
EECE488: Analog CMOS Integrated Circuit Design

Set 3

Differential Amplifiers

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Overview

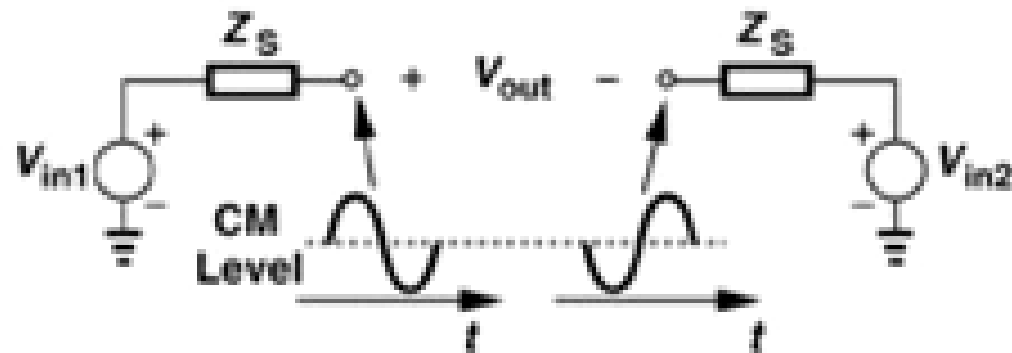
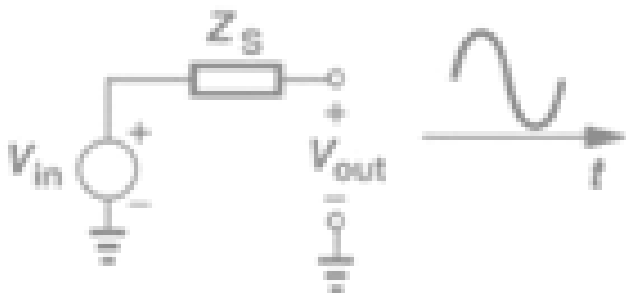
- Reading: Chapter 4 of the Textbook most important circuit inventions.
- Their invention dates back to vacuum tube era (1930s).
- Alan Dower Blumlein (a British Electronics Engineer, 1903-1942) is regarded as the inventor of the vacuum-tube version of differential pair.



- Differential operation offers many useful properties and is widely used in analog and mixed-signal integrated circuits

Single-ended and Differential Signals

- A “single-ended” signal is a signal that is measured with respect to a fixed potential (typically ground).
- “Differential signal” is generally referred to a signal that is measured as a difference between two nodes that have equal but opposite-phase signal excursions around a fixed potential (the fixed potential is called common-mode (CM) level).

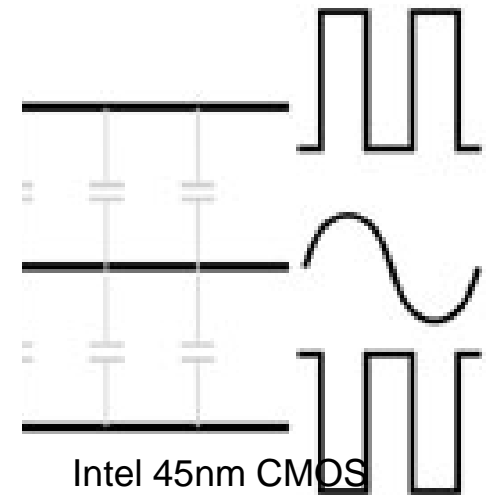
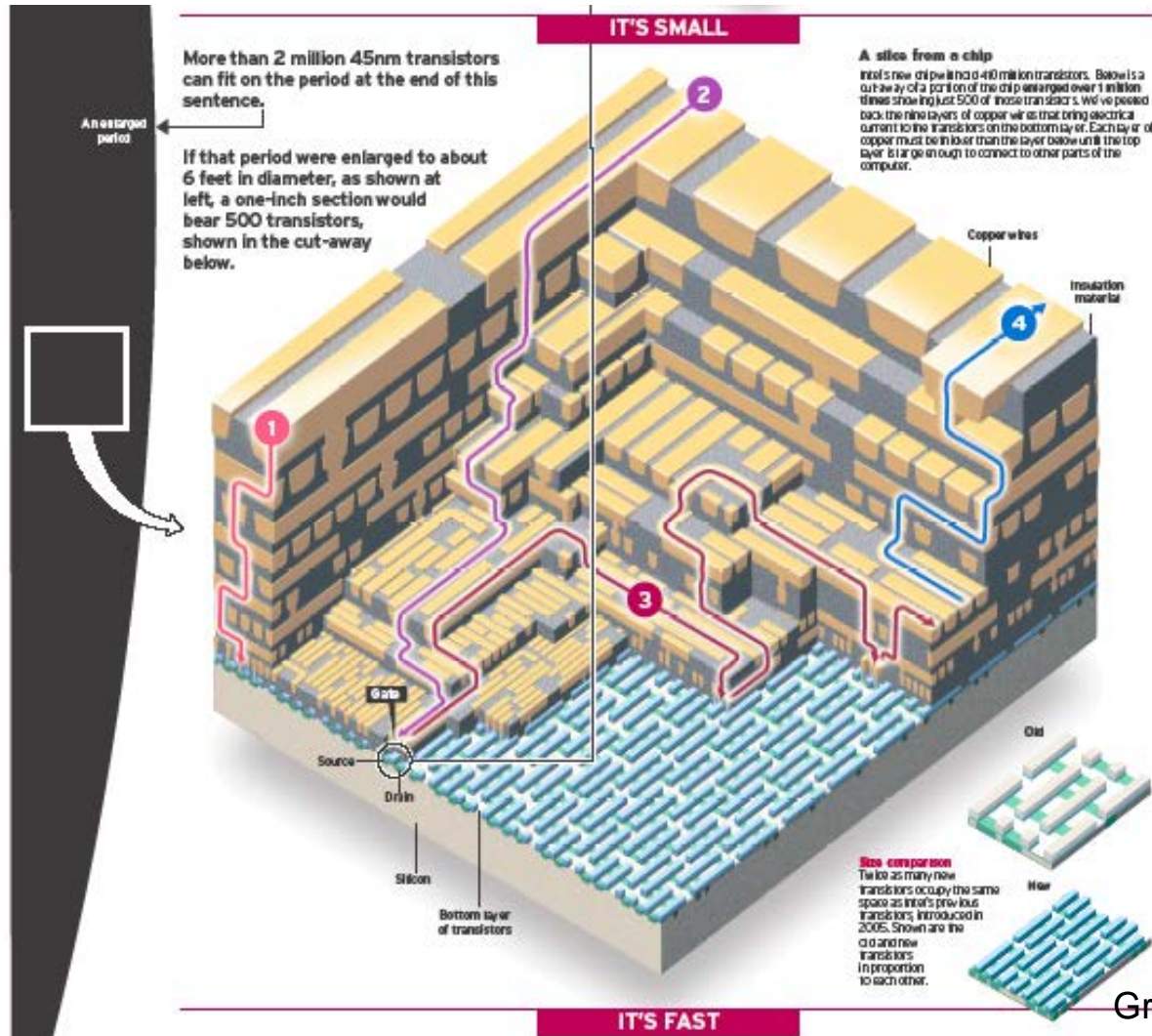


Board Notes (Differential Amplifiers)

Why Differential?

