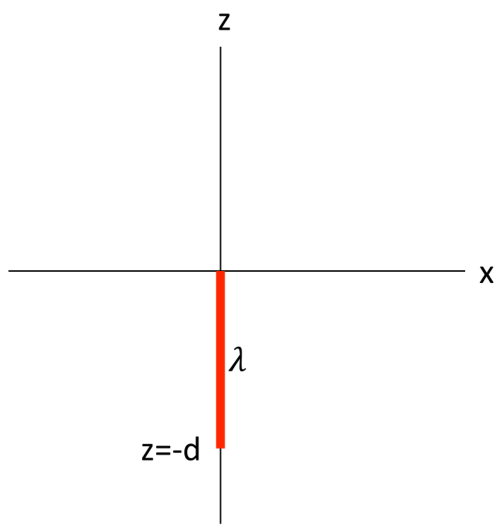
PHYS 301 - Tutorial 7

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

Part 1 – Potential from a line of charge

A uniform line charge density λ extends from the origin to the point $(0, 0, -d)$.



a) Find an expression for the electric potential along the positive z axis, $V(z)$. Recall:

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d\tau' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}. \tag{1}$$

Your answer should be $V(z) = \frac{\lambda}{4\pi\epsilon_0} \ln\left(1 + \frac{d}{z}\right)$. (If not, find out where you went wrong.) At large z, i.e. $z \gg d$, the potential can be approximated by the first few terms of an expansion in powers of $\frac{d}{z}$.

b) Expand the potential $V(z)$ into a Taylor series at the origin. Find the first two non-zero terms. The Taylor expansion of a given function $f(x)$ about the origin is

$$f(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + ... = \sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} x^n. \tag{2}$$

In spherical coordinates, the general solution to the Laplace equation has the form

$$V(r,\theta) = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}} \right) P_\ell(\cos\theta), \tag{3}$$

where $P_\ell(\cos\theta)$ are the Legendre polynomials:

$$\begin{aligned} P_0(\cos\theta) &= 1 \\ P_1(\cos\theta) &= \cos\theta \\ P_2(\cos\theta) &= \frac{1}{2}\left(3\cos^2\theta - 1\right) \\ P_3(\cos\theta) &= \frac{1}{2}\left(5\cos^3\theta - 3\cos\theta\right) \end{aligned} \tag{4}$$

This problem does not have a spherical symmetry. However, we can still use the general solution to the Laplace equation in all regions where $\nabla^2 V = 0$.

c) Describe all regions where the Laplace equation holds.

d) Assuming the potential can be expressed in the above form in this region, find the two leading non-zero A's and B's. (Hint: you already found the answer along the positive z axis, so your answers must match when you set $\theta = 0$.)

Any potential can be expanded into the form

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{\text{monopole}}{r} + \frac{\text{dipole}}{r^2} + \frac{\text{quadrupole}}{r^3} + \ldots \right). \tag{5}$$

e) What are the monopole and dipole moments of this system?

f) How would you expect the monopole and dipole moments to change if the charge distribution were shifted up by $\frac{d}{2}$, so that it was centered at the origin? Check to see if you get what you expect.