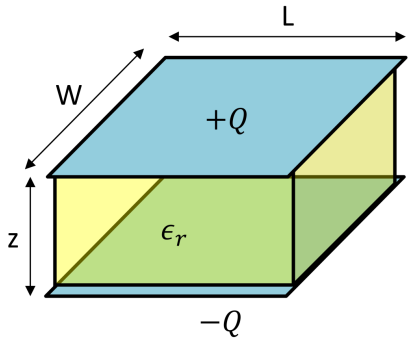
PHYS 301 - Tutorial 6

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

Problem 1 – Parallel plate capacitor

A parallel plate capacitor with surface area LW and charge $\pm Q$ is filled with an insulator that has a relative dielectric constant ϵ_r. The plates are separated by a distance z.



a) Find the **E** and **D** fields between the plates, as well as the bound charge density σ_b. Hint: derive an expression for **D** in terms of the free charge Q and use it to determine **E**. Then, express **E** in terms of the total charge density (free and bound) and solve for σ_b.

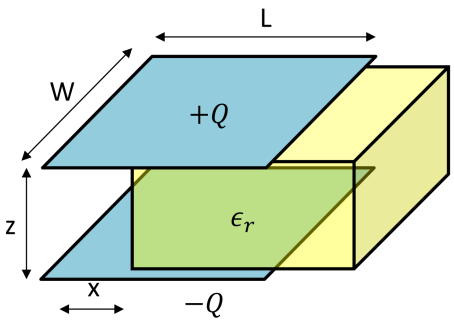
b) Calculate the electric potential difference ΔV between the capacitor plates.

c) Find the capacitance.

d) Find the stored energy of the system.

e) Which of b), c), and d) will increase as ϵ_r increases?

Now, we will begin to pull the dielectric material out from between the two plates.



f) Find the capacitance of the configuration when the dielectric has been pulled a distance x. (Hint: you can treat the two sections as separate capacitors connected in parallel.)

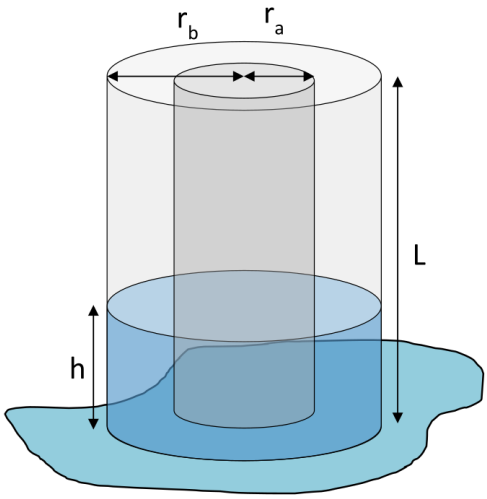
g) Find the stored energy.

h) Is there a force on the dielectric? (Hint: yes.) Explain why. Calculate the magnitude of the force. Which way does it point – i.e. is the dielectric pulled back inside the capacitor or pushed out?

i) What can you conclude in general about how dielectrics behave when exposed to electric fields?

Part 2 – Cylindrical capacitor

Consider a cylindrical capacitor in a dielectric liquid bath. The inner cylinder is held at voltage V while the outer one is grounded. The dielectric liquid has susceptibility χ_e and mass density ρ. Assume $\epsilon_r = 1$ in air.



a) First, find the capacitance. To save time, remember that in a previous tutorial we showed that the electric potential due to a single uniformly charged cylinder of radius s_0 is given by

$$V(s) = \begin{cases} 0 & , \quad s \leq s_0 \\ -\dfrac{\sigma s_0}{\epsilon_0} \ln\left(\dfrac{s}{s_0}\right) & , \quad s > s_0 \end{cases} \tag{1}$$

b) Next, calculate the stored energy. Does it increase or decrease with the height of the liquid?

c) Given what you concluded about dielectrics and capacitors in the previous problem, calculate the force raising the liquid in the cylinder.

d) Finally, determine the height of the column in equilibrium with gravity. How much work was needed to raise it to that level? What provided the work?