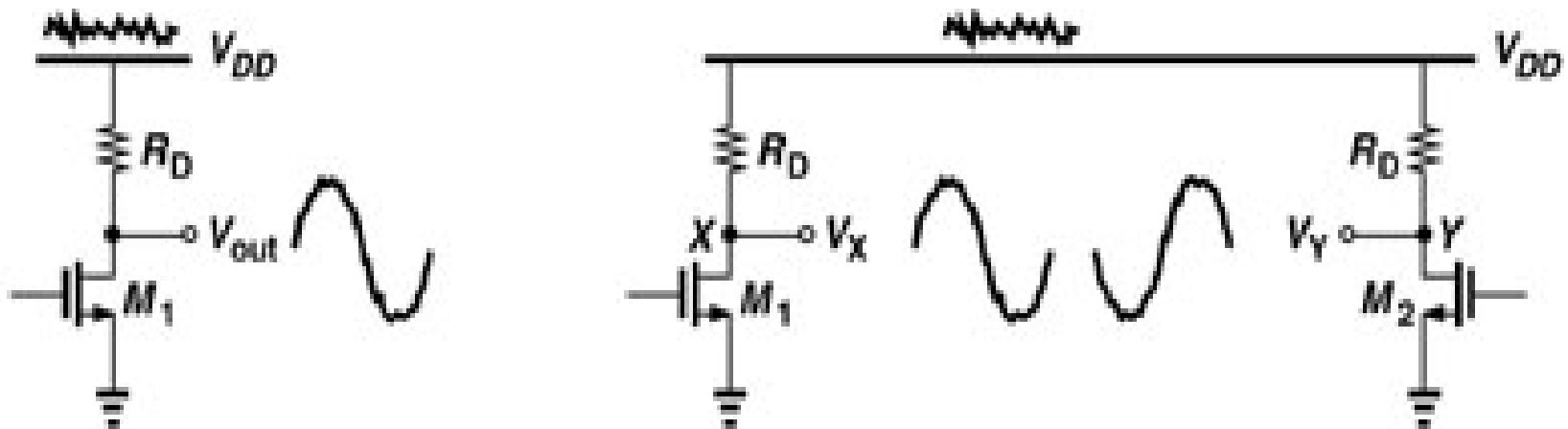
- Better immunity to environmental noise
- Improved linearity
- Higher signal swing compared to single-ended

Higher Immunity to Noise Coupling



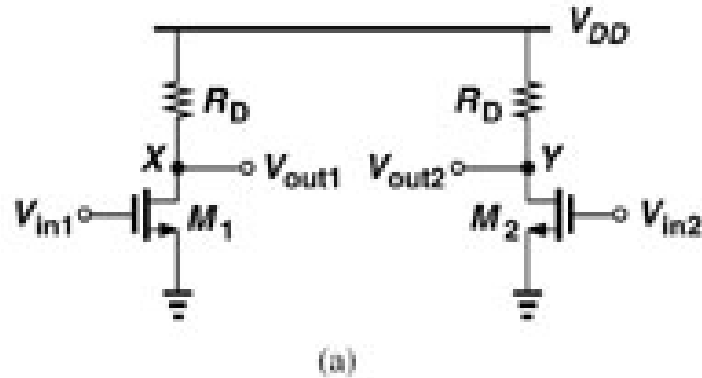
Graphics: Steve Cowden, Oregonian, 2007

Supply Noise Reduction

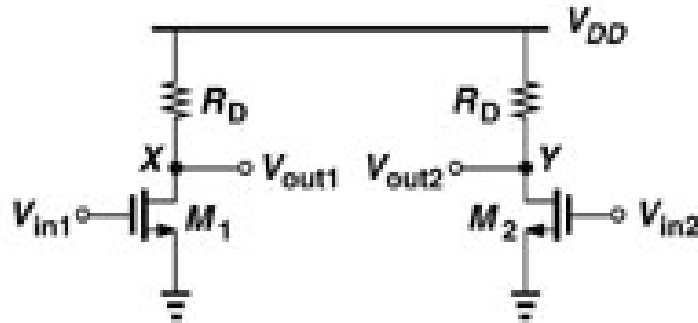


Board Notes (Improved Linearity)

Basic Differential Pair

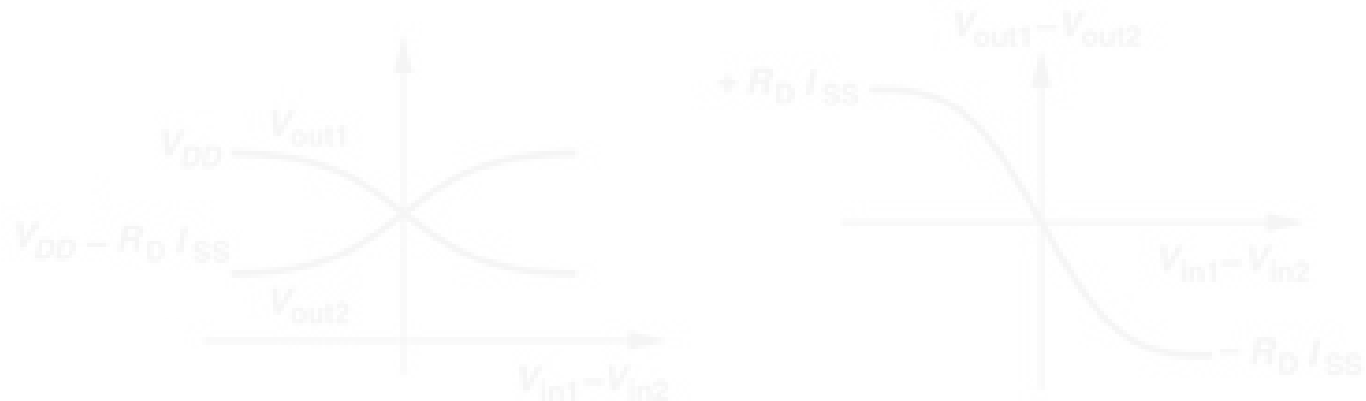
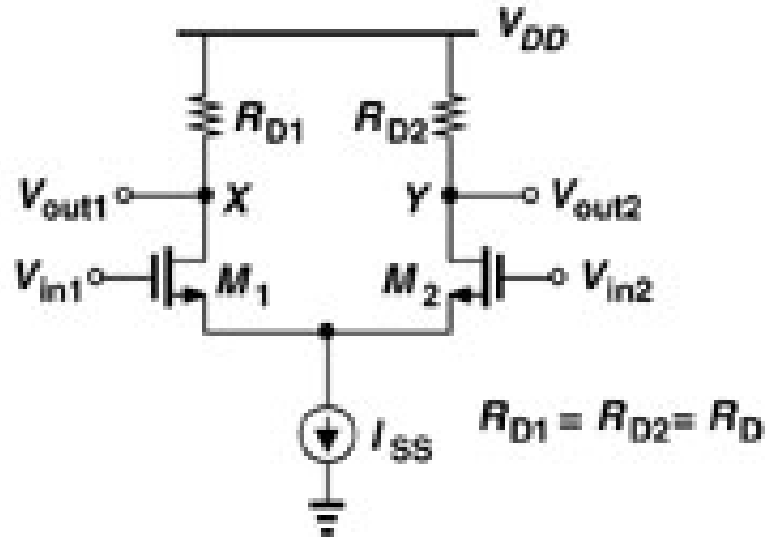


