PHYS 301 - Tutorial 4 (Review)

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

- ➢ You may hand in your tutorial with only Parts 1 and 2 completed and receive full marks.
- ➢ In these parts, you can first compute the field and the potential, and then sketch them, but: Pay attention at (and comment on!) jumps / continuity / corners, and at consistency of all your results.
- ➢ It is strongly recommended to explore Part 3 (not for marks) to better understand which routes and approaches are more useful and which are less useful, and what are possible pitfalls.
- ➢ Explore Part 4 (not for marks) to better understand how to compute electric energy, and which surprises you can meet with when you do it.

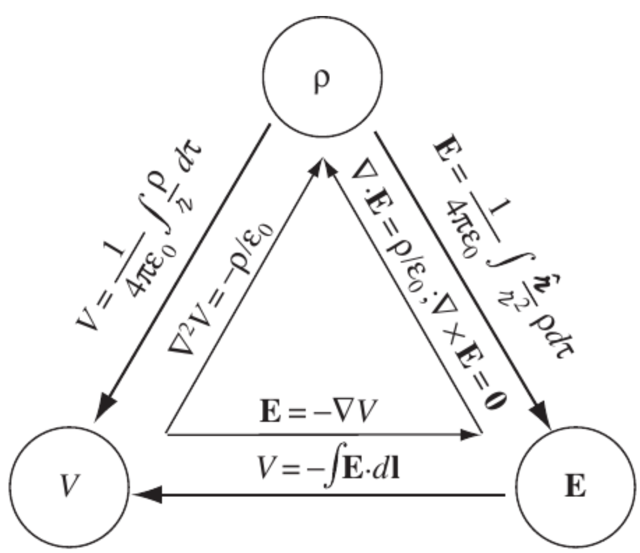
In this tutorial, we will consider an infinitely long cylinder, with radius s_0 and charge distribution

$$\rho = As\theta\,(s_0 - s) + \sigma\delta(s - s_0), \tag{1}$$

where $\theta(x)$ is the Heaviside step function and $\delta(x)$ is the delta function. The step function is defined according to

$$\theta(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases} \tag{2}$$

The θ function is quite useful: it lets you write down functions that turn on and off at specific places. In our case, there is a specific charge density inside the cylinder, and in regions outside the cylinder (i.e. where $s > s_0$) this contribution turns off.



We will work out everything we know about such a charge distribution, recalling the triangle diagram from Griffiths. Your job is to follow every arrow in this diagram: we'll go from the charge distribution to the **E** field and back again, from the **E** field to the potential and back, and from the charge distribution to the potential and back again.

Part 1

First, we establish some preliminary properties of the field.

1. Draw the charge distribution given in Equation 1.

2. Draw the cylinder, and sketch the **E** field everywhere. In what direction does it point? Why?

3. Sketch a graph of the magnitude of the **E** field as a function of the radial coordinate s. Is the **E** field continuous?

4. Sketch a graph of the potential V as a function of s. Is the function $V(s)$ continuous?

Part 2

Next, we move on to following each direction in the Griffiths triangle. We'll start by going around the triangle in a clockwise direction.

1. Determine the **E** field from the charge distribution. Notice that there are two easy ways to do this: using a direct integration approach, or using Gauss' law. Which is easier? Why?

2. Determine the potential V by starting with the **E** field.

3. Determine the charge distribution by starting with the potential. Do you get the same charge distribution that you started with?

Part 3

Now go around the triangle in the opposite direction.

1. Determine the potential V by starting with the charge distribution. Do you get the same V as you found in Part 2?

2. Determine the **E** field by starting with the potential V. Do you find the same thing as you did in Part 2?

3. Determine the charge distribution by starting with the **E** field. Do you get back what you started with?

Part 4

We can study a few more aspects of the cylindrical charge distribution. Recall that we had two expressions for the energy stored in a collection of charges. One is

$$W = \frac{1}{2}\int d\tau \rho V, \tag{3}$$

and the other is

$$W = \frac{\epsilon_0}{2}\int d\tau E^2. \tag{4}$$

To remind yourself of how to apply these formulas, try the following:

1. Calculate the energy density per unit length of the cylinder from the ρV formula.

2. Calculate the energy density per unit length of the cylinder from the E^2 formula. Does this agree or disagree with the solution from 1)? Why or why not?

   Spoiler: From these two exercises, you will see that the E^2 formula and the ρV formula need not always agree. In fact, they are calculating slightly different things! Explain in words what each of these formulas calculates, and when they will return the same result. Include an example where the two formulas would agree.