PHYS 301 - Tutorial 1

- Complete all problems and submit ONE set of answers as a group of 4 students
- One group member **uploads a pdf to gradescope** and specifies the other group members there.
- Upload is due b**y the end of the day of the tutorial**. We will accept late submission till **Thursday 11:59 pm**, after which the submission will be closed.
- All group members, please **ensure that you have created your gradescope account**, even if you are not submitting, otherwise you cannot be added to the submission.

Part 1 – Dirac Delta Function

The 1-dimensional Dirac delta function is defined as

$$\delta(x) = \begin{cases} 0, & x \neq 0 \\ \infty, & x = 0 \end{cases} \tag{1}$$

such that

$$\int_a^b \delta(x)dx = \begin{cases} 1, & a < 0 < b \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

with units $[\delta(x)] = \frac{1}{[x]}$.

a) Sketch the function $f(x) = \delta(x-3)$. (can you *really* sketch it?)

b) Evaluate the integral

$$\int_{-\infty}^{\infty} cx\delta(x-1)dx. \tag{3}$$

c) The Dirac delta function is especially useful in writing expressions for charge densities. Match each description of a charge density with a possible mathematical description.

- $\rho(x, y, z) = A\delta(x-2)\delta(z)$
- $\rho(x, y, z) = B\delta(x+2)\delta(y)\delta(z-1)$
- $\rho(x, y, z) = C\delta(z)$
- $\rho(s, \phi, z) = D\delta(s-s_0)\delta(z-z_0)$
- $\rho(r, \phi, \theta) = E\delta(r-r_0)$

- Uniformly charged infinite sheet
- Uniformly charged spherical shell
- Uniformly charged infinitely long wire
- Uniformly charged circular loop
- Point charge

d) Determine the units of A, B, and C.

e) Describe the location of the uniformly charged infinitely long wire. In which direction does the wire point?

f) Now consider the uniformly charged infinite sheet. Consider the electric field at the point (x, y, z).

1. By symmetry arguments ONLY, (no Gauss' law calculation here!), in what direction could the E field point? Choose ALL that apply:
   (a) $\pm\hat{x}$
   (b) $\pm\hat{y}$
   (c) $\pm\hat{z}$
   (d) None of the above.

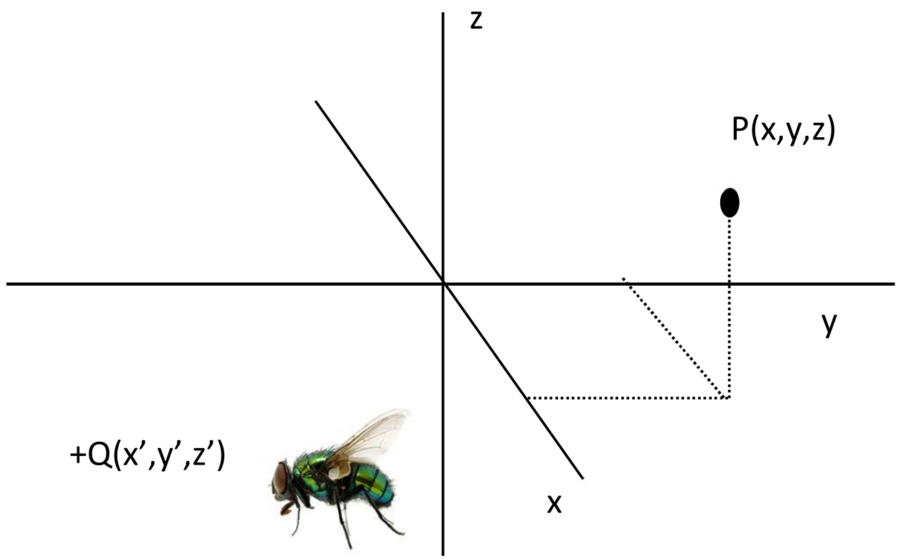
2. By symmetry arguments ONLY, which coordinates could the E field depend on? Choose ALL that apply:
   (a) x
   (b) y
   (c) z
   (d) None of the above.

Explain your answers.

Part 2 – Separation Vector R

In your previous electromagnetism courses up until now, you have probably worked with quantities (electric/magnetic fields, etc.) that were expressed as a function of a position $P = (x, y, z)$, treating the location of the point charge(s) responsible for these fields as fixed. In general, we'd also like to treat the position of all charged objects (x', y', z') as a variable. The position (x, y, z) is called the observation location, or *field point*, while (x', y', z') is called the *source point*.

