PHYS 301 - Tutorial 10

- Complete all problems and submit ONE set of answers as a group (max 4 students)
- One group member uploads a pdf to gradescope and specifies the other group members there.
- Upload is due by the end of the day of the tutorial. We will accept late submission till Thursday 11:59 pm, after which the submission will be closed.

Problem 1 – Sketching the Vector Potential

Consider the equation $\nabla \times \mathbf{E} = 0$ (one of Maxwell's equations). This means that the electric field is conservative, meaning it can be written as the gradient of some scalar field. This allows us to define the scalar potential V where $\mathbf{E} = \nabla V$. Similarly, another one of Maxwell's equations allows us to define the vector potential, $\mathbf{A}$.

a) Which of Maxwell's equations does $\mathbf{A}$ come from? How does it lead to $\mathbf{A}$?

b) What current density $\mathbf{J}$ would create the $\mathbf{B}$ field in Figure 2 below? Can you write an explicit mathematical formula for it?

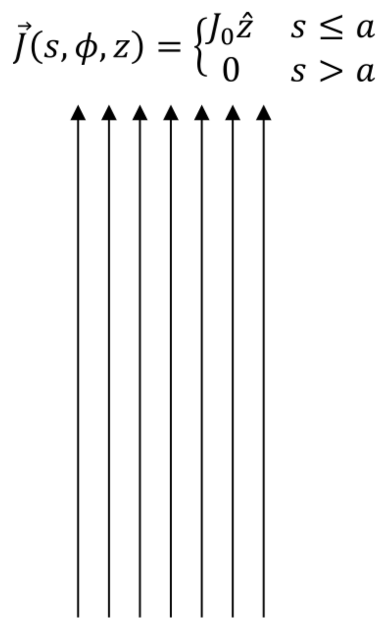

Figure 1

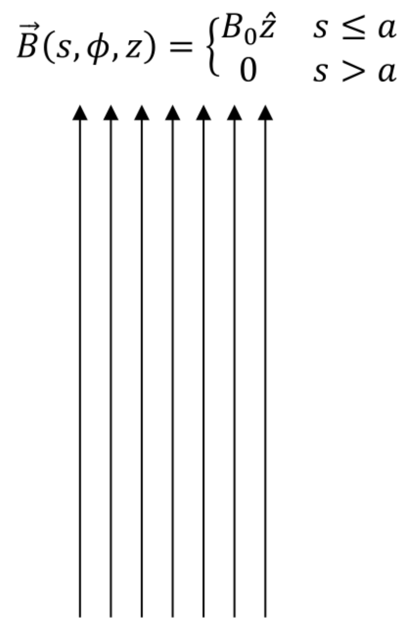

Figure 2

c) Notice that the equations defining $\mathbf{A}$ are mathematically analogous to Maxwell's equations for $\mathbf{B}$:

$$\begin{aligned} \nabla \cdot \mathbf{B} = 0 \;&\Leftrightarrow\; \nabla \cdot \mathbf{A} = 0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \;&\Leftrightarrow\; \nabla \times \mathbf{A} = \mathbf{B} \end{aligned} \tag{1}$$

First, sketch the field **B** in Figure 1. Then, using the mathematical similarities above, sketch **A** in Figure 2. Also make a sketch representing their magnitude as a function of radius: $B(s)$ and $A(s)$.

d) One way to conceptually check your answer to part c) is using an Ampere's law analogy. Ampere's law tells you that the J-flux (I_{enc}) is equal to $\oint \mathbf{B} \cdot d\boldsymbol{\ell}$. What is the analogous relationship between the vector potential and magnetic field? Try using this "Ampere's law analogy" to check your sketch of **A**.

Part 2 – Calculating the Vector Potential

Consider an infinite conducting sheet located in the xy plane ($z = 0$), with a uniform current density K_0 flowing in the positive x direction.

a) Make a sketch of the current density.

b) Write a mathematical expression for the current density **J**. Hint: since the current density is located on a 2D surface, you will need to use a delta function.

c) Your answer should be $\mathbf{J} = K_0\delta(z)\hat{x}$. Show that this current density produces the magnetic field:

$$B = \begin{cases} -\frac{\mu_0 K_0}{2}\hat{y}, & z > 0 \\ +\frac{\mu_0 K_0}{2}\hat{y}, & z < 0\ . \end{cases} \tag{2}$$

(Griffiths ex. 5.8)

d) Sketch your best guess of what **A** looks like for the uniform surface current. Which components (x, y, or z) does **A** have (it might be helpful to look at the relationship between **A**, **B**, and **J** in the examples in Part 1)? Which variables (x, y, or z) does **A** depend on?

e) Using your assumptions for which components **A** has and which variables it depends on, calculate **A**. Does your sketch of **A** resemble the answer you calculated?