

## MIDTERM #1, MATH 300

Monday, February 6, 2006

Student No: \_\_\_\_\_ Name (Print): \_\_\_\_\_

1. (9 marks) Answer true or false to the following questions by putting either true or false in the boxes. If the answer is true give a proof, and if the answer is false give a counter-example.

(a)  $\text{Log } e^z = z \ \forall$  complex numbers  $z$ .

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(b)  $\left(\sqrt{2} \cos \frac{\pi}{6} + i\sqrt{2} \sin \frac{\pi}{6}\right)^4 = -2 + 2\sqrt{3}i$ .

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(c)  $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2) \ \forall$  complex numbers  $z$ .

2. (9 marks) The following questions require little or no computation.

- (a) Let  $f(z) = u(x, y) + v(x, y)$  be an entire function. What are the Cauchy-Riemann equations?

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- (b) Express  $\text{Log}(\sqrt{3} + i)$  in the form  $a + bi$ .

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- (c) Find the principal value of  $(1 + i)^{1+i}$ .

3. (9 marks) Find all solutions of the following equations. Express your answers in the form  $a + bi$ .

(a)  $\frac{1 + z^2}{1 - z^2} = i$ .

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(b)  $z^3 + 1 = 0$ .

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(c)  $\cos z = 2i \sin z$ .

4. (3 marks) Suppose  $u(x, y)$  is harmonic  $\forall (x, y)$  and  $v(x, y)$  is a harmonic conjugate of  $u(x, y)$ . Show that  $u^2(x, y) - v^2(x, y)$  is harmonic  $\forall (x, y)$ .

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5. (6 marks) Describe the image of the rectangle  $\{z \mid 0 \leq x \leq 1, 0 \leq y \leq \pi\}$  under the mapping  $f(z) = e^z$ . Hint: plot the images of the curves  $x = \text{constant}$ ,  $y = \text{constant}$ .