Imagine you're in your room and you notice a fly trapped inside, bumping into the window. Each time the fly crashes into the window, a small amount of charge is transferred onto its body. After a little while, the fly gives up on its escape plans and decides to explore your room instead, having accumulated a total charge $+Q$. Again, we want to find the potential V.



1. On the diagram above, draw the vector $\mathbf{r}$ from the origin to the field point $P = (x, y, z)$ and the vector $\mathbf{r}'$ from the origin to the source point (the fly), located at point (x', y', z').

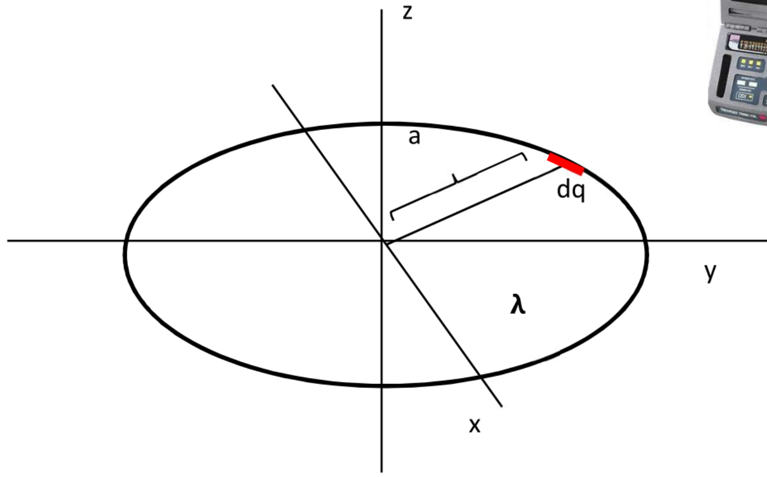
2. We define the separation vector $\mathbf{R}$ as the vector pointing from the source point (primed coordinates) to the field point. Draw the vector $\mathbf{R}$ on the diagram and express it in terms of $\mathbf{r}$ and $\mathbf{r}'$.

3. Express the potential V as a function of $\mathbf{R}$ only. Notice the advantage of expressing V in this way. Instead of needing to specify two positions, we only need a single vector.

4. Express the Cartesian components of $\mathbf{R}$ (i.e. (R_x, R_y, R_z)) in terms of the Cartesian components of $\mathbf{r}$ and $\mathbf{r}'$.

Part 3 – Charged Bike Tires

In the year 2240, a bicyclist, named Doug, gets lost east of Burnaby mountain and gets a flat tire. Doug pulls off the tire and then consults his Tricorder to find out what life forms are nearby. However, the flat tire has somehow been charged with uniform charge density λ. The Tricorder, which has a charge of $+Q$ complains that there is a force from the tire pushing the Tricorder away. Your goal is to calculate the force on the Tricorder produced by the electrically charged tire. (You can treat the tire as having infinitesimal cross-section, i.e. λ is a linear density.)



1. The origin is at the centre of the tire. Label source and field points on the diagram.

2. Now, draw and label the vectors $\mathbf{r}, \mathbf{r}', \mathbf{R}$. What coordinate system is most convenient for this problem?

3. To set up the integral, first write an expression for $d\mathbf{E}$, the electric field due to an infinitesimal section of the charged ring containing charge dq. (Hint: the electric field for a point charge is $+q$ is $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{R}$ – how should you modify this to find $d\mathbf{E}$?). Now express dq in terms of the coordinate system you chose in part 2.

4. Express the Cartesian components of $\mathbf{r}$ and $\mathbf{r}'$ in the coordinate system you chose in part 2. For instance, if you chose cylindrical coordinates, you should express (x, y, z) and (x', y', z') in terms of cylindrical variables (s, ϕ, z) and (s', ϕ', z') respectively. Now do the same for $\mathbf{R}$.

5. With your results from parts 3 and 4, set up three integrals representing the three Cartesian components of the force on the Tricorder: (F_x, F_y, F_z). Notice that these integrals could (at least in principle) be solved by a dumb computer or calculator.

6. Now, simplify by placing the Tricorder on the z-axis, and evaluate the integral. Of course, there are really three integrals (on for each Cartesian component), but you only need to evaluate one of them. Why?