Example: Bound currents

A solid sphere has uniform magnetization field $\mathbf{M}$ along the z axis as shown.

1) Write down expressions for the bound surface and volume currents. ← A.
2) Sketch the $\mathbf{B}$ field outside the sphere. ← B.

C. :(



Example: Bound currents

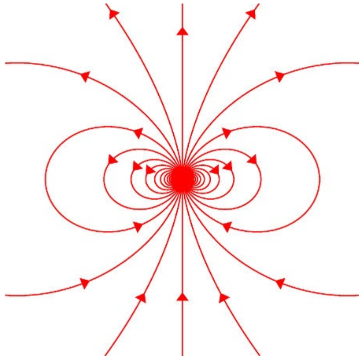
A solid sphere has uniform magnetization field $\mathbf{M}$ along the z axis as shown.

1) Write down expressions for the bound surface and volume currents.
2) Sketch the $\mathbf{B}$ field outside the sphere.

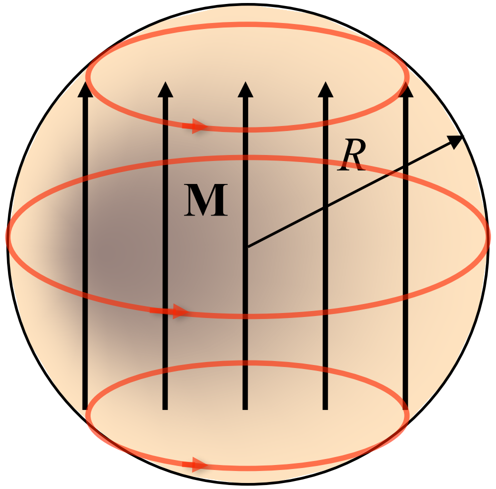
$$\mathbf{J}_b = \nabla \times \mathbf{M} = 0 \qquad \mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = M\,\hat{\mathbf{z}} \times \hat{\mathbf{r}} = M \sin\theta\, \hat{\boldsymbol{\varphi}}$$

B field is dominated by a dipole field with:

$$\mathbf{m} = M \frac{4\pi R^3}{3}\, \hat{\mathbf{z}}$$



We can say that this magnetic field is sourced by bound currents due to magnetization



Auxiliary field **H**

...or the art of bookkeeping fields and currents in magnetics

(Ch 6.3)

- Auxiliary magnetic field, **H**
- Magnetic susceptibility, magnetic permeability, relative permeability
- Finding **H**, **B**, **M** and bound currents in paramagnetics and diamagnetics

Auxiliary field $\mathbf{H}$

As with the displacement field, $\mathbf{D}$, we introduce an auxiliary field in magnetism

Divide the currents into free and bound parts, so that:

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J}_f + \mathbf{J}_b) = \mu_0(\mathbf{J}_f + \nabla \times \mathbf{M})$$

Then: $$\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M}\right) \equiv \nabla \times \mathbf{H} = \mathbf{J}_f$$

$\underbrace{\qquad}_{} = \vec{H}$

enough symmetry!

Compare*: $$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \qquad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

We then have Ampère's "free current" law: $$\nabla \times \mathbf{H} = \mathbf{J}_f \rightarrow \oint_C \mathbf{H} \cdot d\mathbf{l} = I_{f,\,\text{enc}}$$

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* Note the sign flip between $\mathbf{M}$ and $\mathbf{P}$. Traces back to: $\mathbf{J}_b = +(\nabla \times \mathbf{M}) \quad \rho_b = -(\nabla \cdot \mathbf{P})$

Auxiliary field $\mathbf{H}$ – usage notes

- $\mathbf{H}$ turns out to be somewhat more useful than $\mathbf{D}$ because, in electric circuits, we can easily control potential differences and free currents, which relate to $\mathbf{E}$ and $\mathbf{H}$, rather than $\mathbf{D}$ and $\mathbf{B}$.

- Rule of thumb - $\mathbf{H}$ is useful whenever the situation is symmetrical enough to use Ampère's law (infinite cylinders, planes, solenoids, toroids). Nonetheless, $\mathbf{B}$ is the fundamental field.

- There is no a Biot-Savart law or Lorentz force law for $\mathbf{H}$.

Linear magnetic materials

As with dielectric materials, we have paramagnetic and diamagnetic materials which are often linear:

$$\mathbf{M} = \chi_m \mathbf{H}$$

χ_m = susceptibility

where χ_m is the magnetic susceptibility.

Then:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m)\mathbf{H} = \mu_0\mu_r\mathbf{H} = \mu\,\mathbf{H}$$

μ = permeability

μ_r = relative permeability

Paramagnetic materials (dominated by spin $\mathbf{m}$) have:

$$\chi_m > 0; \quad \mu_r > 1; \quad \mu > \mu_0$$

Diamagnetic materials (dominated by orbital $\mathbf{m}$) have:

$$\chi_m < 0; \quad \mu_r < 1; \quad \mu < \mu_0$$

Note the difference!

