

TABLE OF INTEGRALS

1. ELEMENTARY INTEGRALS

All of these follow immediately from the table of derivatives. They should be memorized.

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int cf(x) \, dx = c \int f(x) \, dx$$

$$\int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx$$

$$\int c \, dx = cx + C$$

$$\int x^r \, dx = \frac{x^{r+1}}{r+1} + C \quad (r \neq -1)$$

$$\int \frac{1}{x} \, dx = \ln |x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \frac{1}{x^2 + 1} \, dx = \arctan x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \arcsin x + C$$

$$\int \tan x \, dx = -\ln |\cos x| + C = \ln |\sec x| + C$$

$$\int \cot x \, dx = \ln |\sin x| + C = -\ln |\csc x| + C$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$

2. A SELECTION OF MORE COMPLICATED INTEGRALS

These begin with the two basic formulas, change of variables and integration by parts.

$$\int f(g(x))g'(x) dx = \int f(u) du \text{ where } u = g(x), du = g'(x)dx \text{ (change of variables)}$$

$$\int f(g(x)) dx = \int f(u) \frac{dx}{du} du \text{ where } u = g(x) \text{ (different form of the same change of variables)}$$

$$\int e^{cx} dx = \frac{1}{c}e^{cx} + C \quad (c \neq 0)$$

$$\int a^x dx = \frac{1}{\ln a}a^x + C \text{ (for } a > 0, a \neq 1)$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C, a \neq 0$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C, a > 0$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

To compute $\int \frac{1}{x^2 + bx + c} dx$ we complete the square

$$x^2 + bx + c = x^2 + bx + \frac{b^2}{4} + c - \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$$

If $c - b^2/4 > 0$, set it equal to a^2 ; if < 0 equal to $-a^2$; and if $= 0$ forget it. In any event you will arrive after the change of variables $u = x + \frac{b}{2}$ at one of the three integrals

$$\int \frac{1}{u^2 + a^2} du, \quad \int \frac{1}{u^2 - a^2} du, \quad \int \frac{1}{u^2} du$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left(x\sqrt{x^2 \pm a^2} \pm a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \right) + C$$

$\int x^n e^{cx} dx = x^n \frac{e^{cx}}{c} - \frac{n}{c} \int x^{n-1} e^{cx} dx, c \neq 0$. This is to be used repeatedly until you arrive at the case $n = 0$, which you can do easily.