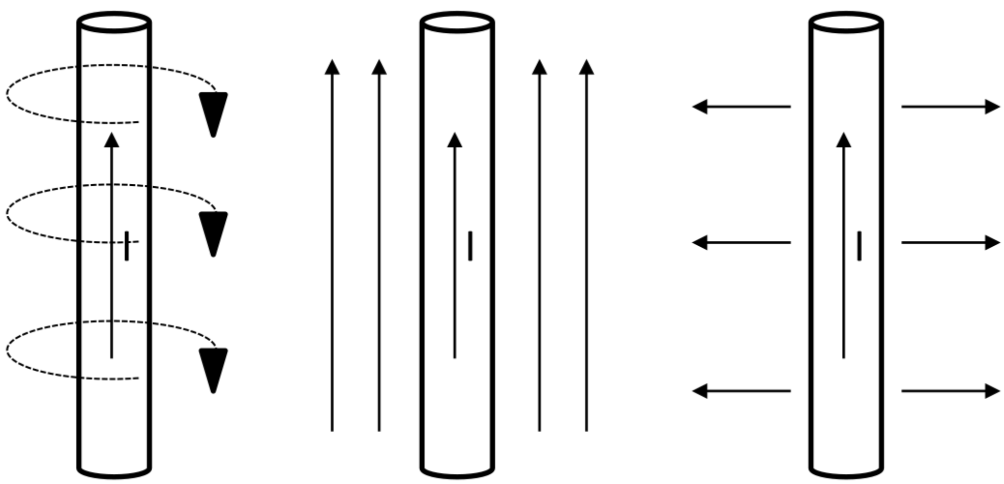
PHYS 301 - Tutorial 9

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

Problem 1 – Magnetic field of a current-carrying wire

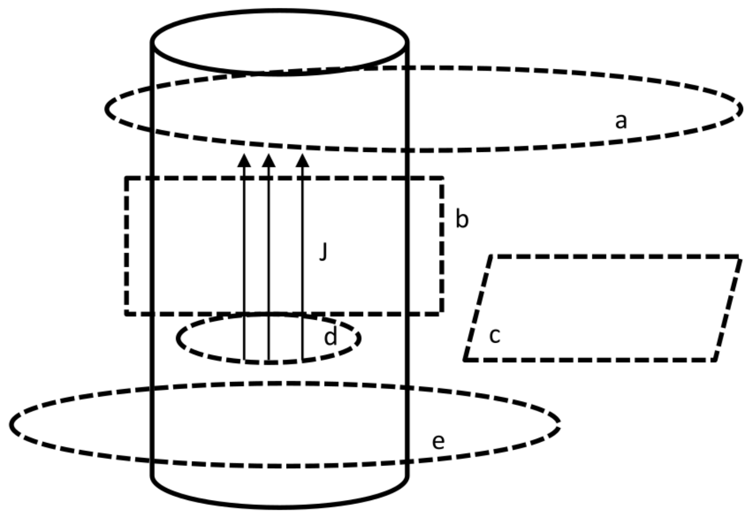
A thin wire carries a uniform current I. This current produces a magnetic field $\mathbf{B}$. Up until now, you've always been told that magnetic fields loop around a current-carrying wire, but how do you know that there are no other components of the magnetic field? E.g., perhaps the magnetic field has a z-component or a radial component.



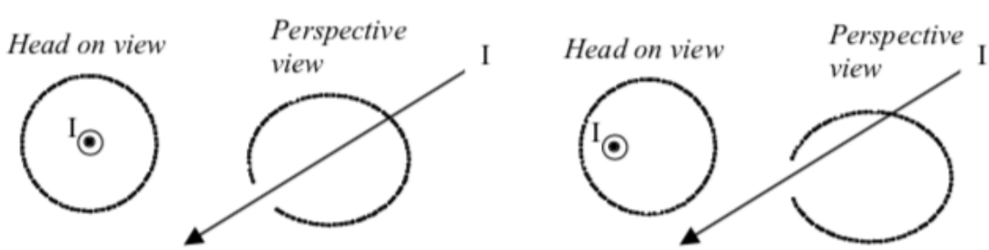
a) Can you think of any convincing arguments for why there can't be any z or s component to the magnetic field? It might be useful to consider symmetry, Maxwell's equations, and any laws that have recently been covered in class.

b) Write down Ampere's Law in integral form.

c) When using Ampere's Law, you must choose an Amperian loop. Consider a fat cylindrical wire with a known current density $\mathbf{J} = J\hat{z}$. For each of the various loops shown in the figure, decide what information, if any, Ampere's law applied to each loop might provide about $\mathbf{B}$. Is Ampere's law invalid for any of them? Can you think of any *other* Amperian loops that might be useful in this situation?

