PHYS 301 - Tutorial 2

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member **uploads a pdf to gradescope** and specifies the other group members there.
- Upload is due **by the end of the day of the tutorial**. We will accept late submission till **Thursday 11:59 pm**, after which the submission will be closed.

Part 1 – Dirac Delta Function Refresher

The 1-dimensional Dirac delta function is defined as

$$\delta(x) = \begin{cases} 0, & x \neq 0 \\ \infty, & x = 0 \end{cases} \tag{1}$$

such that

$$\int_a^b \delta(x)dx = \begin{cases} 1, & a < 0 < b \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

with units $[\delta(x)] = \frac{1}{[x]}$.

What is the physical situation represented by the volume charge density $\rho(x, y, z) = c\delta(x - 3)$? Make a 3D sketch of the charge distribution. What are the units of c?

Part 2 – SLAC Lightning Strike

SLAC (Stanford Linear Accelerator Center) is where quarks (including the charm quark), and the Z boson were discovered. Charged particles are accelerated inside a long metal cylindrical pipe, which is 2 miles long and has a 12 cm diameter. All the air is pumped out of this pipe, known as the "beam line".

One afternoon, the beam line is struck by lightning, which results in a uniform surface charge density σ on the outer surface of the beam line. After the lightning strike, Stanford physicists want to start accelerating particles in the beam line, but they are concerned that the charge density might affect the beam particles, causing them to crash into the wall of the pipe and burn a hole through it. Air and dirt would rush into the empty pipe, causing months of expensive delay. You will investigate whether the surface charge of the beam line could affect the beam particles.

a) Make a sketch of the 2D surface charge.

b) Determine an expression for the volume charge density $\rho(x, y, z)$ of the beam line. Don't worry about the ends of the cylinder for this question – you can assume it is infinite. (Hint: in Part 1, we asked you to interpret a mathematical expression as a physical arrangement – math to physics. In this question, you are going the other way – physics to math!)

c) Using your answer in part **b)**, integrate over the surface to find the total charge on the beam line. Are your units correct?

d) What is the direction of the E-field at all points in space? Explain in detail how you know. (Again, you can assume the cylinder is infinite.)

Now, we will use Gauss' Law ($\Phi_E = \oiint_{\partial V} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enc}}{\epsilon_0}$) to find the E-field at all points in space.

e) Determine which Gaussian surface you should use and include it in your sketch from part **a)**.

f) Write an expression for the flux through each face of the surface.

g) Now write an expression for the quantity $d\mathbf{A}$. What direction does $d\mathbf{A}$ point?

h) Evaluate the integral to find the E-field at all points in space.

i) Does the charge density σ on the beam line affect the particles being accelerated inside it? Could it affect the electronic equipment outside the tunnel?

j) *Bonus problem:* Calculate the electric potential $V = \int_P^{P_0} \mathbf{E} \cdot d\mathbf{l}$ at all points in space. (Hint: set the potential V equal to zero at some radial point in cylindrical coordinates $s = s_0$. Does it mask sense to set $V = 0$ at $s = \infty$? How about at $s = 0$?)

k) *Bonus Problem:* In part **h)** you found the E-field at all points in space due to the SLAC beam line. Now, find all points in space where the E-field has non-zero divergence.