Note the difference between how magnetic and electric susceptibilities are introduced:

$$\mathbf{M} = \chi_m \mathbf{H} \qquad\qquad \mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$$

$\mathbf{M}$ is proportional to the auxiliary field, while $\mathbf{P}$ is proportional to the total field. The magnetic case is more intuitive: the induced magnetization is proportional to the "external" field sourced by the "free" currents that we control in the laboratory!

We *could* have defined: $\mathbf{P} \equiv \chi_e \mathbf{D}$

Equations in red are not correct!

Then with $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ we'd have:

$$\mathbf{E} = \frac{1 - \chi_e}{\epsilon_0} \mathbf{D}$$

This would have made the shielding effect of polarization more explicit. But since we control voltages, V, in the lab, it is empirically more convenient to define $\mathbf{P}$ in terms of $\mathbf{E}$.

Magnetic susceptibility

$\mathbf{M} = \chi_m \mathbf{H}$

The range of χ_m for the three forms of magnetism:

Ferromagnetism: $\chi_m > 0$ and *very* large.

For instance, pure, annealed iron has μ ~10,000.

Paramagnetism: $0 < \chi_m \ll 1$

Typically, $\chi_m \sim 10^{-6}$

Diamagnetism: $\chi_m < 0, \;\; |\chi_m| \ll 1$

Typically somewhat even smaller than is typical in paramagnetism.

The fact that μ is so close to 1 for anything that isn't ferromagnetic is why one could easily avoid noticing magnetism in everyday life.

Auxiliary field

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction.

$\mathbf{B}$ will be counter-clockwise (ccw) as viewed from above. (Right?)

What is the sense of $\mathbf{H}$ and $\mathbf{M}$ inside the cylinder?

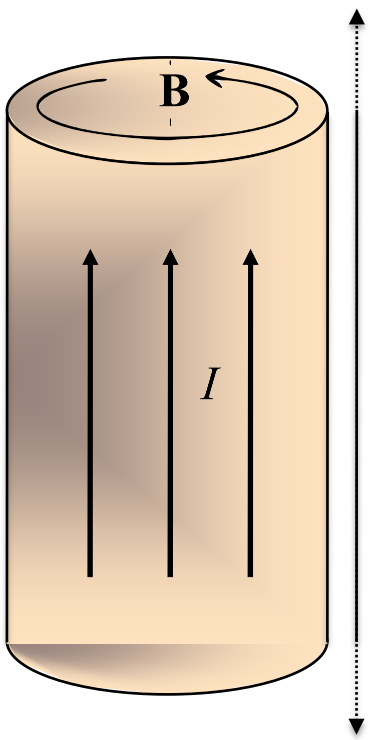
A. Both are ccw

B. Both are cw

C. $\mathbf{H}$ is ccw, $\mathbf{M}$ is cw

D. $\mathbf{M}$ is ccw, $\mathbf{H}$ is cw

E. Both are zero



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What is the sense of $\mathbf{H}$ and $\mathbf{M}$ inside the cylinder?

$$\nabla \times \mathbf{H} = \mathbf{J}_f \;\rightarrow\; \mathbf{H} \text{ is ccw}$$

$$\mathbf{M} = \chi_m \mathbf{H} \qquad \chi_m > 0 \;\rightarrow\; \mathbf{M} \text{ is ccw}$$

A. Both are ccw
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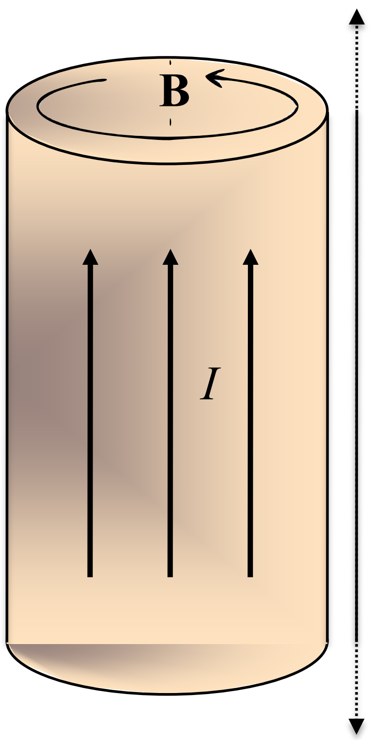
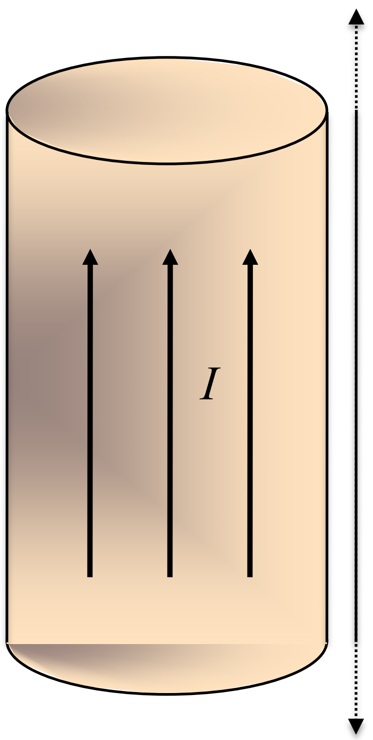


Bound volume current

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction.