Basic Differential Pair

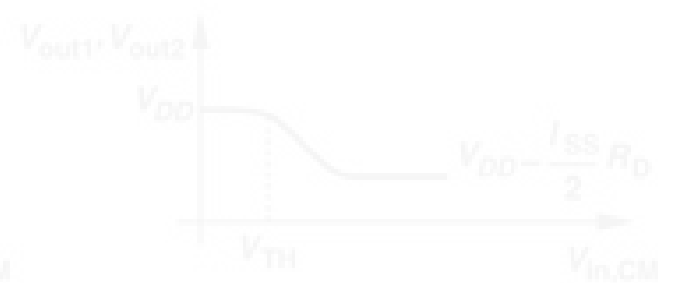
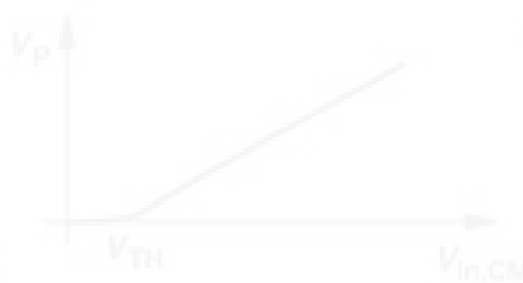
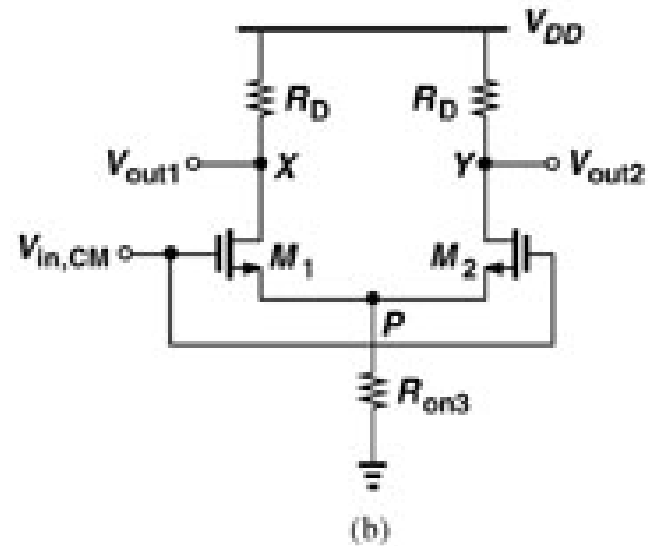
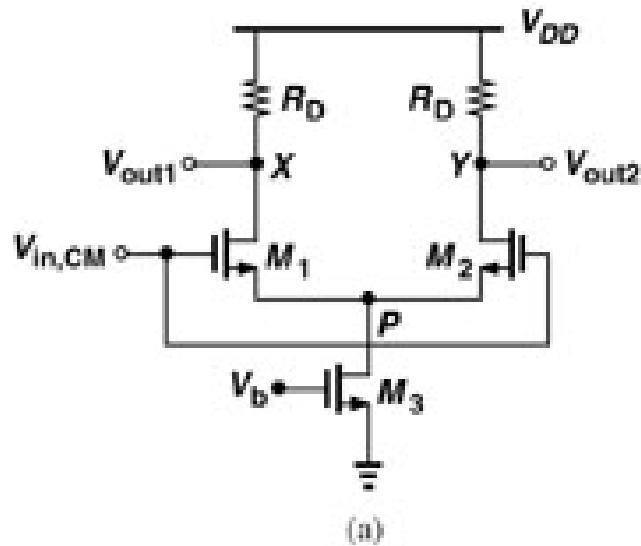


- Problem: Sensitive to input common-mode (CM) level
 - Bias currents of the transistors M_1 and M_2 changes as the input CM level changes
 - g_m of the devices as well as output CM level change
- Can we think of a solution?

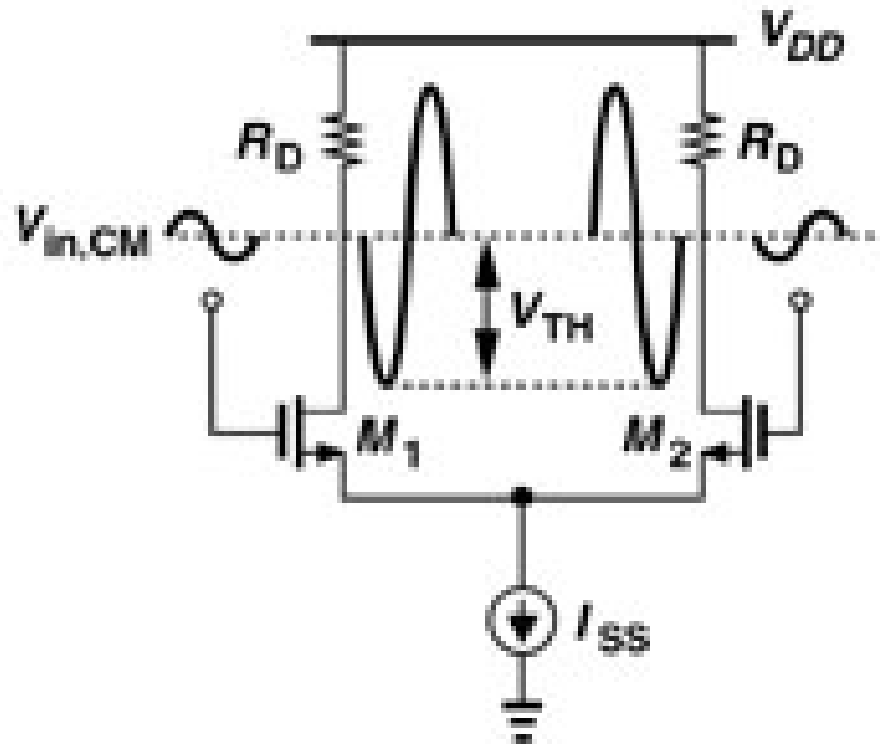
Differential Pair



Common-Mode Response



Common-Mode Input versus Output Swing

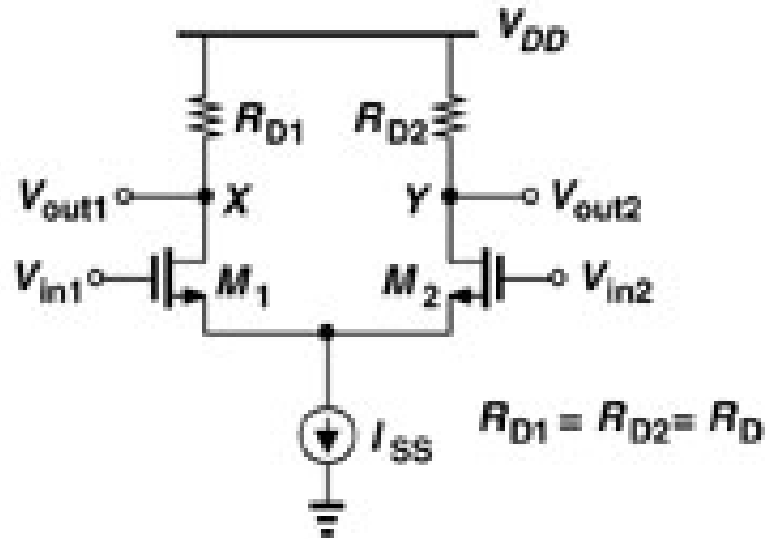


Board Notes (“Half-Circuit” Concept)

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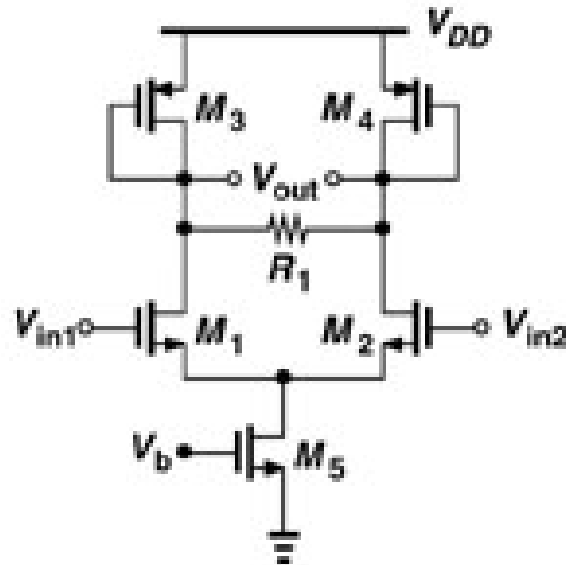
Example

- Using the half-circuit concept, calculate the small-signal differential gain of the following circuit (for two cases of $\lambda=0$ and $\lambda \neq 0$).



Example

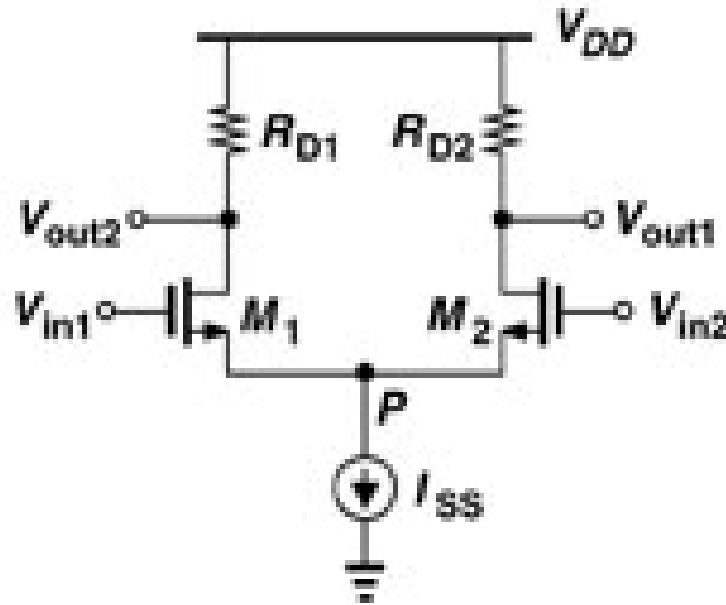
- Using the half-circuit concept, calculate the small-signal differential gain of the following circuit (for two cases of $\lambda=0$ and $\lambda \neq 0$).



