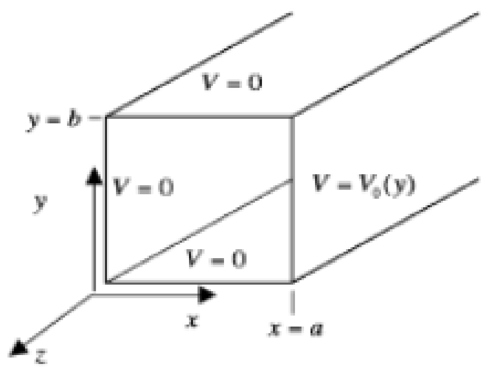
PHYS 301 - Tutorial 8

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

Problem 1 – Laplace's equation and separation of variables

Within a very long, rectangular, hollow pipe, there are no electric charges. The walls of this pipe are kept at a known voltage. Three of the walls are grounded: $V(x=0,y,z)=0; V(x,y=0,z)=0; V(x,y=b,z)=0$. The fourth wall maintains a potential that varies with y: $V(x=a,y,z)=V_0(y)$, which will be specified later.



In order to find the voltage inside the pipe, you will need to solve Laplace's equation:

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0. \tag{1}$$

a) What does it mean to "separate variables" for $V(x,y,z)$? Can you use that approach here? If so, write down the general form that V must have.

b) Plug this general form of V into Laplace's equation. After doing so, you should have several terms. Simplify them as much as possible.

- Are any of the terms zero?
- What must be true about the remaining terms in order to satisfy Laplace's equation?
- Write down the ordinary differential equations you need to solve to find V.

c) Solve the differential equations generally. Do **not** apply the boundary conditions yet. Is there any ambiguity? Could this ambiguity be resolved by considering the boundary conditions? Now, we'll be more specific about the voltage on the fourth wall (the only wall not grounded).

What does each boundary condition below tell you? (You may want to write down the new function $V(x, y, z)$ after applying each boundary condition.)

$$\begin{aligned} V(x, y = 0, z) &= 0 \\ V(x, y = b, z) &= 0 \\ V(x = 0, y, z) &= 0 \\ V(x = a, y, z) &= V_0 \sin \frac{17\pi y}{b} \end{aligned} \tag{2}$$

Use these boundary conditions to find the voltage everywhere inside the pipe.

d) Explicitly check that your answer for $V(x, y, z)$ satisfies all boundary conditions. If your solution satisfies the boundary conditions, could there be another (or more than one) solution for $V(x, y, z)$ that would also work?

Problem 2 – Fourier's trick

a) After applying the boundary conditions of the three grounded walls, what did your answer look like? Write it down in the form of a sum, i.e.

$$V(x, y, z) = \sum_n C_n(...) \tag{3}$$

b) Suppose the fourth wall now maintains a constant voltage: $V(x = a, y, z) = V_0$. What is the new voltage everywhere inside the pipe? To find it,

1. Evaluate the equation above at the $x = a$ boundary.
2. What do you multiply both sides by? (Fourier's trick)
3. What are the limits of integration?
4. Perform the integration. What happens to the infinite sum after you perform the integration?
5. Now find all the C_n's.
6. Write down $V(x, y, z)$.