What is the direction of the bound volume current?



.A J_b is parallel to I

B. J_b is anti-parallel to I

C. J_b is zero

D. None of the above

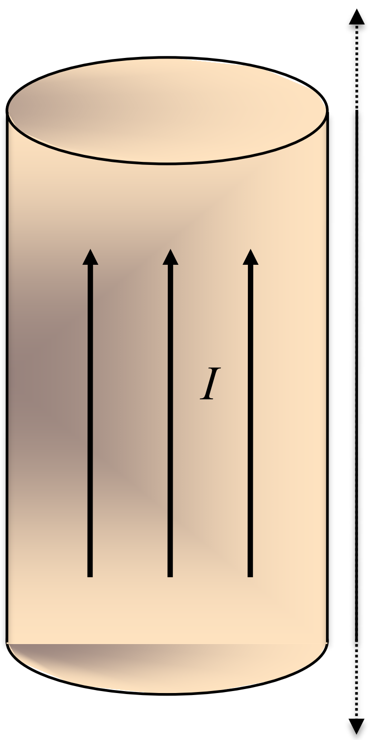
E. No idea

Bound volume current

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction. What is the direction of the bound volume current?

$$\nabla \times \mathbf{H} \parallel \mathbf{I} \qquad \text{and} \qquad \mathbf{M} = \chi_m \mathbf{H} \quad (\chi_m > 0)$$

$$\rightarrow \mathbf{J}_b = \nabla \times \mathbf{M} \parallel \nabla \times \mathbf{H} \parallel \mathbf{I}$$



.A $\mathbf{J}_b$ is parallel to I

B. $\mathbf{J}_b$ is anti-parallel to I

C. $\mathbf{J}_b$ is zero

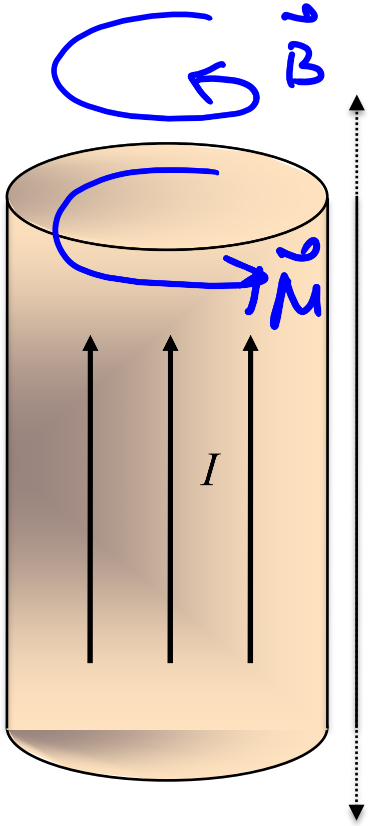
D. None of the above

E. No idea

$$\nabla \times \mathbf{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

Bound surface current

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction. What is the direction of the bound surface current?



.A K_b is parallel to I

B. K_b is anti-parallel to I

C. K_b is zero

D. None of the above

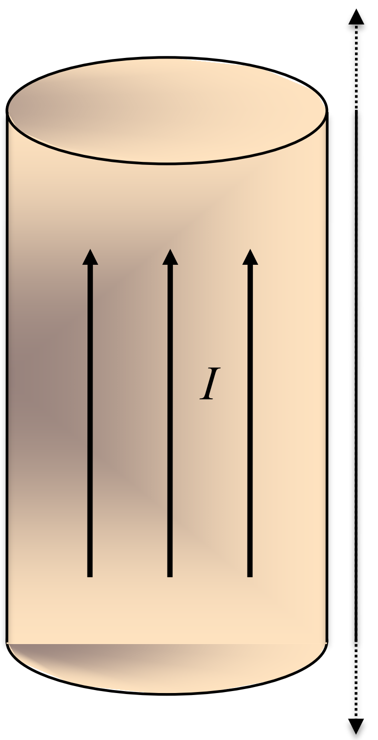
E. No idea

Bound surface current

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction. What is the direction of the bound surface current?

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} \parallel -\hat{\mathbf{z}}$$

A. K_b is parallel to I
B. K_b is anti-parallel to I
C. K_b is zero
D. None of the above
E. No idea



Total bound current

Q: A very long aluminum (paramagnetic) rod carries a uniform current I in the $+z$ direction.

What is the direction and the magnitude of the total (surface + volume) bound current?

do this on your own!