Example

- Sketch the small-signal gain of a differential pair as a function of its input common-mode level.

Analysis of Differential Amplifier



$$V_{in1} - V_{in2} = V_{GS1} - V_{GS2}, \quad V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH}$$

Analysis of Differential Amplifier

$$V_{in1} - V_{in2} = \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$(V_{in1} - V_{in2})^2 = \frac{2}{\mu_n C_{ox} \frac{W}{L}} (I_{D1} + I_{D2} - 2\sqrt{I_{D1}I_{D2}})$$

$$\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 - I_{SS} = -2\sqrt{I_{D1}I_{D2}}$$

$$\frac{1}{4} \left(\mu_n C_{ox} \frac{W}{L} \right)^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2 = 4I_{D1}I_{D2}$$

Analysis of Differential Amplifier

Using:

$$4I_{D1}I_{D2} = (I_{D1} + I_{D2})^2 - (I_{D1} - I_{D2})^2 = I_{SS}^2 - (I_{D1} - I_{D2})^2$$

and

$$4I_{D1}I_{D2} = \frac{1}{4}(\mu_n C_{ox} \frac{W}{L})^2 (V_{in1} - V_{in2})^4 + I_{SS}^2 - I_{SS} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2})^2$$

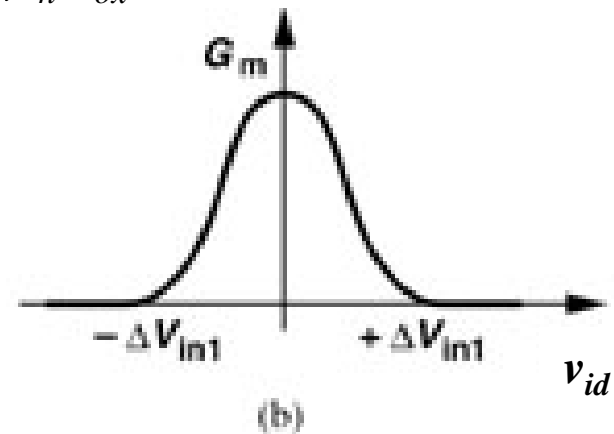
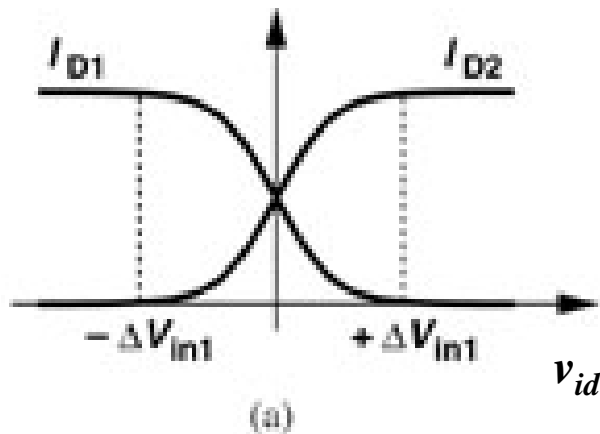
We have:

$$I_{D1} - I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

Analysis of Differential Amplifier

$$i_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} v_{id} \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - v_{id}^2}$$

$$\frac{\partial i_d}{\partial v_{id}} = G_m = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\frac{4I_{SS}}{\mu_n C_{ox} W / L} - v_{id}^2}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} W / L} - v_{id}^2}}$$



Analysis of Differential Amplifier

- For small v_{id} :

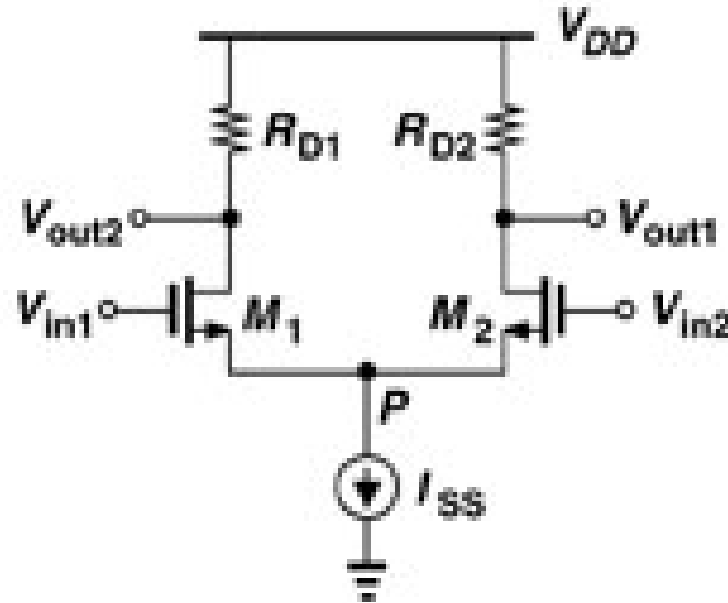
$$G_m = \frac{\partial i_d}{\partial v_{id}} = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} = g_{m1} = g_{m1}$$

- We have:

$$V_{out1} - V_{out2} = R_D (I_{D1} - I_{D2}) = R_D G_m (V_{in1} - V_{in2})$$

$$\frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = g_{m1} R_D$$

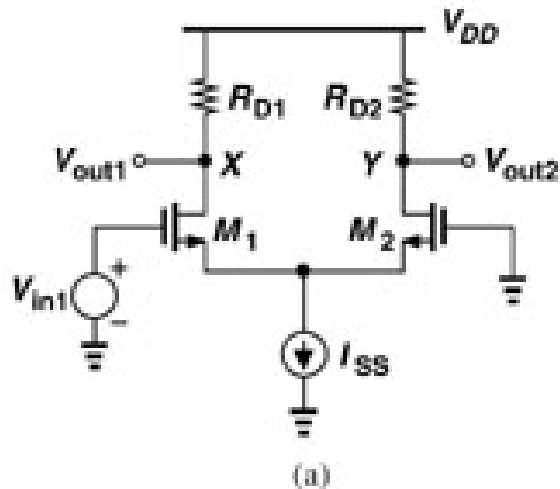
Differential Gain



$$A_v(\text{diff}) = \frac{V_{out1} - V_{out2}}{V_{in1} - V_{in2}} = g_m R_D$$

$$A_v(\text{Single-Ended}) = \frac{g_m}{2} R_D$$

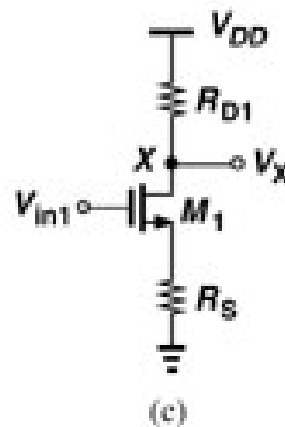
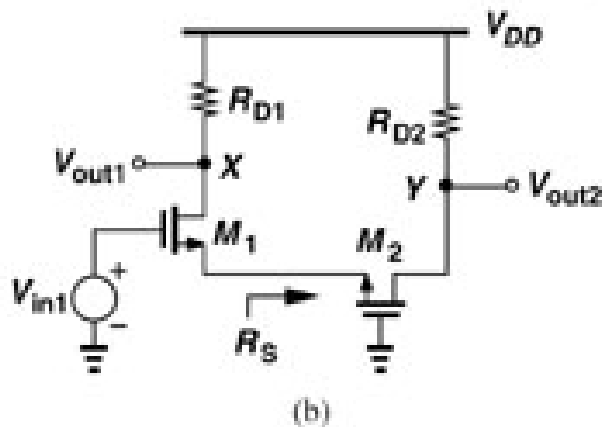
Differential Pair as a CS and CD-CG Amplifier