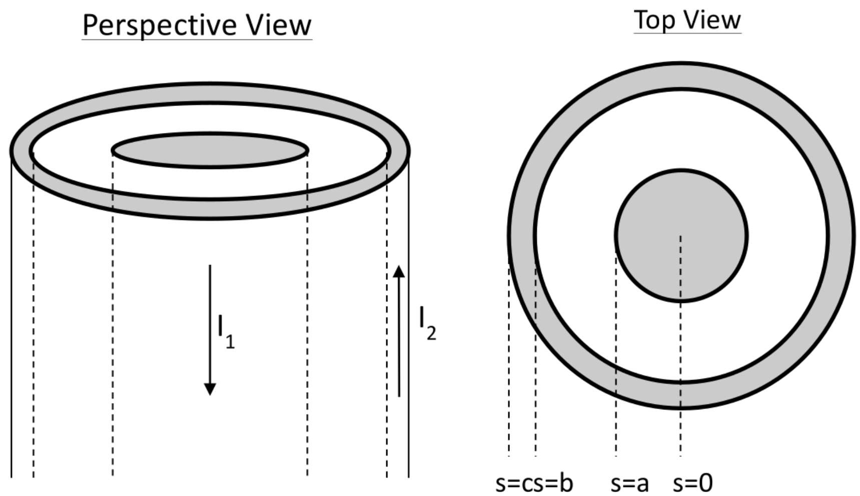


d) Compare $\oint \mathbf{B} \cdot d\mathbf{l}$ for two Amperian loops shown below. Which has a larger magnitude?



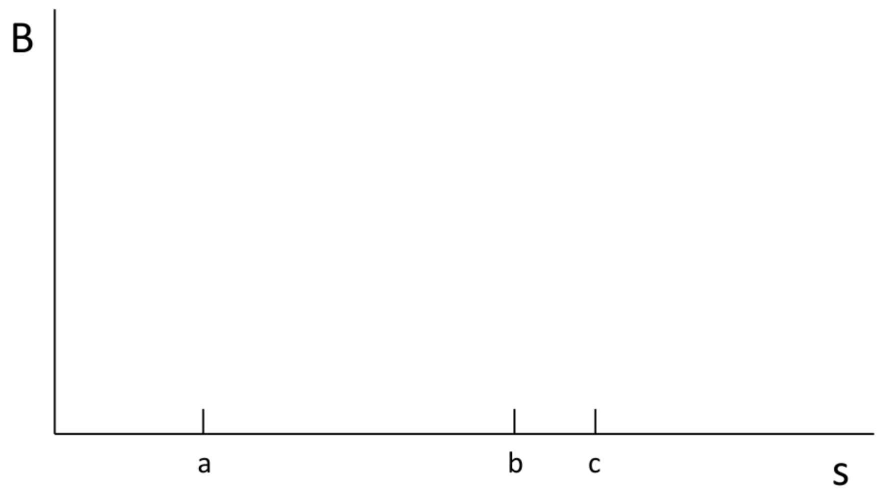
Part 2 Application of Ampere's Law

While studying intensely for your physics final, you decide to take a break and listen to your stereo. As you unwind to a little Hootie and the Blowfish, your thoughts drift to newspaper stories about the dangers of household magnetic fields on the body. You examine your stereo wires and find that most of them are coaxial cable: essentially one thin conducting cylinder surrounded by a conducting cylindrical shell (the shell has some finite thickness). Current travels up the inside conductor, and back down the conducting shell. As a way to practice for your physics final you decide to calculate the magnetic field at different radii.



Before you start calculating, think about what answers you expect.

a) Sketch a qualitative graph of $|\mathbf{B}|$ in four regions: $s < a,\ a < s < b,\ b < s < c,$ and $c < s$.



b) Is there a magnetic field inside either of the two conductors?

c) How should I_1 and I_2 compare so that there is no magnetic field outside the coaxial cable?

d) Ohm's law for a wire says that Resistance $\propto \frac{\text{Length}}{\text{Area}}$. Does this dependence imply anything about the current density J throughout the body of a conducting wire? (i.e. is all the current concentrated right at the centre of the wire? Does it only flow on the outer edges? Does it spread out uniformly across the cross-sectional area? Is it proportional to the radius? etc.)

e) Using your model for J from the previous part, calculate $\mathbf{B}$ in all four regions. Does your sketch from part a) resemble your answers?

f) If $I = 1$ Amp (not an unreasonable number), and the inner conductor is about 1 cm in diameter, what is the maximum possible value of $\mathbf{B}$ anywhere in this problem? What is it at your location in the room? Compare your answer with the Earth's magnetic field (~ 0.0005 T). What do you think about the newspaper's concerns?