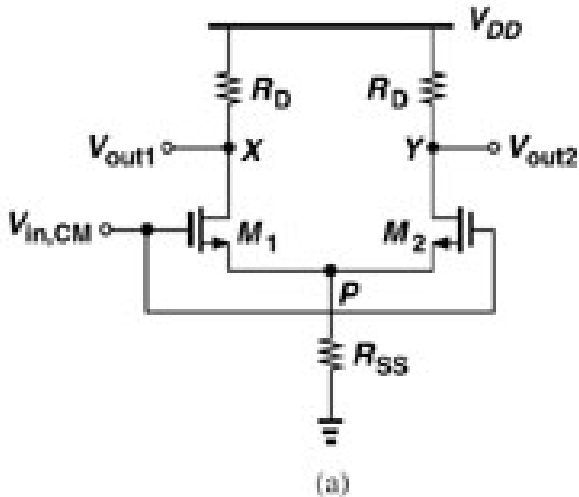
$$A_v = -\frac{g_m R_D}{1 + g_m R_S}$$

$$= -\frac{g_m}{2} R_D, \quad (S.E.)$$

$$= g_m R_D, \quad (diff)$$

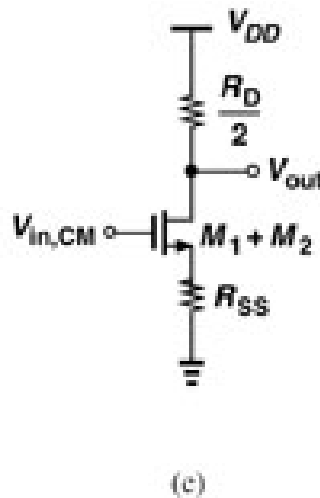
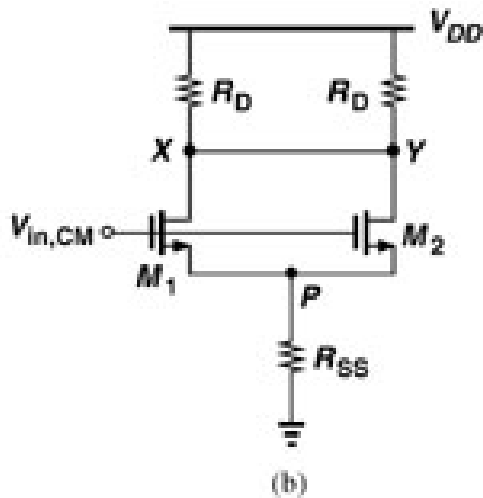


Common-Mode Response



$$A_v = \frac{V_{out}}{V_{in,CM}}$$

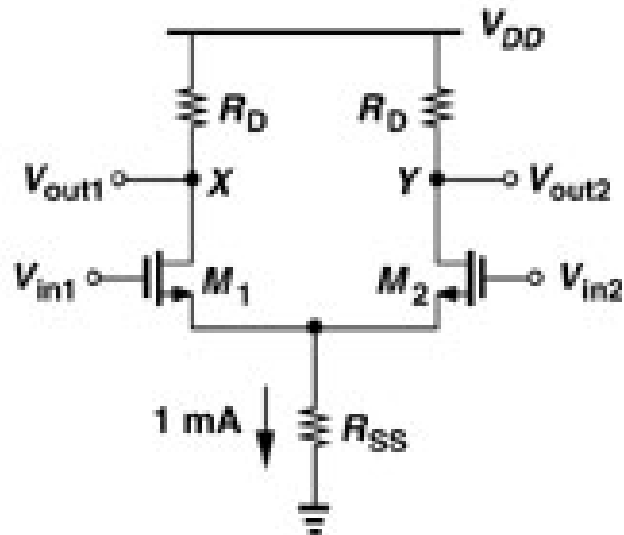
$$= -\frac{R_D / 2}{1/(2g_m) + R_{SS}}$$



Board Notes

Example

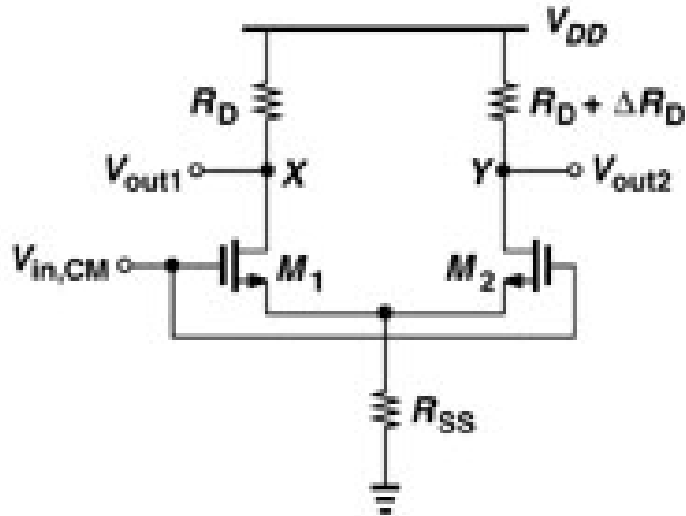
- In the following circuit assume that $R_{SS}=500\Omega$ and $W/L=25/0.5$, $\mu_n C_{ox}=50\mu A/V^2$, $V_{TH}=0.6$, $\lambda=\gamma=0$ and $V_{DD}=3V$.



- What is the required input CM for which R_{SS} sustains $0.5V$?
- Calculate R_D for a differential gain of $5V/V$.
- What happens at the output if the input CM level is 50mV higher than the value calculated in part (a)?

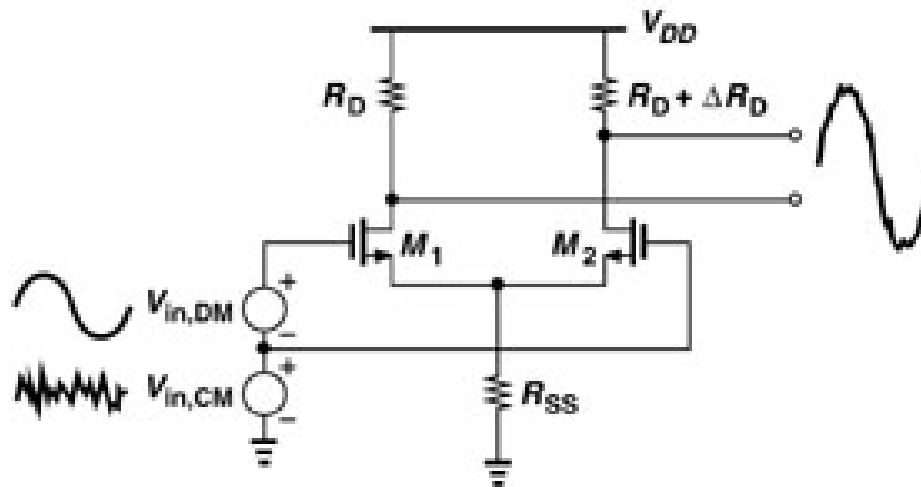
Board Notes

Common-Mode Response

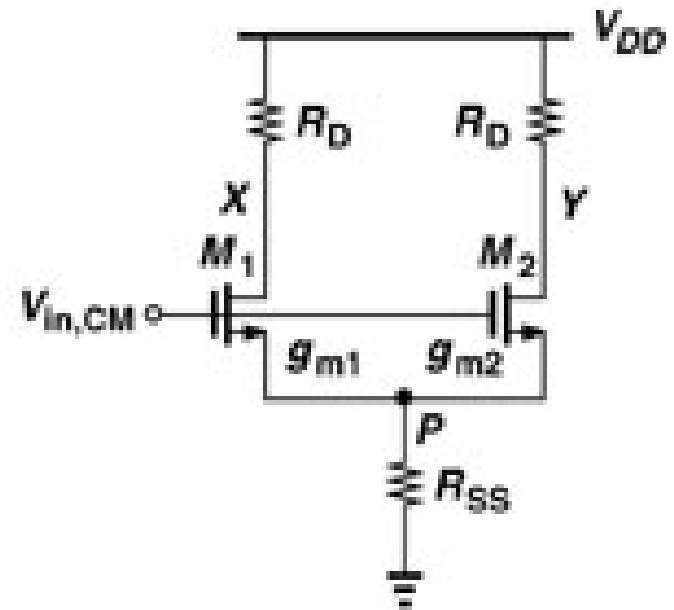
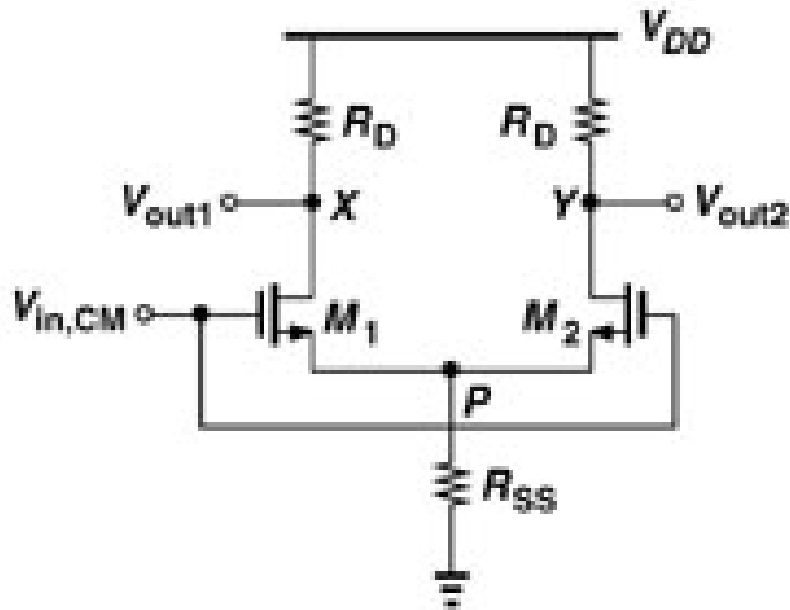


$$\frac{\Delta V_X}{\Delta V_{in,CM}} = -\frac{g_m}{1 + 2g_m R_{SS}} R_D$$

$$\frac{\Delta V_Y}{\Delta V_{in,CM}} = -\frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

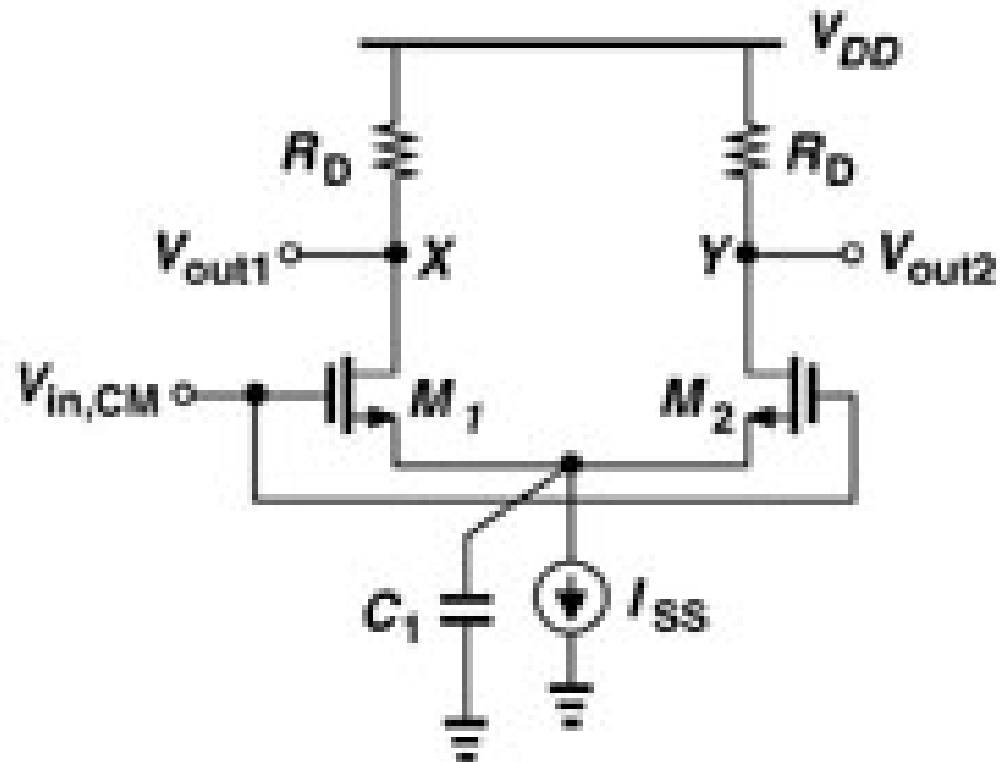


Common-Mode Response

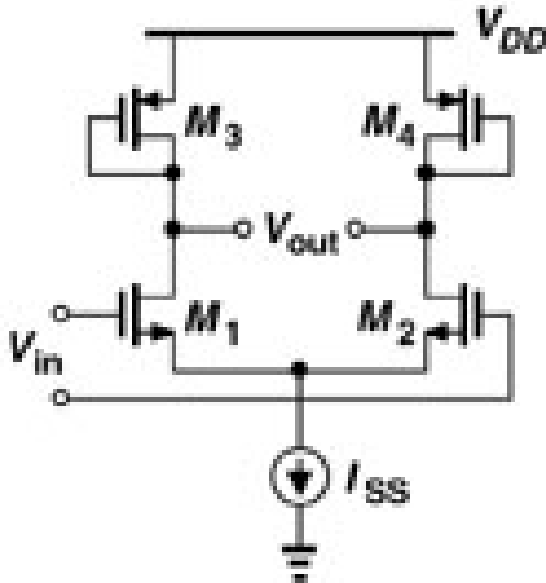


$$\frac{V_X - V_Y}{V_{in,CM}} = - \frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D$$

Common-Mode Response

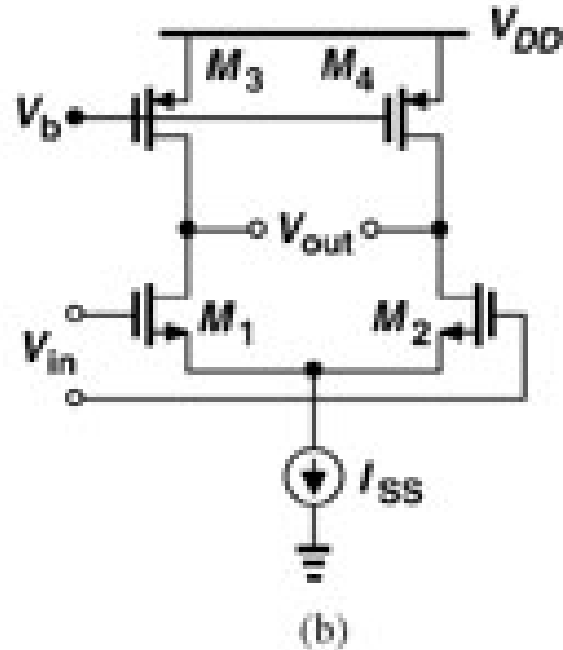


Differential Pair with MOS Loads



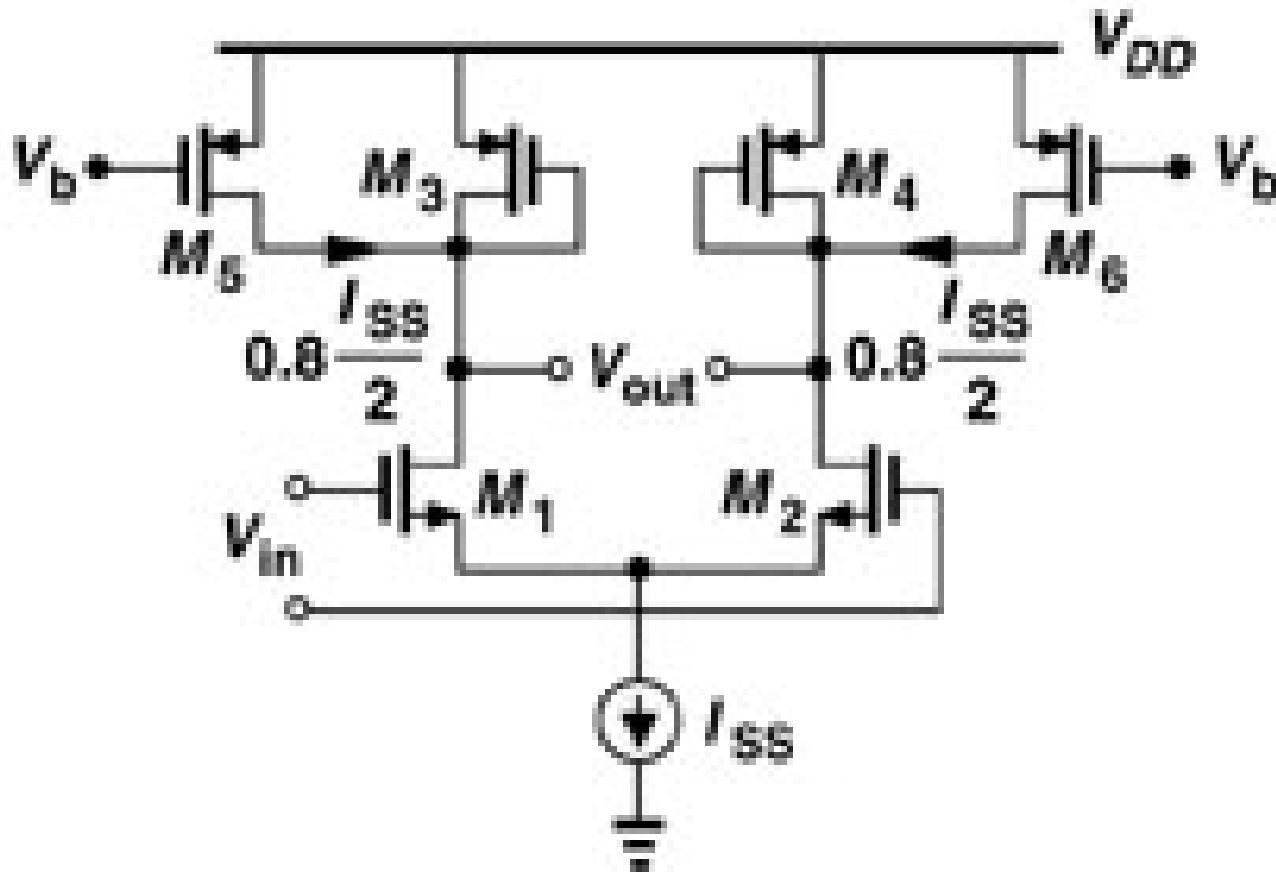
$$A_v = -g_{mN}(g_{mP}^{-1} \parallel r_{oN} \parallel r_{oP})$$

$$\approx -\frac{g_{mN}}{g_{mP}}$$

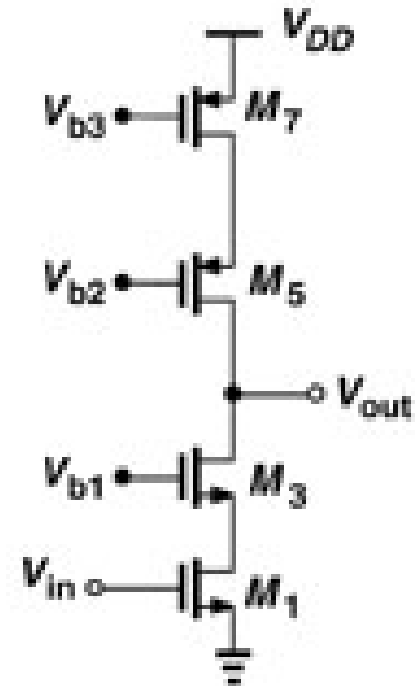
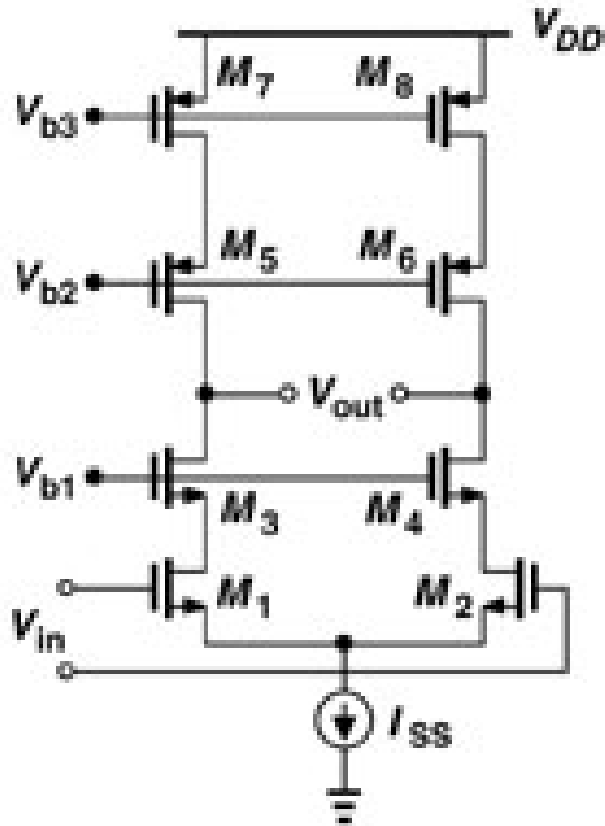


$$A_v = -g_{mN}(r_{oN} \parallel r_{oP})$$

MOS Loads

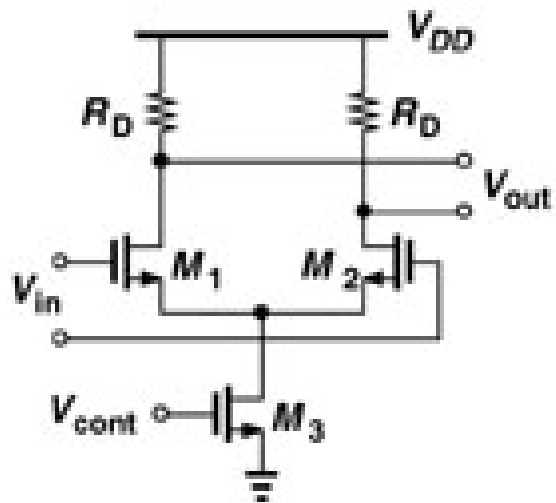


MOS Loads

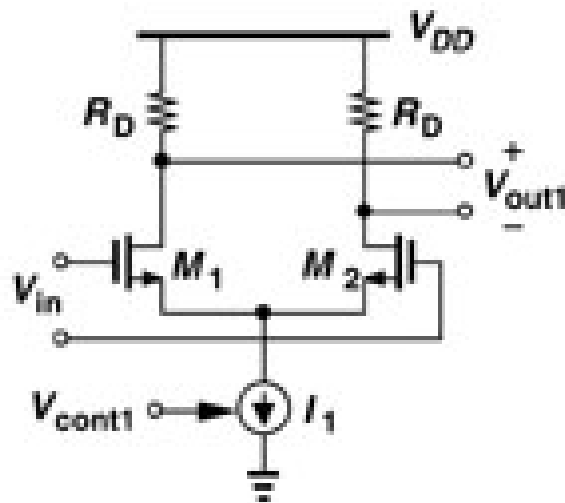


$$A_v \approx g_{m1}[(g_{m3}r_{o3}r_{o1}) \parallel (g_{m5}r_{o5}r_{o7})]$$

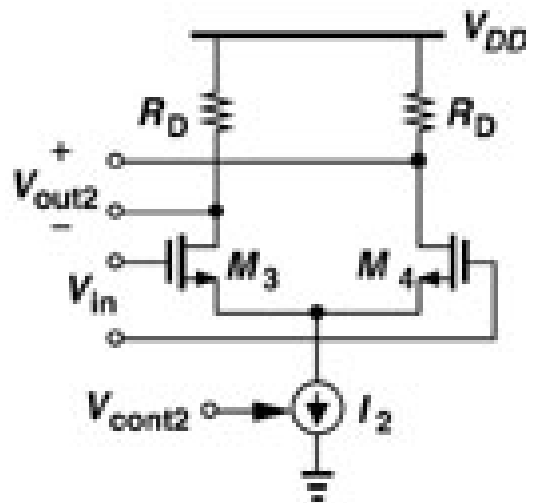
Gilbert Cell



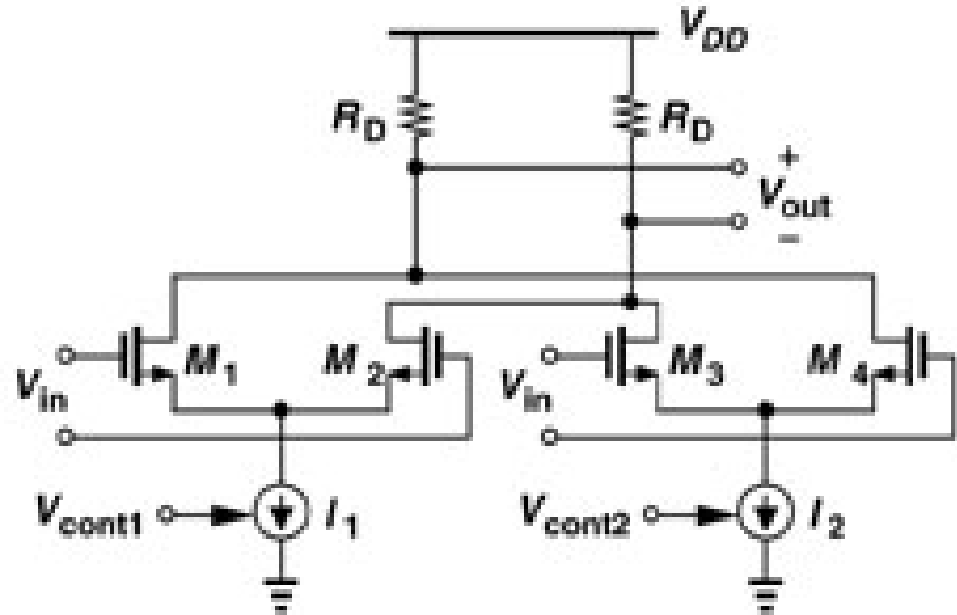
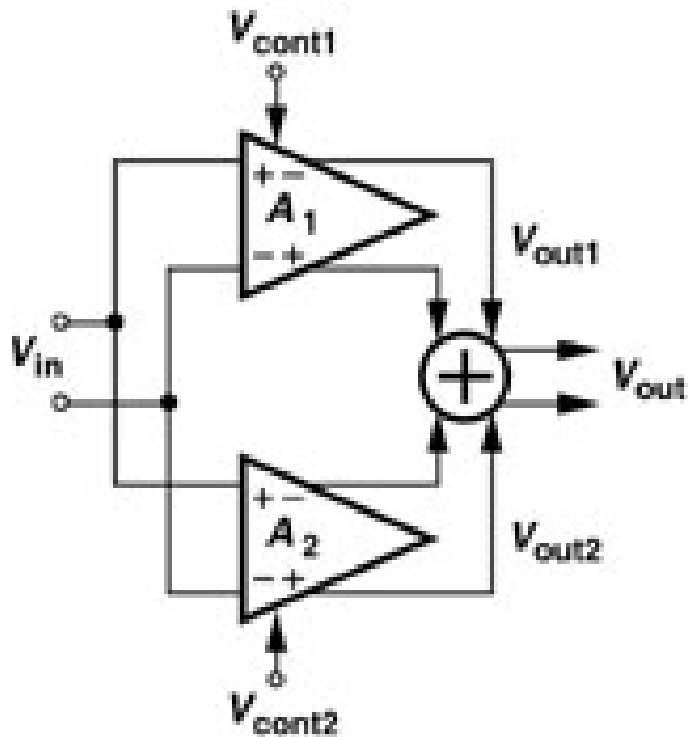
(a)



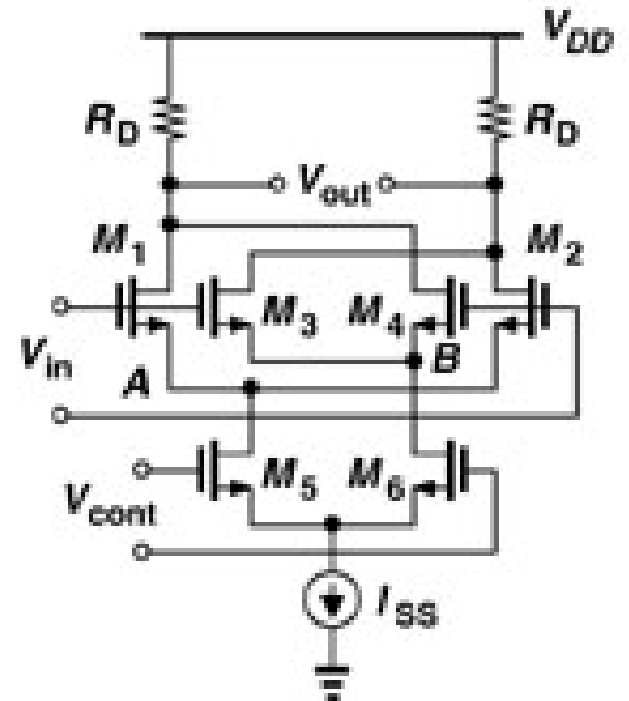
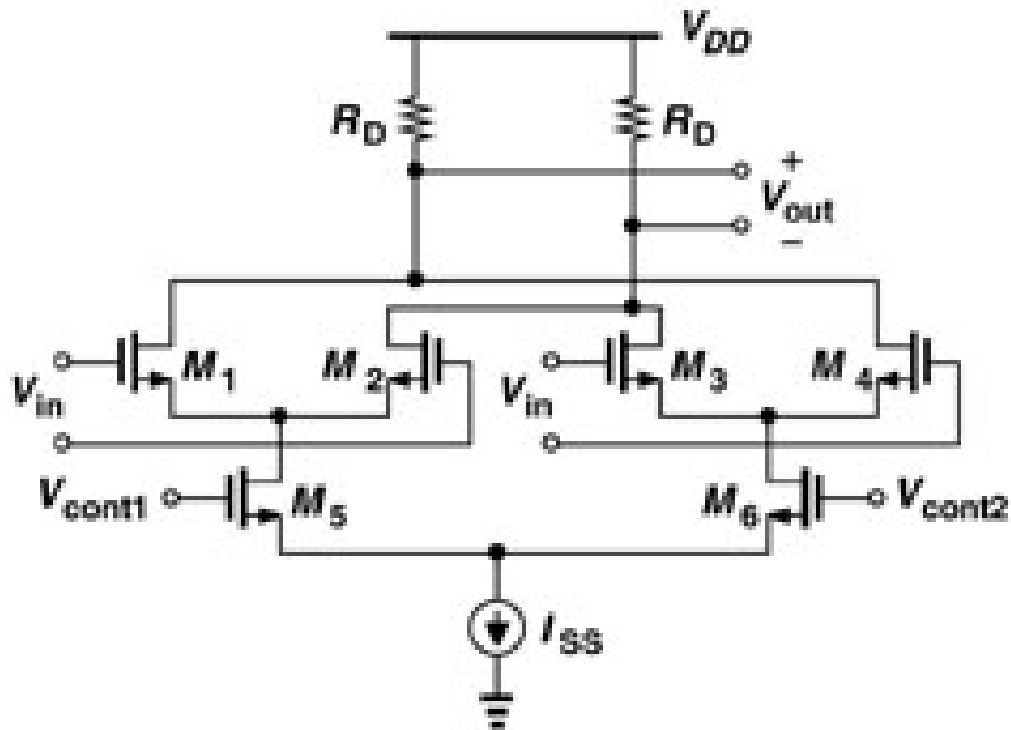
(b)



Gilbert Cell



Gilbert Cell



$$V_{OUT} = k V_{in} V_{cont}$$